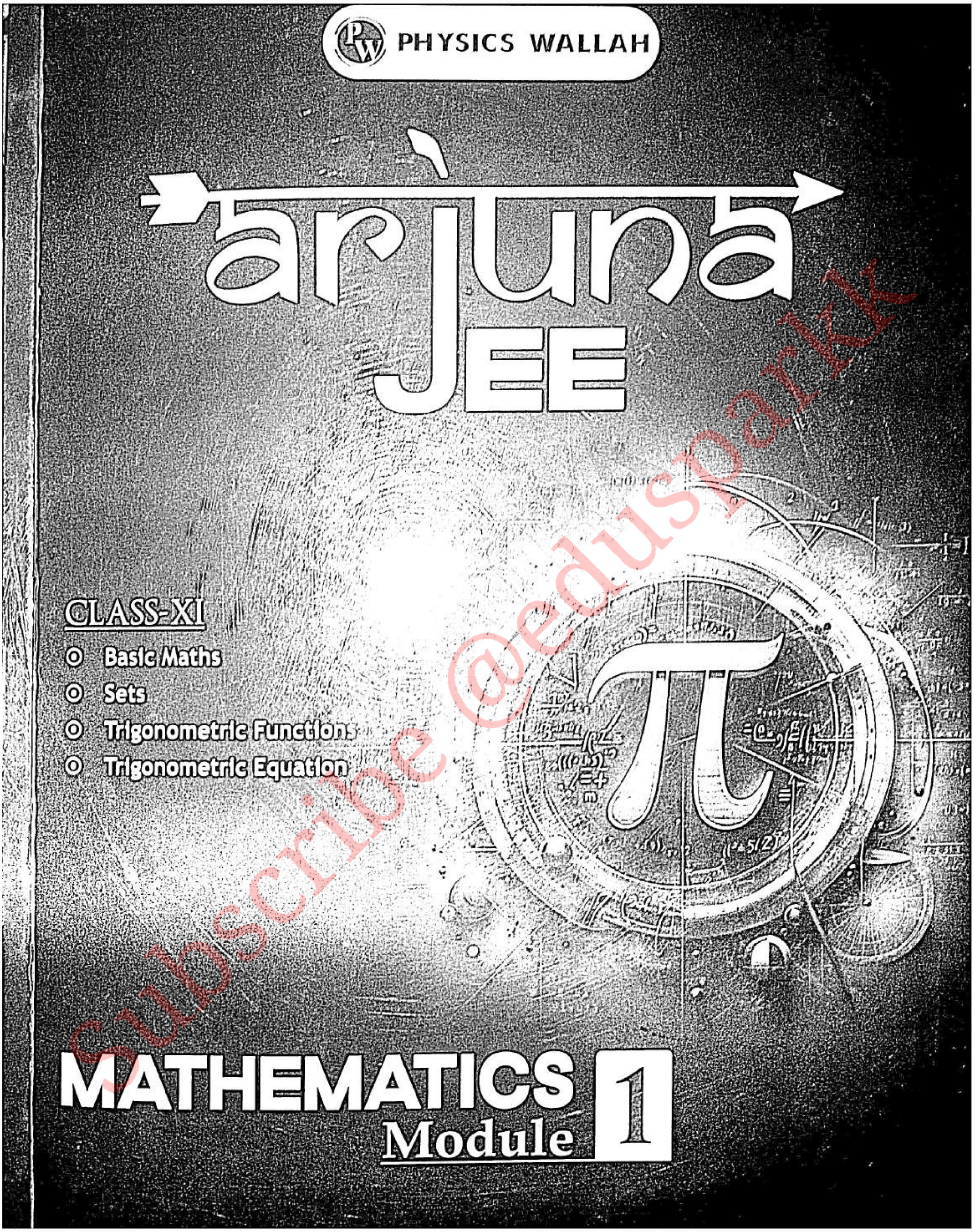


PHYSICS WALLAH

arjuna JEE

CLASS-XI

- Basic Maths
- Sets
- Trigonometric Functions
- Trigonometric Equation

MATHEMATICS

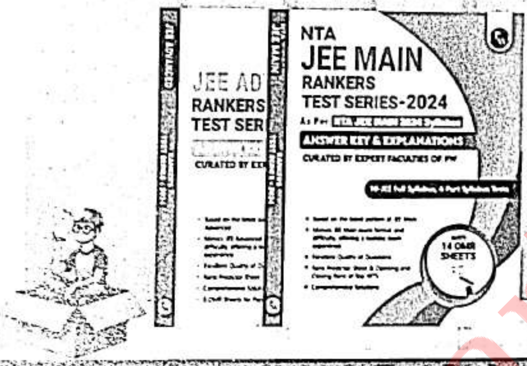
Module 1

Credit - Akshat

Make Your Study Table More Resourceful

Explore Book Insights wit
Sample PDF

Your
Preparation
Ladder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RANKERS
TEST SERIES-2024
ANSWER KEY & EXPLANATIONS
14 OMR SHEETS

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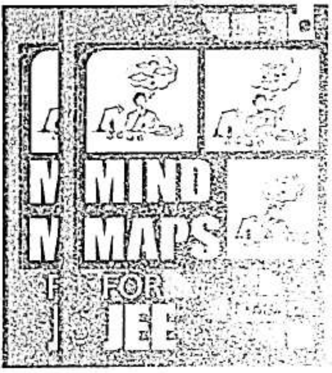
MIND
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FOR
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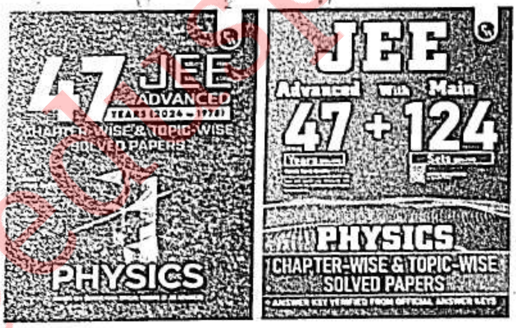
47 JEE
ADVANCED
YEARS (2024 – 1978)
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SOLVED PAPERS
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JEE
Advanced With Main
47 + 124
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CHAPTER-WISE & TOPIC-WISE
SOLVED PAPERS

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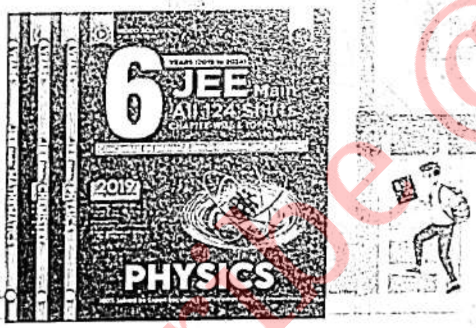
6 JEE Main
All 124 Shifts
2019
PHYSICS

Solve More JEE Main PYQs

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Module 1

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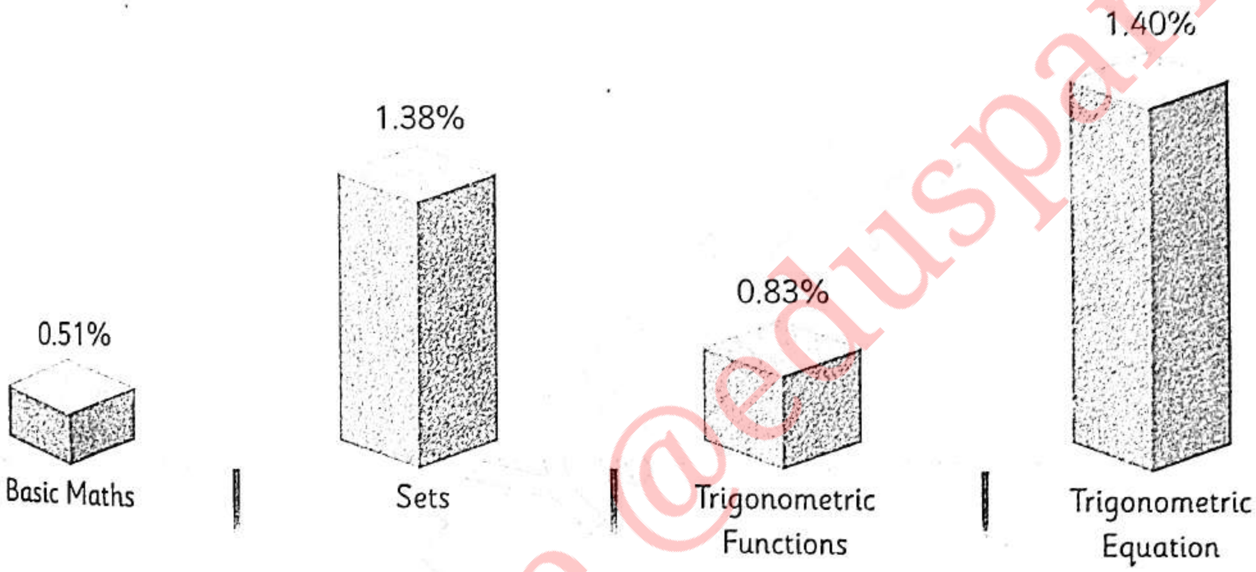

Chapter Wise Weightage (in %) Based on Past 6 Years JEE Main Papers

MATHEMATICS

Module 1

Relaxation: In this module for the emotional well being we will practice relaxation along with our regular academic learning

Credit - Akshat

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PREFACE

A highly skilled professional team of Physics Wallah (PW) works arduously to ensure that the students receive the best content for the JEE exam.

From the beginning, the whole content team comprising Associate faculties, DTP operators, Proofreaders and others are involved in shaping the material to their best knowledge and experience to produce powerful content for the students.

Faculties have adopted a new style of presenting the content in easy-to-understand language and have provided the team with their guidance and supervision throughout the creation of this Study Material.

Physics Wallah (PW) strongly believes in conceptual and fun-based learning. PW provides highly exam-oriented content to bring quality and clarity to the students.

A plethora of **JEE Study Material** is available in the market but PW professionals are continuously working to provide the supreme Study Material for our **JEE** students.

This Study Material adopts a multi-faceted approach to master and understanding the concepts by having a rich diversity of questions asked in the examination and equip the students with the knowledge for the competitive exam.

The main objective of the study material is to provide a large number of quality problems with varying cognitive levels to facilitate the teaching-learning of concepts that are presented through the book.

It has become popular among aspirants because of its easy-to-understand language and clear illustrations with diagrams, mnemonics, flow charts and tables.

Students can benefit themselves by attempting the exercise given in this Study Material for the self-assessment and also mastering the basic techniques of problem-solving.

The questions are strictly designed in accordance with the exam relevant topics that helps to develop examination temperament within the students.

Mastering the Physics Wallah (PW) study material curated by the PW team, the students can easily qualify for the exam with a Top Rank in the **JEE**.

In each chapter, for better understanding, questions have been classified according to the latest syllabus of **JEE**

- The nature and diversity of the questions help students to ace the examination
- Quality questions to strengthen the concept of the topic at the zenith level

BOOK FEATURES

- Detailed theory with Train Your Brain and Concept Application Questions
- "Aarambh" Section for Solved Miscellaneous Examples
- School Level Problems for School Going Students
- "Prarambh" Section for Topicwise, "Prabal" Section for JEE Main Level and "Parikshit" Section for Advance Level MCQs covered in exercises
- Match the Column, Comprehension Based, Numerical Type and Multicorrect questions covered for complete practice
- JEE Past Year Questions
- PW Challengers Problems for Difficult Level Questions
- Elaborated Solutions Booklet you can get separately

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3P PROGRESSOR

PLANNER

CHAPTER NAMES	Start Date of Module	End Date of Module	Solve More PYQs	Revision Period	Re-revision Period (Before Test Series)
Basic Maths					
Sets					
Trigonometric Functions					
Trigonometric Equation					

PROGRESS TRACKING

CHAPTER NAMES	Solved Examples	School Level	Topicwise	Main Level	Advanced Level	PYQs	Challengers
Basic Maths	☐	☐	☐	☐	☐	☐	☐
Sets	☐	☐	☐	☐	☐	☐	☐
Trigonometric Functions	☐	☐	☐	☐	☐	☐	☐
Trigonometric Equation	☐	☐	☐	☐	☐	☐	☐

PREP POINTERS

Note down the topic you want to revise or section you want to re attempt while your preparation.

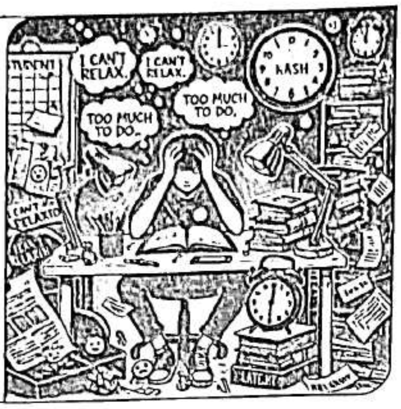
The Power of Relaxation: Turning Stress into Strength

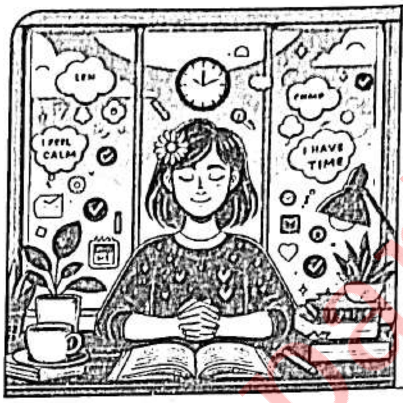
> "Almost everything will work again if you unplug it for a few minutes, including you."
> — **Anne Lamott**

Relaxation is essential for maintaining focus, mental clarity, and emotional well-being during intense exam preparation. For a JEE aspirant, incorporating relaxation techniques, such as mindful breathing, taking short breaks, or practicing meditation, helps reduce stress, boost concentration, and enhance performance. It's not just about pausing from study—it's about recharging your mind and body, so you can approach each session with renewed energy.

Without Relaxation

Without relaxation, stress can build up, clouding your mind and making it harder to stay focused and calm under pressure.


I CAN'T RELAX.
I CAN'T RELAX.
KASH
TOO MUCH TO DO.
TOO MUCH TO DO.
CAN'T RELAX



With Relaxation

Relaxation clears your mind, refreshing your energy and focus, helping you stay calm and approach challenges with clarity and confidence.

Here is a table comparing scenarios that illustrate how practicing relaxation versus not practicing relaxation can impact a student' productivity and mental state during exam preparation:

Scenario	*Without Relaxation*	*With Relaxation*
After Hours of Continuous Study	Feels mentally exhausted, struggling to absorb new information.	Takes a 5-minute break for deep breathing, returning to studies refreshed and more focused.
Dealing with Stress Before an Exam	Feels anxious, with thoughts spiraling, "What if I forget everything? I'm not ready."	Takes a moment to do mindful breathing, thinking, "I've prepared well, and I'll do my best."
Facing a Mental Block	Becomes frustrated, thinking, "I can't figure this out; I'm wasting time."	Steps away for a short walk or stretches, comes back with a clearer mind, and approaches the problem with fresh energy.
Studying Late into the Night	Stays up late, pushing through tiredness, leading to lower retention and feeling burnt out the next day.	Takes short breaks during study sessions and gets enough sleep, waking up refreshed and ready to study more effectively.
Preparing for a Tough Concept	Feels overwhelmed, keeps reading the same material without making progress.	Takes a brief relaxation exercise (e.g., deep breathing), clears the mind, and approaches the concept with better focus.
After a Bad Study Session	Feels frustrated and discouraged, thinking, "I'll never get this right."	Practices progressive muscle relaxation to release tension and resets the mood for the next session with a calm mindset.
Handling Distractions	Tries to force focus despite distractions, becoming increasingly irritated.	Takes a 2-minute mindfulness break to center themselves, refocusing on the task with greater attention and calm.
Balancing Study and Breaks	Feels guilty about taking breaks, thinking, "I can't afford to stop studying."	Recognizes the importance of breaks, thinking, "This short break will make my next study session more productive."
During Study Sessions	Feels overwhelmed and stressed out, unable to sustain long hours of study.	Incorporates relaxation techniques like meditation or breathing exercises, keeping stress in check and enhancing endurance.

This table shows how taking regular breaks and incorporating relaxation practices can make studying more effective, prevent burnout and improve overall well-being. By balancing intense study sessions with moments of calm and recharge, students can enhance thei productivity and maintain their motivation throughout their preparation journey.

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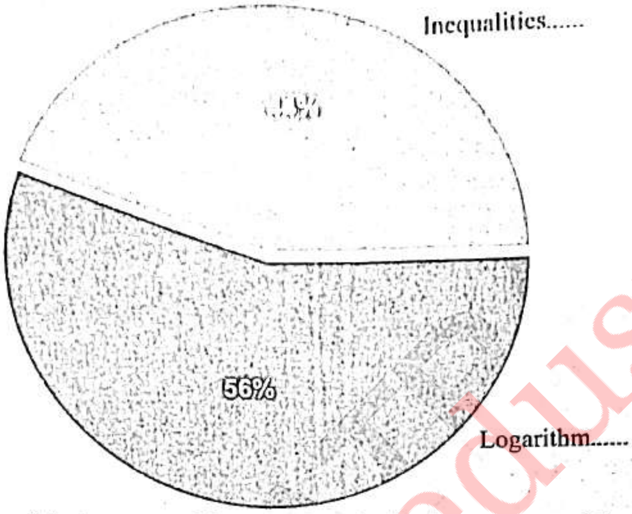
CHAPTER

1

Basic Maths

Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)



"How's the Josh?" for these Topics: *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

NUMBER SYSTEM

(*i*) **Natural numbers:** The counting numbers 1, 2, 3, 4, ... are called Natural Numbers. The set of natural numbers is denoted by N.

Thus $N = \{1, 2, 3, 4, ...\}$.

(*ii*) **Whole numbers:** Natural numbers including zero are called whole numbers. The set of whole numbers is denoted by W.

Thus $W = \{0, 1, 2, ...\}$

(*iii*) **Integers:** The numbers ... – 3, – 2, – 1, 0, 1, 2, 3 .... are called integers and the set is denoted by I or Z. Thus I (or Z) = $\{.. -3, -2, -1, 0, 1, 2, 3...\}$

Note: (*a*) Positive integers $I^+ = \{1, 2, 3....\} = N$

(*b*) Negative integers $I^- = \{....., -3, -2, -1\}$.

(*c*) Non-negative integers (whole numbers) = $\{0, 1, 2, ...\}$.

(*d*) Non-positive integers = $\{......, -3, -2, -1, 0\}$.

(*iv*) **Even integers:** Integers which are divisible by 2 are called even integers.

e.g. 0, ± 2, ± 4,.......

(*v*) **Odd integers:** Integers which are not divisible by 2 are called odd integers.

e.g. ± 1, ± 3, ± 5, ± 7......

(*vi*) **Prime numbers:** Natural numbers which are divisible by 1 and itself only are called prime numbers.

e.g. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ...

(*vii*) **Composite number:** Let 'a' be a natural number, '*a*' is said to be composite if, it has atleast three distinct factors.

e.g. 4, 6, 8, 9, 10, 12, 14, 15 ....

Note: (*a*) 1 is neither a prime number nor a composite number.

(*b*) Numbers which are not prime are composite numbers (except 1).

(*c*) '4' is the smallest composite number.

(*d*) '2' is the only even prime number.

(*viii*) **Co-prime numbers:** Two natural numbers (not necessarily prime) are called coprime, if their H.C.F (Highest common factor) is one.

e.g. (1, 2), (1, 3), (3, 4), (3, 10), (3, 8), (5, 6), (7, 8) (15, 16) etc.

These numbers are also known as **relatively prime** numbers.

Note:

(*a*) Two prime numbers are always co-prime but converse need not be true.

(*b*) Consecutive natural numbers are always co-prime numbers.

(*ix*) **Twin prime numbers:** If the difference between two prime numbers is two, then the numbers are called twin prime numbers.

e.g. {3, 5}, {5, 7}, {11, 13}, {17, 19}, {29, 31}

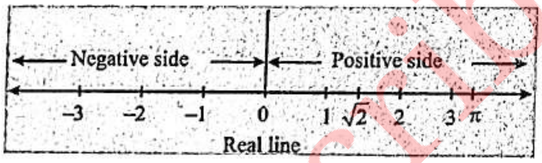
(*x*) **Rational numbers:** All the numbers that can be represented in the form p/q, where p and q are integers and $q \neq 0$, are called rational numbers and their set is denoted by Q. Thus $Q = \{p/q : p, q \in \text{I and } q \neq 0\}$. It may be noted that every integer is a rational number since it can be written as p/q. It may be noted that all recurring decimals are rational numbers.

Note: Maximum number of different decimal digits in $\frac{p}{q}$ is equal to q, i.e. $\frac{11}{9}$ will have maximum of 9 different decimal digits.

(*xi*) **Irrational numbers:** The numbers which can not be expressed in p/q form where $p, q \in \text{I}$ and $q \neq 0$ i.e. the numbers which are not rational are called irrational numbers and their set is denoted by Q^c. (i.e. complementary set of Q) e.g. $\sqrt{2}$, $1 + \sqrt{3}$ etc. Irrational numbers can not be expressed as recurring decimals.

Note: $e \approx 2.71$ (is called Napier's constant) and $\pi \approx 3.14$ are irrational numbers.

(*xii*) **Real numbers:** Numbers which can be expressed on number line are called real numbers. The complete set of rational and irrational numbers is the set of real numbers and is denoted by R. Thus $R = Q \cup Q^C$.



All real numbers follow the order property i.e. if there are two distinct real numbers a and b then either $a < b$ or $a > b$.

Note:

(*a*) Integers are rational numbers, but converse need not be true.

(*b*) Negative of an irrational number is an irrational number.

(*c*) Sum of a rational number and an irrational number is always an irrational number

e.g. $2 + \sqrt{3}$

(*d*) The product of a non zero rational number and an irrational number will always be an irrational number.

(*e*) If $a \in Q$ and $b \notin Q$, then ab = rational number, only if $a = 0$.

(*f*) Sum, difference, product and quotient of two irrational numbers need not be a irrational number or we can say, result may be a rational number also.

ADVANCED LEARNING

(*xiii*) **Complex number:** A number of the form $a + ib$ is called a complex number, where $a, b \in R$ and $i = \sqrt{-1}$. Complex number is usually denoted by z and the set of complex number is represented by C. Thus $C = \{a + ib : a, b \in R \text{ and } i = \sqrt{-1}\}$

Note: It may be noted that $N \subset W \subset I \subset Q \subset R \subset C$.

Train Your Brain

Example 1: The value of $1.\overline{285714} \div 1.\overline{714285}$ = ______.

(*a*) $\frac{3}{4}$ (*b*) $\frac{7}{8}$

(*c*) $\frac{7}{12}$ (*d*) $\frac{3}{7}$

Sol. (*a*)

$$1.\overline{285714}$$
$$= 1 + 0.\overline{285714}$$
$$= 1 + \frac{2}{7} = \frac{9}{7}$$
$$1.\overline{714285}$$
$$= 1 + \frac{5}{7} = \frac{12}{7}$$
$$\therefore 1.\overline{285714} \div 1.\overline{714285}$$
$$= \frac{9}{7} \div \frac{12}{7}$$
$$= \frac{9}{7} \times \frac{7}{12}$$
$$= \frac{3}{4}$$

Example 2: Prove that the difference $10^{25} - 7$ is divisible by 3.

Sol. Write the given difference in the form $10^{25} - 7 = (10^{25} - 1) - 6$. The number $10^{25} - 1 = \underbrace{99..9}_{25\text{ digits}}$ is divisible by 3 (and 9). Since the numbers $(10^{25} - 1)$ and 6 are divisible by 3, the number $10^{25} - 7$, being their difference, is also divisible by 3 without a remainder.

Concept Application

1. The product of $1.\overline{142857}$ and $0.\overline{63}$ = ______.
   (a) $\frac{8}{11}$
   (b) $\frac{7}{11}$
   (c) $\frac{11}{7}$
   (d) $\frac{8}{7}$

2. If $x=\sqrt{12}-\sqrt{9}, y=\sqrt{13}-\sqrt{10},$ and $z=\sqrt{11}-\sqrt{8},$ then which of the following is true?
   (a) $z>x>y$
   (b) $z>y>x$
   (c) $y>x>z$
   (d) $y>z>x$

SOME IMPORTANT IDENTITIES

1. $(a+b)^2=a^2+2ab+b^2=(a-b)^2+4ab$
2. $(a-b)^2=a^2-2ab+b^2=(a+b)^2-4ab$
3. $a^2-b^2=(a+b)(a-b)$
4. $(a+b)^3=a^3+b^3+3ab(a+b)$
5. $(a-b)^3=a^3-b^3-3ab(a-b)$
6. $a^3+b^3=(a+b)^3-3ab(a+b)=(a+b)(a^2+b^2-ab)$
7. $a^3-b^3=(a-b)^3+3ab(a-b)=(a-b)(a^2+b^2+ab)$
8. $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca=a^2+b^2+c^2+2abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$
9. $a^2+b^2+c^2-ab-bc-ca=\frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]$
10. $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$
    $=\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$
    If $a+b+c=0$, then $a^3+b^3+c^3=3abc$
11. $a^4-b^4=(a+b)(a-b)(a^2+b^2)$
12. $a^4+a^2+1=(a^2+1)^2-a^2=(1+a+a^2)(1-a+a^2)$

Train Your Brain

Example 3: Show that the expression, $(x^2-yz)^3+(y^2-zx)^3+(z^2-xy)^3-3(x^2-yz).(y^2-zx).(z^2-xy)$ is a perfect square and find its square root.

Sol. $(x^2-yz)^3+(y^2-zx)^3+(z^2-xy)^3-3(x^2-yz)(y^2-zx)(z^2-xy)=a^3+b^3+c^3-3abc$

(where $a=x^2-yz, b=y^2-zx, c=z^2-xy$)

$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$=\frac{1}{2}(a+b+c)((a-b)^2+(b-c)^2+(c-a)^2)$

$=\frac{1}{2}(x^2+y^2+z^2-xy-yz-zx)[(x^2-yz-y^2+zx)^2+(y^2-zx-z^2+xy)^2+(z^2-xy-x^2+yz)^2]$

$=\frac{1}{2}(x^2+y^2+z^2-xy-yz-zx)[\{x^2-y^2+z(x-y)\}^2+\{y^2-z^2+x(y-z)\}^2+\{z^2-x^2+y(z-x)\}^2]$

$=\frac{1}{2}(x^2+y^2+z^2-xy-yz-zx)(x+y+z)^2[(x-y)^2+(y-z)^2+(z-x)^2]$

$=(x+y+z)^2(x^2+y^2+z^2-xy-yz-zx)^2$

$=(x^3+y^3+z^3-3xyz)^2$

(which is a perfect square) its square roots are $\pm(x^3+y^3+z^3-3xyz)$

Example 4: If $x^2-4x+1=0$, then what is the value of $x^3+\frac{1}{x^3}$?

Sol. $x^2-4x+1=0 \Rightarrow x+\frac{1}{x}=4$

$$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)=4^3-3\times4=52$$

Example 5: If $x+\frac{1}{x}=a$, then what is the value of $x^3+x^2+\frac{1}{x^3}+\frac{1}{x^2}$?

(a) a^3+a^2
(b) a^3+a^2-5a
(c) a^3+a^2-3a-2
(d) a^3+a^2-4a-2

Sol. (c)

Given, $x+\frac{1}{x}=a$

Now, $x^3+x^2+\frac{1}{x^3}+\frac{1}{x^2}=\left(x^3+\frac{1}{x^3}\right)+\left(x^2+\frac{1}{x^2}\right)$

$=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)+\left(x+\frac{1}{x}\right)^2-2$

$=a^3-3a+a^2-2=a^3+a^2-3a-2.$

Concept Application

3. If $x^{1/3} + y^{1/3} + z^{1/3} = 0$, then $(x + y + z)^3$ is equal to?
   (*a*) 1 (*b*) 3
   (*c*) $3xy$ (*d*) $27xyz$

4. If $a + b + c = 0$, then what is the value of

$$\frac{a^2+b^2+c^2}{(a-b)^2+(b-c)^2+(c-a)^2}$$

   (*a*) 1 (*b*) 3 (*c*) $\frac{1}{3}$ (*d*) 0

5. If $x+\frac{1}{x}=p$ then $x^6+\frac{1}{x^6}$ equals to :
   (*a*) $p^6 + 6p$ (*b*) $p^6 - 6p$
   (*c*) $p^6 + 6p^4 + 9p^2 + 2$ (*d*) $p^6 - 6p^4 + 9p^2 - 2$

6. If $x+\frac{1}{x}=4$, then find values of
   (*i*) $x^2+\frac{1}{x^2}$ (*ii*) $x^3+\frac{1}{x^3}$
   (*iii*) $x^4+\frac{1}{x^4}$

7. Prove that $(1 + x)(1 + x^2)(1 + x^4)(1 + x^8)(1 + x^{16}) = \frac{(1-x^{32})}{(1-x)}$

8. If x, y, z are all different real numbers, then prove that

$$\frac{1}{(x-y)^2}+\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}=\left(\frac{1}{x-y}+\frac{1}{y-z}+\frac{1}{z-x}\right)^2$$

9. If $\frac{a}{b}+\frac{b}{a}=-1$, then find value of $a^3 - b^3$.

10. If $a - b = -8$, $ab = -12$ then $a^3 - b^3$ will be

11. The product $(x + y)(x - y)(x^2 + xy + y^2)(x^2 - xy + y^2)$ simplifies to

12. Find the real values of p, q, r satisfying $(2p - 3)^8 + (1 - q)^6 + (4 - 3r)^4 = 0$.

ADVANCED LEARNING

INDICES

If 'a' is any non zero real or imaginary number and 'm' is the positive integer, then $a^m = a \cdot a \cdot a. \ldots a$ (m times). Here a is called the base and m is called the index, power or exponent.

Law of indices:

1. $a^0 = 1, (a \neq 0)$
2. $a^{-m} = \frac{1}{a^m}, (a \neq 0)$
3. $a^{m+n} = a^m \cdot a^n$, where m and n are rational numbers
4. $a^{m-n} = \frac{a^m}{a^n}$, where m and n are rational numbers, $a \neq 0$
5. $(a^m)^n = a^{mn}$
6. $a^{p/q} = \sqrt[q]{a^p}$
7. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$, where $m, n \in N$ and $(m, n \geq 2)$ and a is positive rational number
8. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, $a, b \in R$ and atleast one of a and b should be positive

SURDS

If a is a positive rational number, which is not the nth power (n is any natural number) of any rational number, then the irrational number $\pm\sqrt[n]{a}$ are called simple surds or monomial surds.

Every surd is an irrational number (but every irrational number is not a surd). So, the representation of monomial surd on a number line is same as that of irrational numbers.

Examples:

1. $\sqrt{3}$ is a surd and $\sqrt{3}$ is an irrational number.
2. $\sqrt[3]{5}$ is a surd and $\sqrt[3]{5}$ is an irrational number.
3. π is an irrational number, but it is not a surd.
4. $\sqrt[3]{3+\sqrt{2}}$ is an irrational number. It is not a surd, because $3+\sqrt{2}$ is not a rational number.

Train Your Brain

Example 6: Simplify $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4$

Sol. $a^{9(1/6)(1/3)4} \cdot a^{9(1/3)(1/6)4} = a^2 \cdot a^2 = a^4$.

Example 7: Arrange the following in ascending or descending order of magnitude:

$\sqrt[4]{6}, \sqrt[3]{7}, \sqrt{5}$

Sol. $\sqrt[4]{6} = 6^{1/4}, \sqrt[3]{7} = 7^{1/3}, \sqrt{5} = 5^{1/2}$

LCM of the denominators of the exponents of these three terms, 4, 3 and 2 is12.

Now express the exponent of each term, as a fraction in which then denominator is 12.

$$6^{\frac{1}{4}} = 6^{\frac{3}{12}} = \left(6^3\right)^{\frac{1}{12}} = \sqrt[12]{216}$$

$$7^{\frac{1}{3}} = 7^{\frac{4}{12}} = \left(7^4\right)^{\frac{1}{12}} = \sqrt[12]{2401}$$

$$5^{\frac{1}{2}} = 5^{\frac{6}{12}} = \left(5^6\right)^{\frac{1}{12}} = \sqrt[12]{15625}$$

Now, $\sqrt[4]{6}=\sqrt[12]{216}, \sqrt[3]{7}=\sqrt[12]{2401}, \sqrt{5}=\sqrt[12]{15625}$

Hence, their ascending order is

$\sqrt[12]{216}, \sqrt[12]{2401}, \sqrt[12]{15625}$, *i.e.*, $\sqrt[4]{6}, \sqrt[3]{7}, \sqrt{5}$

$\therefore$ The descending order of magnitude of the given radical is $\sqrt{5}, \sqrt[3]{7}, \sqrt[4]{6}$.

Example 8: Find the square root of $10+\sqrt{24}+\sqrt{60}+\sqrt{40}$

Sol. $=\sqrt{10+\sqrt{24}+\sqrt{60}+\sqrt{40}}$

$=\sqrt{10+2\sqrt{6}+2\sqrt{15}+2\sqrt{10}}$

$=\sqrt{(2+3+5)+2\sqrt{2(3)}+2\sqrt{3(5)}+2\sqrt{2(5)}}$

$=\sqrt{\left(\sqrt{2}+\sqrt{3}+\sqrt{5}\right)^2}$

$=\sqrt{2}+\sqrt{3}+\sqrt{5}.$

Concept Application

13. If the surds $\sqrt[4]{4}, \sqrt[6]{5}, \sqrt[8]{6}$ *and* $\sqrt[12]{8}$ are arranged in ascending order from left to right, then the third surd from the left is

(*a*) $\sqrt[12]{8}$ (*b*) $\sqrt[4]{4}$ (*c*) $\sqrt[8]{6}$ (*d*) $\sqrt[6]{5}$

14. $\sqrt[6]{15-2\sqrt{56}}\,\sqrt[3]{\sqrt{7}+2\sqrt{2}}=$ ______.

(*a*) 0 (*b*) $\sqrt{2}$ (*c*) 1 (*d*) $\sqrt[6]{2}$

15. If $x=2^{1/3}-2$, then $x^3+6x^2+12x=$ ________.

(*a*)6 (*b*) -6 (*c*) 8 (*d*) -8

16. If $a=4^{\frac{-3}{2}}, b=(125)^{\frac{-2}{3}}, c=81^{\frac{-3}{4}}$ then $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=$

17. If $p=8^{\frac{2}{3}}, q=9^{\frac{3}{2}}, r=16^{\frac{5}{4}}, s=\frac{1}{6^{-2}}$ then $p+q+r+s=$

18. If $u=\left(\frac{1}{27}\right)^{\frac{-4}{3}}, v=\left(\frac{1}{216}\right)^{\frac{-2}{3}}, w=(8)^{5/3}\,.\,(4)^{-5/2}$, $t=(9)^{3/2}\,.\,(81)^{-3/4}$ then $\frac{vwt}{\sqrt{u}}=$

19. Which of the following number is greater than 1

(*a*) $\left(\frac{1}{216}\right)^{-2/3}\cdot\left(\frac{1}{6^2}\right)$ (*b*) $(125)^{-2/3}\,.\,(625)^{1/2}$

(*c*) $\log_{(1/2)} 16$ (*d*) $\log_{16} 32$

20. Simplify $a\left(\frac{\sqrt{a}+\sqrt{b}}{2b\sqrt{a}}\right)^{-1}+b\left(\frac{\sqrt{a}+\sqrt{b}}{2a\sqrt{b}}\right)^{-1}$

ADVANCED LEARNING

RATIO

(*i*) If A and B be two quantities of the same kind, then their ratio is $A : B$; which may be denoted by the fraction $\frac{A}{B}$ (This may be an integer or fraction)

(*ii*) A ratio may represented in a number of ways e.g. $\frac{a}{b}=\frac{ma}{mb}=\frac{na}{nb}=$...... where m, n,...... are non-zero numbers.

(*iii*) To compare two or more ratio, reduce them to common denominator.

(*iv*) Ratio between two ratios may be represented as the ratio of two integers

e.g. $\frac{a}{b}:\frac{c}{d}:\frac{a/b}{c/d}=\frac{ad}{bc}$ or $ad : bc$

(*v*) Ratios are compounded by multiplying them together i.e. $\frac{a}{b}\cdot\frac{c}{d}\cdot\frac{e}{f}\ldots\ldots=\frac{ace}{bdf}\ldots\ldots$

(*vi*) If $a : b$ is any ratio then its duplicate ratio is $a^2 : b^2$; triplicate ratio is $a^3 : b^3$..... etc.

(*vii*) If $a : b$ is any ratio, then its sub-duplicate ratio is $a^{1/2} : b^{1/2}$; sub-triplicate ratio is $a^{1/3} : b^{1/3}$ etc.

PROPORTION

When two ratios are equal, then the four quantities compositing them are said to be proportional. If $\frac{a}{b}=\frac{c}{d}$, then it is written as $a : b = c : d$ or $a : b :: c : d$

(*i*) 'a' and 'd' are known as extremes and 'b and c' are known as means.

(*ii*) An important property of proportion Product of extremes = product of means.

(*iii*) If $a : b = c : d$, then $b : a = d : c$ (Invertando)

(*iv*) If $a : b = c : d$, then $a : c = b : d$ (Alternando)

(*v*) If $a : b = c : d$, then $\frac{a+b}{b}=\frac{c+d}{d}$ (Componendo)

(*vi*) If $a : b = c : d$, then $\frac{a-b}{b}=\frac{c-d}{d}$ (Dividendo)

(*vii*) If $a : b = c : d$, then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$ (Componendo and dividendo) (*viii*) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ...$, then each $\frac{a+c+e+.....}{b+d+f+......}$

$= \frac{\text{Sum of the numerators}}{\text{Sum of the denominators}}$

(*ix*) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ...$, then each $= \frac{xa+yc+ze+......}{xb+yd+zf+......}$

(*x*) If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ...$, then each $= \left(\frac{xa^n+yc^n+ze^n}{xb^n+yd^n+zf^n}\right)^{1/n}$

Train Your Brain

Example 9: If $\frac{x+y}{2} = \frac{y+z}{3} = \frac{z+x}{4}$, then find $x : y : z$.

Sol. Each $= \frac{\text{Sum of the numerators}}{\text{Sum of the denominators}}$

$$= \frac{2(x+y+z)}{9} = \frac{x+y+z}{9/2} \text{ and therefore each}$$

$$= \frac{(x+y+z)-(y+z)}{\frac{9}{2}-3} = \frac{(x+y+z)-(x+z)}{\frac{9}{2}-4}$$

$$= \frac{(x+y+z)-(x+y)}{\frac{9}{2}-2}$$

$$= \frac{x}{3/2} = \frac{y}{1/2} = \frac{z}{5/2} \Rightarrow x : y : z = 3 : 1 : 5$$

Example 10: If $\frac{a+3b+2c+6d}{-a-3b+2c+6d} = \frac{3a+b+6c+2d}{-3a-b+6c+2d}$, then the correct statement is

(*a*) $ad = bc$ (*b*) $ac = bd$

(*c*) $c = \frac{ab}{d}$ (*d*) $a + d = b + c$

Sol. (*a*) Apply *C* and *D* the the given equation

$$\Rightarrow \frac{4c+12d}{2a+6d} = \frac{4d+12c}{2b+6a} \quad \Rightarrow \frac{c+3d}{a+3b} = \frac{d+3c}{b+3a}$$

$$\Rightarrow bc + 3ac + 3bd + 9ad = ad + 3ac + 3bd + 9bc$$

$$\Rightarrow 8ad = 8bc$$

$$\Rightarrow ad = bc$$

Concept Application

21. Solve the equations $\frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}} = \frac{2x^2-1}{2}$

22. Solve: $\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}} = 3$

23. If $(4x - 3y) : (2x + 5y) = 12 : 19$ then $x : y$ is ......

(*a*) 2 : 3 (*b*) 1 : 2

(*c*) 9 : 4 (*d*) 2 : 1

INTERVALS

Intervals are basically subsets of *R* and are commonly used in solving inequalities or in finding domains. If there are two numbers $a, b \in R$ such that $a < b$, we can define four types of intervals as follows :

Open Interval (a, b)
$\{x : a < x < b\}$ i.e. extreme points are not included
Closed Interval $[a, b]$
$\{x : a \le x \le b\}$ i.e. extreme points are included
It can possible when *a* and *b* are finite
Semi-Open Interval $(a, b]$
$\{x : a < x \le b\}$ i.e. *a* is not included and *b* is included
Semi-Closed Interval $[a, b)$
$\{x : a \le x < b\}$ i.e. *a* is included and *b* is not included

Note:

1. The infinite intervals are defined as follows :
   (*i*) $(a, \infty) = \{x : x > a\}$
   (*ii*) $[a, \infty) = \{x : x \ge a\}$
   (*iii*) $(-\infty, b) = \{x : x < b\}$
   (*iv*) $(\infty, b] = \{x : x \le b\}$
   (*v*) $(-\infty, \infty) = \{x : x \in R\}$
2. $x \in \{1, 2\}$ denotes some particular values of *x*, i.e. $x = 1, 2$
3. If there is no value of *x*, then we say $x \in \phi$ (null set)

GENERAL METHOD TO SOLVE INEQUALITIES

Method of Intervals (Wavy Curve Method)

Let $g(x) = \left(\frac{(x-b_1)^{k_1}(x-b_2)^{k_2} - - - (x-b_n)^{k_n}}{(x-a_1)^{r_1}(x-a_2)^{r_2} - - - (x-a_n)^{r_n}}\right)$ ...(*i*)

Where $k_1, k_2 ..... k_n$ and $r_1, r_2 .... r_n \in N$ and $b_1, b_2..... b_n$ and $a_1, a_2 ... a_n$ are real numbers.

Then to solve the inequality following steps are taken.

Steps: Points where numerator becomes zero are called zeros or roots of the function and where denominator becomes zero are called poles of the function.

(*i*) First we find the zeros and poles of the function.

(*ii*) Then we mark all the zeros and poles on the real line and form a curve to divide the real line in many intervals.

(*iii*) Determine sign of the function in any of the interval and then alternates the sign in the neghbouring interval if the poles or zeros dividing the two interval has appeared odd number of times otherwise retain the sign.

(*iv*) Thus we consider all the intervals. The solution of the $g(x) > 0$ is the union of the intervals in which we have put the plus sign and the solution of $g(x) < 0$ is the union of all intervals in which we have put the minus sign.

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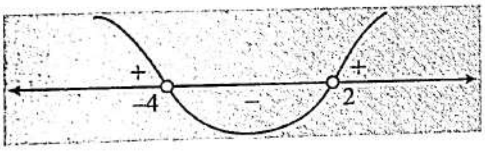
Example 11: Solution $\dfrac{(x-1)^2(x+4)}{(2-x)} < 0$ is

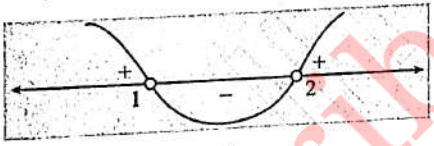
Sol. $\dfrac{(x-1)^2(x+4)}{(2-x)} < 0 \Rightarrow \dfrac{(x+4)}{(x-2)} > 0$



$\Rightarrow (-\infty, -4) \cup (2, \infty)$.

Example 12: The solution of $\dfrac{\sqrt[3]{(x+4)^4}(x-1)^3}{(x-2)} > 0$ is

Sol. $\dfrac{(x+4)^{4/3}(x-1)^3}{(x-2)} > 0 \Rightarrow \dfrac{x-1}{x-2} > 0$



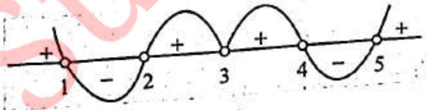
$\Rightarrow (-\infty, 1) \cup (2, \infty)$ Excluding -4.

Example 13: Find the range of x, so that following expressions are defined.

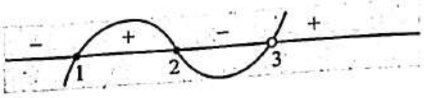
(*a*) $(x-1)(x-2)(x-3)^2(x-4)^5(x-5) > 0$

(*b*) $\dfrac{(x-1)(x-2)}{(x-3)} \geq 0$

Sol. (*a*) $\Rightarrow x \in (-\infty, 1) \cup (2, 3) \cup (3, 4) \cup (5, \infty)$



(*b*) $\Rightarrow x \in [1, 2] \cup (3, \infty)$



Concept Application

24. Solve the inequality $\dfrac{x^3-4x^2-12x}{x+3} \geq 0$

25. Solve the inequality $\dfrac{(x+1)^4(x-2)(x-3)^3(x-4)^2}{x^3-36x} \geq 0$

26. Find the number of integer values of variable x satisfying the following pair of inequalities.

$$\frac{(x-1)(x+4)}{x-3} < 0 \,\&\, x^2+6x-27 \leq 0$$

27. The solution of the inequality $2x-1 \leq x^2+3 \leq x-1$ is

(*a*) $x \in R$
(*b*) $(-2, 2]$
(*c*) $(-2, 2)$
(*d*) $x \in \phi$

28. Solve for x: $-1 \leq \dfrac{1+x^2}{2x} \leq 1$

29. Solve for x: $\dfrac{(x-\sin 1)(x-\sin 2)}{(x-\sin 3)(x-\sin 4)} \leq 0$

30. Solve for x: $\sqrt{(x-1)}(x-2)x \leq 0$

31. Solve for x: $\sqrt{x-5}-\sqrt{9-x} > 1 : x \in Z$

32. Solve for x: $\sqrt{x-1} > \sqrt{3-x}$

LOGARITHM FUNCTION

Definition

The logarithm of the number N to the base 'a' is the exponent indicating the power to which the base 'a' must be raised to obtain the number N.

This number is designated as $\log_a N$.

Hence $\log_a N = x \Leftrightarrow a^x = N, a > 0, a \neq 1$ and $N > 0$

If $a = 10$, then we write $\log b$ rather than $\log_{10} b$

$a = e$, we write $\ln b$ rather than $\log_e b$

The existence and uniqueness of the number $\log_a N$ follows from the properties of an exponential functions.

Domain

The existence and uniqueness of the number $\log_a N$ can be determined with the help of set of conditions, $a > 0$ & $a \neq 1$ & $N > 0$.

FUNDAMENTAL IDENTITY

(*i*) $\log_a 1 = 0$ $(a > 0,\ a \neq 1)$

(*ii*) $\log_a a = 1$ $(a > 0,\ a \neq 1)$

(*iii*) $\log_{1/a} a = -1$ $(a > 0,\ a \neq 1)$

Remember

$\log_{10}2 \approx 0.3010;\quad \log_{10}3 \approx 0.4771$

$\ln 2 \approx 0.693;\quad \ln 10 \approx 2.303$

FUNDAMENTAL LOGARITHMIC IDENTITY

$a^{\log_a N} = N,\ a > 0,\ a \neq 1\ \&\ N > 0$

Proof :

$\log_a N = x$ ... (i)

$N = (a)^x$ ... (ii)

by equation (i) & (ii)

$N = (a)^{\log_a N}$

PRINCIPAL PROPERTIES

Let M & N are arbitrary positive numbers, $a > 0$, $a \neq 1$, and x, y are any real numbers, then:

(*i*) $\log_a(M \cdot N) = \log_a M + \log_a N$;

Proof:

Let $\log_a M = x$ and $\log_a N = y$

$\Rightarrow M = a^x$ and $N = a^y$

Now, $MN = a^x a^y = a^{x+y}$

$\Rightarrow \log_a MN = x + y$

In general

$\log_a(x_1 x_2 \ldots x_n) = \log_a x_1 + \log_a x_2 + \ldots + \log_a x_n$

(*ii*) $\log_a(M/N) = \log_a M - \log_a N$

Proof:

Let $\log_a M = x$ and $\log_a N = y$

$\Rightarrow M = a^x$ and $N = a^y$

Now, $M/N = a^x/a^y = a^{x-y}$

$\Rightarrow \log_a(M/N) = x - y$

(*iii*) $\log_a M^\alpha = \alpha \cdot \log_a M$

BASE CHANGING THEOREM

It states that ratio of logarithm of two numbers is independent of their common base

Symbolically

$$\frac{\log_a M}{\log_a b} = \log_b M\ (a > 0, M > 0, b > 0)$$

Proof:

Let $\log_b M = x$

$\Rightarrow M = b^x$

$\Rightarrow \log_a M = \log_a b^x$

$\Rightarrow \log_a M = x \cdot \log_a b$

$$\Rightarrow \frac{\log_a M}{\log_a b} = x = \log_b M$$

Important Results

(*i*) **Base power formula:** $\log_{a^k} M = \dfrac{1}{k}\log_a M$

Proof:

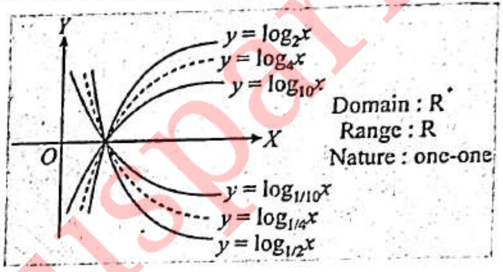
$$\log_{a^k}(M) = \frac{\log_a M}{\log_a a^k} = \frac{\log_a M}{k\log_a a} = \frac{1}{k}\log_a M$$

(*ii*) $a^{\log_b c} = c^{\log_b a}$

Proof: $a^{\log_b c} = a^{\log_a c \cdot \log_b a} = (a^{\log_a c})^{\log_b a} = (c)^{\log_b a}$

GRAPH OF LOGARITHMIC FUNCTIONS

If $a > 0, a \neq 1$, then the function $y = \log_a x, x \in R^+$ (set of positive real numbers) is called the logarithmic Function with base a.



Note: (*i*) If the number and the base are on the same side of the unity, then the logarithm is positive.

(*ii*) If the number and the base are on the opposite sides of unity, then the logarithm is negative.

EXPONENTIAL FUNCTION

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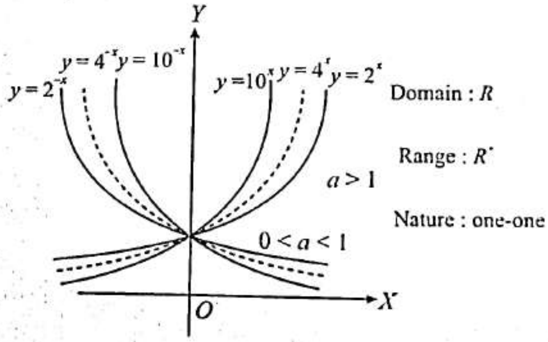
If $a > 0$, $a \neq 1$ then the function defined by $f(x) = a^x$, $x \in R$ is called an Exponential Function with base a.



LOGARITHMIC EQUATION

The equality $\log_a x = \log_a y$ is possible if and only if $x = y$

i.e. $\log_a x = \log_a y \iff x = y$

Always check validity of given equation, $(x > 0, y > 0, a > 0, a \neq 1)$

LOGARITHMIC INEQUALITY

Let 'a' is a real number such that

(i) If $a > 1$, then $\log_a x > \log_a y \Rightarrow x > y$

(ii) If $a > 1$, then $\log_a x < \alpha \Rightarrow 0 < x < a^\alpha$

(iii) If $a > 1$, then $\log_a x > \alpha \Rightarrow x > a^\alpha$

(iv) If $0 < a < 1$, then $\log_a x > \log_a y \Rightarrow 0 < x < y$

(v) If $0 < a < 1$, then $\log_a x < \alpha \Rightarrow x > a^\alpha$

Form - I: $f(x) > 0, g(x) > 0, g(x) \neq 1$

Form	Collection of system
(a) $\log_{g(x)} f(x) \geq 0 \Leftrightarrow$	$\begin{cases} f(x) \geq 1, & g(x) > 1 \\ 0 < f(x) \leq 1, & 0 < g(x) < 1 \end{cases}$
(b) $\log_{g(x)} f(x) \leq 0 \Leftrightarrow$	$\begin{cases} f(x) \geq 1, 0 < g(x) < 1 \\ 0 < f(x) \leq 1, g(x) > 1 \end{cases}$
(c) $\log_{g(x)} f(x) \geq a \Leftrightarrow$	$\begin{cases} f(x) \geq g(x)^a, g(x) > 1 \\ 0 < f(x) \leq g(x)^a, 0 < g(x) < 1 \end{cases}$
(d) $\log_{g(x)} f(x) \leq a \Leftrightarrow$	$\begin{cases} 0 < f(x) \leq g(x)^a, g(x) > 1 \\ f(x) \geq g(x)^a, 0 < g(x) < 1 \end{cases}$

From - II: $f(x) > 0, g(x) > 0, \phi(x) > 0, \phi(x) \neq 1$

Form	Collection of system
(a) $\log_{\phi(x)} f(x) \geq \log_{\phi(x)} g(x) \Leftrightarrow$	$\begin{cases} f(x) \geq g(x), \phi(x) > 1, \\ 0 < f(x) \leq g(x); 0 < \phi(x) < 1 \end{cases}$
(b) $\log_{\phi(x)} f(x) \leq \log_{\phi(x)} g(x) \Leftrightarrow$	$\begin{cases} 0 < f(x) \leq g(x), \phi(x) > 1, \\ f(x) \geq g(x) > 0, 0 < \phi(x) < 1 \end{cases}$

COMMON AND NATURAL LOGARITHM

$\log_{10} N$ is referred as a common logarithm and $\log_e N$ is called as natural logarithm of N to the base Napierian and is popularly written as $\ell n\, N$. Note that e is an irrational quantity lying between 2.7 to 2.8 **Note that** $e^{\ell n\, x} = x$.

Train Your Brain

Example 14: How many solutions are there for equation

$\log_4 (x - 1) = \log_2 (x - 3)$?

Sol. $\log_4 (x - 1) = \log_2 (x - 3)$

$\Rightarrow \log_{2^2} (x - 1) = \log_2 (x - 3)$

$\Rightarrow \frac{1}{2} \log_2 (x - 1) = \log_2 (x - 3)$

$\Rightarrow \log_2 (x - 1)^{1/2} = \log_2 (x - 3)$

$\Rightarrow (x - 1)^{1/2} = (x - 3)$

$\Rightarrow x - 1 = x^2 - 6x + 9$

$\Rightarrow (x - 2)(x - 5) = 0$

$\Rightarrow x = 2, 5$

But $x - 1 > 0$ and $x - 3 > 0$

$x > 1$ and $x > 3$

So only one solution $x = 5$

Example 15: Solve the logarithmic inequality $\log_{1/5} \frac{4x+6}{x} \geq 0$.

Sol. Since $\log_{1/5} 1 = 0$, the given inequality can be written as.

$$\log_{1/5} \frac{4x+6}{x} \geq \log_{1/5} 1$$

When the domain of the function is taken into account the inequality is equivalent to the system of inequalities.

$$\begin{cases} \frac{4x+6}{x} > 0, \\ \frac{4x+6}{x} \leq 1 \end{cases}$$

Solving the inequalities by using method of intervals

$$x \in \left[-2, \frac{-3}{2}\right)$$

Example 16: For $x \geq 0$, what is the smallest possible value of the expression $\log(x^3 - 4x^2 + x + 26) - \log(x + 2)$?

Sol. $\log \frac{(x^3 - 4x^2 + x + 26)}{(x+2)}$

$= \log \frac{(x^2 - 6x + 13)(x+2)}{(x+2)}$

$= \log (x^2 - 6x + 13)$ $\quad [\because x \neq -2]$

$= \log \{(x - 3)^2 + 4\}$

$\therefore$ Minimum value is $\log 4$ when $x = 3$

Example 17: Given $\log_2 a = s$, $\log_4 b = s^2$ and $\log_{c^2}(8) = \frac{2}{s^3+1}$.

Write $\log_2 \frac{a^2 b^5}{c^4}$ as a function of 's' ($a, b, c > 0, c \neq 1$).

Sol. Given $\log_2 a = s$ ...(i)

$\log_2 b = 2s^2$ ...(ii)

$\log_8 c^2 = \frac{s^3+1}{2}$ ...(iii)

$\Rightarrow \frac{2 \log c}{3 \log 2} = \frac{s^3+1}{2} \Rightarrow 4 \log_2 c = 3(s^3 + 1)$ ...(iv)

to find $2 \log_2 a + 5 \log_2 b - 4 \log_2 c$

$\Rightarrow \quad 2s + 10s^2 - 3(s^3 + 1)$

Example 18: If $\frac{1}{4}\log_4 M + 4\log_4 N = 1 + \log_{.008} 5$ then the value of $MN^{16} = k.2^{1/3}$, where k is equal to

(a) 8 (b) 32

(c) 36 (d) 40

Sol. (b) $\frac{1}{8}\log_2 M + 2\log_2 N = 1 + \frac{\log_2 5}{\log_2 (.008)}$

$$\Rightarrow \log_2 M^{1/8} + \log_2 N^2 = 1 + \frac{\log_2 10 - 1}{3 - 3\log_2 10}$$

$$\Rightarrow \log_2 \left(MN^{16}\right)^{1/8} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\Rightarrow \left(MN^{16}\right)^{1/8} = 2^{2/3}$$

$$\Rightarrow MN^{16} = 2^{16/3} = 32(2^{1/3})$$

Concept Application

33. Solve the inequality $\log_{1/3}(5x-1) > 0$.

34. Suppose that a and b are positive real numbers such that $\log_{27} a \div \log_9 b = \frac{7}{2}$ and $\log_{27} b + \log_9 a = \frac{2}{3}$. Find the value of the ab.

35. If $m_1 = \log_8 16$, $m_2 = \log_{81} 27$, $m_3 = \log_{1/3} 1/9$, $m_4 = \log_{1/3} 9\sqrt{3}$ then $m_1 . m_2 . m_3 \cdot m_4$

36. If $p = \log_{2\sqrt{3}} 1728$, $q = \log_2 (\cos 45°)$, $r = \log_2 (\log_2 4)$, $s = \log_3 (\tan 30°)$, $t = \log_{625} 125$ then $\frac{prt}{qs} =$

37. If $\log_7 (\log_3 (\log_2 x)) = 0$, then find $\log_{0.125} x$.

38. Solve for x:

(i) $\log_3 x > 0$ (ii) $\log_5 x \geq 0$

(iii) $\log_6 x < 0$ (iv) $\log_2 x \leq 0$

(v) $\log_{1/7} x > 0$ (vi) $\log_{1/8} x \geq 0$

(vii) $\log_{1/9} x < 0$ (viii) $\log_{1/e} x \leq 0$

(ix) $\log_2 (x-1) > 1$ (x) $\log_{1/2} (x-2) \leq 1$

39. Solve for x:

(i) $\log_4 (2x-3) < 2$

(ii) $\log_{1/2} (3x-2) \geq 3$

(iii) $\log_{16} (\log_4 (x)) > 1$

(iv) $\log_{1/2} (\log_{1/4} (x)) < 1$

40. $\log_2 (a^2 - 5) = 2$

41. $\log_{1/3} (a^2 - 1) = -1$

42. Prove that : $2^{\sqrt{\log_2 3}} = 3^{\sqrt{\log_3 2}}$.

43. If $\log_{12} 27 = a$ find the value of $\log_6 16$ in term of a.

44. Solve for x:

(i) $\log_x (2) . \log_{2x} 2 = \log_{4x} 2$

(ii) $5^{\log_a x} + 5x^{\log_a 5} = 3 (a > 0)$

(iii) $x^{(\log_2 x) + 4} = 32$

(iv) $\log_{x+1} (x^2 + x - 6)^2 = 4$

(v) $x + \log_{10} (1 + 2^x) = x \cdot \log_{10} 5 + \log_{10} 6$

45. Prove that $\log_a N \cdot \log_b N + \log_b N \cdot \log_c N + \log_c N \cdot \log_a N = \frac{\log_a N \cdot \log_b N \cdot \log_c N}{\log_{abc} N}$

46. Prove that: $2^{\left(\sqrt{\log_a \sqrt[4]{ab} + \log_b \sqrt[4]{ab}} - \sqrt{\log_a \sqrt[4]{\frac{b}{a}} + \log_b \sqrt[4]{\frac{a}{b}}}\right) \cdot \sqrt{\log_a b}}$

$$= \begin{cases} 2 & \text{if } b \geq a > 1 \\ 2^{\log_a b} & \text{if } 1 < b < a \end{cases}$$

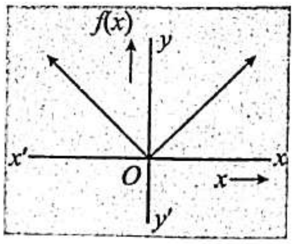
47. Simplify: $\log_{1/3} \sqrt[4]{729 \cdot \sqrt[3]{9^{-1} \cdot 27^{-4/3}}}$

ABSOLUTE VALUE FUNCTION / MODULUS FUNCTION

This is also known as absolute value function and denoted by

$$f(x) = |x| \text{ i.e. } f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Domain of this function is set of all real numbers because $f(x)$ exists for all $x \in R$ but $|x| \geq 0$ so range is all non-negative real numbers.



Domain = R; Range = $[0, \infty)$ or $R^+ \cup \{0\}$

Properties of modulus: For any $a, b \in R$

(i) $|a| \geq 0$

(ii) $|a| = |-a|$

(a) $|a|^n = |a^n|$

(b) $|a^n| = a^n$, where n is even and $n \in z$

(*iii*) $|a| \geq a, |a| \geq -a$

(*iv*) $|ab| = |a||b|$

(*v*) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

Note: $|f(x)| + |g(x)| = |f(x) + g(x)|$

$\Rightarrow f(x) \cdot g(x) \geq 0$

Train Your Brain

Example 19: The absolute value of sum of real solutions of $\log_2 |x^2 + 5x + 4| = \log_2 3 + \log_2 |x + 1|$ is

(*a*) 8 (*b*) 6 (*c*) 7 (*d*) 5

Sol. (*c*) $\log_2 \frac{|(x+1)(x+4)|}{|x+1|} = \log_2 3$

$|x + 4| = 3$

$x + 4 = -3, +3$

$x = -7, -1$ (rejected); $\Rightarrow x = -7$

Example 20: Solve the following linear equation

(*i*) $x|x| = 4$

(*ii*) $|x - 3| + 2|x + 1| = 4$

Sol. (*i*) $x|x| = 4$

If $x > 0$

$\therefore x^2 = 4 \Rightarrow x = \pm 2 \quad \therefore x = 2 \ (\because x \geq 0)$

If $x < 0$

$\Rightarrow -x^2 = 4 \Rightarrow x^2 = -4$ which is not possible

(*ii*) $|x - 3| + 2|x + 1| = 4$

Case-I: If $x \leq -1$

$\therefore -(x - 3) - 2(x + 1) = 4$

$\Rightarrow -x + 3 - 2x - 2 = 4 \Rightarrow -3x + 1 = 4$

$\Rightarrow -3x = 3 \quad \Rightarrow x = -1$

Case-II: If $-1 < x \leq 3$

$\therefore -(x - 3) + 2(x + 1) = 4$

$\Rightarrow -x + 3 + 2x + 2 = 4$

$\Rightarrow x = -1$ which is not possible

Case-III: If $x > 3$

$x - 3 + 2(x + 1) = 4$

$3x - 1 = 4 \Rightarrow x = 5/3$ which is not possible

$\therefore x = -1$

Example 21: Number of real solutions of $|x - 1| = |x - 2| + |x - 3|$ is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) more than 2

Sol. (*c*)

Case-I: $x \leq 1, 1 - x = 2 - x + 3 - x$

$x = 4$ (rejected)

Case-II: $1 < x \leq 2, x - 1 = 2 - x + 3 - x, x = 2$

Case-III: $2 < x < 3, x - 1 = x - 2 + 3 - x, x = 2$

Case-IV: $x \geq 3, x - 1 = x - 2 + x - 3$

$x = 4 \Rightarrow x = 2, 4$

Concept Application

48. Let $x_1, x_2, x_3, x_4, x_5 \in R$ and $|x_1 - x_2| = 2, |x_2 - x_3| = 4, |x_3 - x_4| = 3, |x_4 - x_5| = 5.$
Then the sum of all distinct possible values of $|x_5 - x_1|$ is

49. The number of integers which does NOT satisfy $\log_{|2x|}(|x + 2| + |x - 2|) = 1$ is
(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

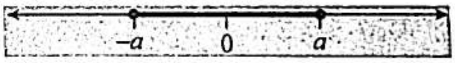
50. The real solutions of the equation where $|x|^2 - 3|x| + 2 = 0$ where $x_1 < x_2 < x_3 < x_4$ then
(*a*) $|x_1| = |x_3|$ (*b*) $|x_2| = |x_3|$
(*c*) $x_1 + x_4 = x_2 + x_3$ (*d*) $-x_2 + x_4 = x_1 - x_3$

51. Solve for x:
(*i*) $||x - 2| - 1| = 2$ (*ii*) $||x - 3| - 5| = 1$
(*iii*) $|||x - 5| - 4| - 3| = 2$

INEQUALITIES INVOLVING ABSOLUTE VALUE

(*i*) $|x| \leq a$ (where $a > 0$)

It implies those values of x on real number line which are at distance a or less than a from zero.



$\Rightarrow -a \leq x \leq a$

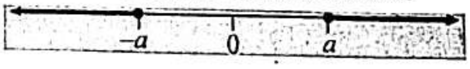
e.g. $|x| \leq 2 \Rightarrow -2 \leq x \leq 2$

$|x| < 3 \Rightarrow -3 < x < 3$

In general, $|f(x)| \leq a$ (where $a > 0$) $\Rightarrow -a \leq f(x) \leq a.$

(*ii*) $|x| \geq a$ (where $a > 0$)

It implies those values of x on real number line which are at distance a or more than a from zero



$\Rightarrow x \leq -a$ or $x \geq a$

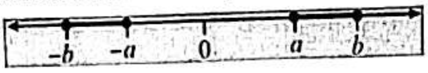
e.g. $|x| \geq 3 \Rightarrow x \leq -3$ or $x \geq 3$

$|x| > 2 \Rightarrow x < -2$ or $x > 2$

In general, $|f(x)| \geq a \Rightarrow f(x) \leq -a$ or $f(x) \geq a$

(*iii*) $a \le |x| \le b$ (where $a, b > 0$)

It implies those value of x on real number line whose distance from zero is equal to a or b or lies between a and b



$\Rightarrow [-b, -a] \cup [a, b]$

e.g. $2 \le |x| \le 4 \Rightarrow x \in [-4, -2] \cup [2, 4]$

(*iv*) If $|x+y| = |x| + |y|, xy \ge 0$

If $|x-y| = |x| + |y|, xy \le 0$

If $|x+y| = ||x| - |y||, xy \le 0$

If $|x-y| = ||x| - |y||, xy \ge 0$

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Example 22: Solve $x^2 - 4|x| + 3 < 0$.

Sol. $x^2 - 4|x| + 3 < 0$

$\Rightarrow (|x| - 1)(|x| - 3) < 0$

$\Rightarrow 1 < |x| < 3$

$\Rightarrow -3 < x < -1$ or $1 < x < 3$

$\Rightarrow x \in (-3, -1) \cup (1, 3)$

Example 23: Solve $1 \le |x - 2| \le 3$

Sol. $1 \le |x - 2| \le 3$

$\Rightarrow -3 \le x - 2 \le -1$ or $1 \le x - 2 \le 3$

$\Rightarrow -1 \le x \le 1$ or $3 \le x \le 5$

$\Rightarrow x \in [-1, 1] \cup [3, 5]$

Example 24: Solve $|x - 1| + |x - 2| + |x - 3| \ge 6$,

Sol. For $x \le 1$, the given inequation becomes

$1 - x + 2 - x + 3 - x \ge 6 \Rightarrow -3x \ge 0$.

$\Rightarrow$ $x \le 0$ and for $x \ge 3$, the given equation becomes

$x - 1 + x - 2 + x - 3 \ge 6 \;\Rightarrow\; 3x \ge 12 \Rightarrow x \ge 4$

For $1 < x \le 2$

we get $x - 1 + 2 - x + 3 - x \ge 6$

$\Rightarrow \; -x + 4 \ge 6$

i.e. $-x \ge 2 \;\Rightarrow x \le -2$ Not possible

For $2 < x < 3$,

We get $x - 1 + x - 2 + 3 - x \ge 6$

$\Rightarrow \; x \ge 6$ not possible

Hence solution set is $(-\infty, 0] \cup [4, \infty)$

i.e. $x \le 0$ or $x \ge 4$

Concept Application

52. Solve $||x - 1| - 2| < 5$

53. Number of non-positive integral values of 'x' satisfying the given inequality, $|x^2 - 1| \le |2x - 1|$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

54. Solve $|x^2 - 2x| + |x - 4| > |x^2 - 3x + 4|$.

55. Solve $\left|\dfrac{x-3}{x+1}\right| \le 1$.

56. Solve for x:

(*i*) $|x| > 1$ (*ii*) $|x| \ge 5$ (*iii*) $|x| < 7$

(*iv*) $|x| \le 10$ (*v*) $|x| \ge 0$ (*vi*) $|x| < -8$

(*vii*) $|x| > -4$ (*viii*) $|x| \ge -5$ (*ix*) $|x| \le -10$

57. Solve for x:

(*i*) $|x - 1| > 1$ (*ii*) $|x - 2| < 1$

(*iii*) $1 < |2x + 1| < 3$ (*iv*) $1 \le |1 - 2x| \le 3$

(*v*) $-1 \le |3x - 1| \le 5$ (*vi*) $-6 \le |1 - 3x| \le -1$

58. Solve for x:

(*i*) $||x - 2| - 1| \le 2$ (*ii*) $||x - 3| - 5| \ge 1$

(*iii*) $|||x - 5| - 4| - 3| \le 2$

59. Solve for x:

(*i*) $\dfrac{1}{2} \le |2x - 1| \le \dfrac{3}{5}$ (*ii*) $-\dfrac{1}{3} \le |3x - 4| \le 2$

(*iii*) $2 \le |4 - 5x| \le \dfrac{10}{3}$ (*iv*) $3 < |x^2 - 1| < 8$

60. Solve for x:

(*i*) $|||x + 5| - 3| - 1| = 2$

(*ii*) $||||x - 5| - 7| - 3| - 2| = 1$

61. Solve for x:

(*i*) $2 \le |||x - 7| - 3| + 2| \le 5$

(*ii*) $||||x - 5| - 7| - 3| - 2| \le 1$

62. Solve for x:

(*i*) $\dfrac{(|x| - 1)}{(|x| - 2)} \le 0$ (*ii*) $\dfrac{(|x| - 1)(|x| - 3)}{|x|^2 - 2|x|} \ge 0$

(*iii*) $\dfrac{(|x|^2 - 5|x| + 6)}{(4 - |x|^2)} \le 0$

(*iv*) $(||x - 1| - 2| - 3)(|x - 2| - 3) \ge 0$

(*v*) $(|||x - 1| - 2| - 1| - 2)(|x - 2|) \ge 0$

63. Solve for x:

(*i*) $\left|\dfrac{x-3}{x+1}\right| \le 1$ (*ii*) $\left|\dfrac{x+1}{x}\right| + |x+1| = \dfrac{(x+1)^2}{|x|}$

(*iii*) $\left|1 + \dfrac{3}{x}\right| > 2$ (*iv*) $|2^x - 1| + |4 - 2^x| < 3$

(*v*) $\left(\dfrac{1}{3}\right)^{\frac{|x+2|}{2-|x|}} > 9$

64. Solve: $||x^2 - 2x + 6| - |x + 6|| = |x^2 - 3x|$

Aarambh (Solved Examples)

1. The value of $81^{(1/\log_5 3)} + 27^{\log_9 36} + 3^{4/\log_7 9}$ is equal to

(*a*) 49 (*b*) 625 (*c*) 216 (*d*) 890

Sol. $81^{(1/\log_5 3)} + 27^{\log_9 36} + 3^{4/\log_7 9}$

$= 3^{4\log_3 5} + 3^{3\cdot\frac{1}{2}\log_3 36} + 3^{4\log_9 7}$

$= 3^{\log_3 5^4} + 3^{\log_3 36^{3/2}} + 3^{\log_3 7^{4/2}}$

$= 5^4 + 36^{3/2} + 7^2 = 890$

Therefore, option (*d*) is the correct answer.

2. The largest integral value of x satisfying

$\sqrt{18^x - 5} \le \sqrt{2(18^x + 12)} - \sqrt{18^x + 5}$ is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) no integral value of x possible

Sol. Let $18^x = p$

$\sqrt{p-5} + \sqrt{p+5} \le \sqrt{2(p+12)}$

$\Rightarrow\ p-5+p+5+2\sqrt{p^2-25} \le 2p+24$

$\Rightarrow\ \sqrt{p^2-25} \le 12 \Rightarrow p^2 \le 169 \Rightarrow p \le 13$

Also $p \ge 5$

Thus $5 \le p \le 13 \Rightarrow \log_{18}5 \le x \le \log_{18}13$

Therefore, option (*d*) is the correct answer.

3. Solve if $|x-5| + |x+4| = 9$

(*a*) $[-4, 5]$ (*b*) $(-4, 5)$ (*c*) $(-4, 5]$ (*d*) $[-4, 5)$

Sol. Given equation is of form $|a| + |b| = |a - b|$

It is true for $ab \le 0$

$(x-5)(x+4) \le 0$

So $x \in [-4, 5]$

Therefore, option (*a*) is the correct answer.

4. Solve $\dfrac{(e-\sin x)(x-2)}{(x+4)} \ge 0.$

(*a*) $(-\infty, -4) \cup [2, \infty)$ (*b*) $(-\infty, -4] \cup (2, \infty)$

(*c*) $(-\infty, -4) \cup (2, \infty)$ (*d*) None of these

Sol. Zeros $x = 2$, Pole $x \ne -4$

$e - \sin x > 0$ always positive

$\dfrac{(e-\sin x)(x-2)}{(x+4)} \ge 0$

Final solution $x \in (-\infty, -4) \cup [2, \infty)$

Therefore, option (*a*) is the correct answer.

5. Values of x satisfying the equation

$\log_5^2 x + \log_{5x}\left(\dfrac{5}{x}\right) = 1$ are

(*a*) 1 (*b*) 5

(*c*) $\dfrac{1}{25}$ (*d*) 3

Sol. $(\log_5 x)^2 + \log_{5x}\dfrac{5}{x} = 1$

$\Rightarrow (\log_5 x)^2 + \log_{5x}5 - \log_{5x}x = 1$

$\Rightarrow (\log_5 x)^2 + \dfrac{\log_5 5}{\log_5 5 + \log_5 x} - \dfrac{\log_5 x}{\log_5 5 + \log_5 x} = 1$

$\Rightarrow (\log_5 x)^2 + \dfrac{1}{1+\log_5 x} - \dfrac{\log_5 x}{1+\log_5 x} = 1$

Let $\log_5 x = t$

$\therefore\ t^2 + \dfrac{1}{1+t} - \dfrac{t}{1+t} = 1$

$\Rightarrow\ \dfrac{t^2(1+t)+1-t}{1+t} = 1$

$\Rightarrow t^3 + t^2 + 1 - t = 1 + t$

$t^3 + t^2 - 2t = 0$

$t\,(t^2 + t - 2) = 0$

$t\,(t-1)\,(t+2) = 0$

$t = 0, 1, -2$

$\therefore \log_5 x = 0, 1, -2$

$\therefore x = 1, 5, \dfrac{1}{25}$

Therefore, option (*a, b, c*) is the correct answers.

6. The equation $\log_{x^2} 16 + \log_{2x} 64 = 3$ has

(*a*) One irrational solution

(*b*) No prime solution

(*c*) Two real solutions

(*d*) One integral solution

Sol. $\dfrac{4}{2}\log_x 2 + \dfrac{\log_x 64}{\log_x 2x}$

$\Rightarrow\ 2\log_x 2 + \dfrac{6\log_x 2}{1+\log_x 2} = 3$

Let $\alpha = \log_x 2$

$\therefore 2\alpha + \dfrac{6\alpha}{1+\alpha} = 3$

$\Rightarrow 2\alpha + 2\alpha^2 + 6\alpha - 3 - 3\alpha = 0$

$\Rightarrow 2\alpha^2 + 5\alpha - 3 = 0$

$\Rightarrow (\alpha + 3)\,(2\alpha - 1) = 0 \Rightarrow a = -3, 1/2$

$\therefore \log_x 2 = -3 \Rightarrow x = 2^{-1/3}$ (Irrational)

or $\log_x 2 = \dfrac{1}{2} \Rightarrow x = 4$ (Integer)

Therefore, option (*a, b, c, d*) is the correct answers.

7. If $\frac{1}{2} \le \log_{0.1} x \le 2$, then

(*a*) Maximum value of x is $\frac{1}{\sqrt{10}}$

(*b*) x lies between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$

(*c*) Minimum value of x is $\frac{1}{10}$

(*d*) Minimum value of x is $\frac{1}{100}$

Sol. $\frac{1}{2} \le \log_{0.1} x \le 2 \Rightarrow \left(\frac{1}{10}\right)^{1/2} \ge x \ge \left(\frac{1}{10}\right)^2$

Therefore, option (*a*, *b*, *d*) is the correct answers.

8. Let $N = \frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3}$. Then N is

(*a*) a natural number	(*b*) a prime number
(*c*) a rational number	(*d*) an integer

Sol. $= \log_3 135 \log_3 15 - \log_3 5 \log_3 405$

$= \log_3(5 \times 3^3) \cdot \log_3(5 \times 3) - \log_3 5 . \log_3 (5 \times 3^4)$

$= (\log_3 5 + \log_3 3^3)(\log_3 5 + \log_3 3) - \log_3 5 (\log_3 5 + \log_3 3^4)$

$= (x + 3)(x + 1) - x(x + 4)$

{Let $\log_3 5 = x$}

$= x^2 + 4x + 3 - x^2 - 4x = 3$

which is Prime, rational Integer and natural number

Therefore, option (*a*, *b*, *c*, *d*) is the correct answers.

9. If $|x - 5| + |x + 5| = 10$, then

(*a*) The number of integral solutions is 10

(*b*) The number of integral solutions is 11

(*c*) The sum of all the integral solutions is 0

(*d*) All the solutions of the equation are rational numbers

Sol. $|x - 5| + |x + 5| = 10$

Case-I: $x \ge 5$, the equation becomes

$(x - 5) + (x + 5) = 10$

$\Rightarrow 2x = 10$

$\Rightarrow x = 5$ which satisfies the case, therefore accepted.

Case-II: $-5 < x < 5$ The above equation becomes

$-(x - 5) + (x + 5) = 10$

$\Rightarrow -x + 5 + x + 5 = 10$.

$\Rightarrow 10 = 10$ which is true.

So, the solution is $x \in (-5, 5)$

Case-III: $x \le -5$, The above equation becomes

$-(x - 5) - (x + 5) = 10$

$\Rightarrow -x + 5 - x - 5 = 10$

$\Rightarrow -2x = 10$

$\Rightarrow x = -5$ which satisfies the above case so, accepted.

$\therefore$ final answer is $x \in [-5, 5]$

Therefore, option (*b*, *c*) is the correct answers.

10. If $\log_a x = b$ for permissible values of a and x then identify the statements(*s*) which can be correct?

(*a*) If a and b are two irrational numbers then x can be rational.

(*b*) If a rational and b irrational then x can be rational.

(*c*) If a irrational and b rational then x can be rational.

(*d*) If a rational and b rational then x can be rational.

Sol. (*a*) $a = (\sqrt{2})^{\sqrt{2}}$ is irrational

$b = \sqrt{2}$ is also irrational

but $a^b = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = 2$ which is rational $\Rightarrow$ (*a*) is correct.

(*b*) $a = 2 \in Q$; $b = \log_2 3 \notin Q$

$a^b = 2^{\log_2 3} = 3 \in Q \Rightarrow$ (*b*) is correct

Therefore, option (*a*, *b*, *c*, *d*) is the correct answers.

11. Match the column:

Column-I		**Column-II**	
A.	The roots of $\log_2(x + e) = \log_2 x + \log_2 e$ is a	p.	Positive Number
B.	The solution of $\log_{1/5}(2x^2 + 5x + 1) < 0$ contains	q.	Rational Number
C.	$\log_{\sin\frac{\pi}{6}} \pi$ is	r.	Irrational Number
D.	$\log_{10} 5 . \log_{10} 20 + \log_{10}^2 2$ simplifies to	s.	Negative Number

(*a*) A→(p, r) ; B→(p, q, r, s) ; C→(r, s) ; D→(p, q)

(*b*) A→(p, q, r, s) ; B→(p, r) ; C→(r, s) ; D→(p, q)

(*c*) A→(r, s) ; B→(p, q, r, s) ; C→(p, r) ; D→(p, q)

(*d*) A→(p, q) ; B→(p, q, r, s) ; C→(r, s) ; D→(p, r)

Sol. (*A*) $x + e = xe$

$x(e - 1) = e \Rightarrow x = \frac{e}{e - 1}$

(*B*) $2x^2 + 5x + 1 > 1$ and $2x^2 + 5x + 1 > 0$

$\Rightarrow 2x^2 + 5x + 1 > 1$

$\Rightarrow (x)(2x + 5) > 0$

$\Rightarrow x \in \left(-\infty, \frac{-5}{2}\right) \cup (0, \infty)$

(*D*) $(1 - \log_{10} 2)(1 + \log_{10} 2) + \log_{10}^2 2$

$\Rightarrow 1 - \log_{10}^2 2 + \log_{10}^2 2 = 1$

Therefore, option (*a*) is the correct answer.

12. Let $y = \sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16} - \log_2 12 \cdot \log_2 48 + 10.$

Find $y \in N$.

Sol. $y = \sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16} - \log_2 12 \cdot \log_2 48 + 10$

$= \sqrt{\log_2 3 \cdot (2 + \log_2 3)(4 + \log_2 3)(6 + \log_2 3) + 16} - (2 + \log_2 3)(4 + \log_2 3) + 10$

Let us put $\log_2 3 = x$

$$= \sqrt{x(2+x)(4+x)(6+x)+16} - (2+x)(4+x)+10$$

$$= \sqrt{(x^2+6x)(x^2+6x+8)+16} - (x^2+6x+8)+10$$

Put again $x^2 + 6x = \alpha$

$$= \sqrt{\alpha(\alpha+8)+16} - (\alpha+8)+10$$

$$= \sqrt{\alpha^2+8\alpha+16} - (\alpha+8)+10$$

$$= \sqrt{(\alpha+4)^2} - (\alpha+8)+10$$

$$= (\alpha+4)-(\alpha+8)+10 = y = 6.$$

Therefore, 6 is the correct answer.

13. If 'x' and 'y' are real numbers such that,

$2\log(2y-3x) = \log x + \log y$, find $\dfrac{y}{x}$.

(truncated upto two decimal)

Sol. $\log(2y-3x)^2 = \log xy$

$\Rightarrow (2y-3x)^2 = xy$

$\Rightarrow 4y^2 - 12xy + 9x^2 = xy$

Dividing the equation by y^2

$$9\left(\frac{x}{y}\right)^2 - 13\frac{x}{y} + 4 = 0$$

$$\left(\frac{x}{y}-1\right)\left(\frac{9x}{y}-4\right) = 0$$

$$\frac{x}{y} = 1, \frac{x}{y} = \frac{4}{9}.$$

$x = y$ disregarded as for $x = y$, $2y - 3x$ is negative.

Hence $\dfrac{y}{x} = \dfrac{9}{4}$.

Therefore, 2.25 is the correct answer.

14. Sum of all the solutions of the equation

$\log_6(x^2-1) - \log_6\sqrt{(x-6)^2} = \log_6(x+1)^2$ is $a + \sqrt{b}$, $(a, b \in N)$.

Then $a + b$ is equal to

Sol. $\log_6(x^2-1) - \log_6\sqrt{(x-6)^2} = \log_6(x+1)^2$

$$\Rightarrow \log_6\frac{(x-1)(x+1)}{(x+1)^2} = \log_6|x-6|$$

$$\Rightarrow \log_6\left[\frac{(x-1)}{(x+1)}\right] = \log_6|x-6|$$

$$\Rightarrow \frac{x-1}{x+1} = |x-6|$$

Case-I: $x \geq 6$

$\Rightarrow x - 1 = x^2 - 5x - 6$

$\Rightarrow x^2 - 6x - 5 = 0$

$\Rightarrow (x-3)^2 = 14$

$\Rightarrow x = 3 \pm \sqrt{14}$

$x = 3 - \sqrt{14} < 1$ rejected

$x = 3 + \sqrt{14}$ accepted

Case-II: $x < 6$

$x - 1 = -(x^2 - 5x - 6)$

$\Rightarrow x^2 - 4x - 7 = 0$

$(x-2)^2 = 11$

$x = 2 \pm \sqrt{11}$

$x = 2 + \sqrt{11}$ (accepted)

$x = 2 - \sqrt{11}$ (accepted)

Sum of roots $= 7 + \sqrt{14}$

$\Rightarrow a = 7, b = 14$

$a + b = 21$

Therefore, 21 is the correct answer.

"FIVE-MINUTE MEDITATION" BEFORE STUDY SESSIONS

- Before starting a study session, set aside five minutes for a brief meditation. Sit in a quiet space, close your eyes, and focus on your breathing, gently clearing your mind of any worries or distractions.
- This helps center your thoughts and prepares you to focus better during your study session.
- Before you open your books, sit still, take deep breaths, and focus on the sensation of air moving in and out of your body for 5 minutes.

School Level Problems

SINGLE CORRECT TYPE QUESTIONS

1. If p_1 and p_2 are two odd prime numbers such that $p_1 > p_2$, then $p_1^2 - p_2^2$ is
   (*a*) an even number (*b*) an odd number
   (*c*) An odd prime number (*d*) a prime number

2. The smallest number by which $\sqrt{27}$ should be multiplied to get a rational number is
   (*a*) $\sqrt{27}$ (*b*) $3\sqrt{3}$ (*c*) $\sqrt{3}$ (*d*) 3

3. If $a + ib = c + id$, then
   (*a*) $a^2 + c^2 = 0$ (*b*) $b^2 + c^2 = 0$
   (*c*) $b^2 + d^2 = 0$ (*d*) $a^2 + b^2 = c^2 + d^2$

4. If $(\sqrt{5})^5 \times 25^2 = 5^x \times 5\sqrt{5}$, then x is equal to
   (*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

5. If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$, then the value of x is
   (*a*) $\frac{1}{2}$ (*b*) 1 (*c*) 2 (*d*) $\frac{7}{2}$

6. If $\frac{2}{3}$ of A = 75% of B = 0.6 of C, then $A : B : C$ is
   (*a*) 2 : 3 : 3 (*b*) 3 : 4 : 5
   (*c*) 4 : 5 : 6 (*d*) 9 : 8 : 10

7. If $\log_{10}5 + \log_{10}(5x + 1) = \log_{10}(x + 5) + 1$, then x is equal to:
   (*a*) 1 (*b*) 3 (*c*) 5 (*d*) 10

8. If $\log_8 p = 25$ and $\log_2 q = 5$, then
   (*a*) $p = q^{15}$ (*b*) $p^2 = q^3$
   (*c*) $p = q^5$ (*d*) $p^3 = q$

9. If $4x + 3 < 6x + 7$, then x belongs to the interval
   (*a*) $(2, \infty)$ (*b*) $(-2, \infty)$
   (*c*) $(-\infty, 2)$ (*d*) $(-4, \infty)$

VERY SHORT ANSWER TYPE QUESTIONS

10. If $\log_7 2 = m$, then $\log_{49}28$ is

11. $\frac{4}{x+1} \le 3 \le \frac{6}{x+1}$

12. Simplify $\frac{11^{n+3} + 3\times 11^{n+1}}{5\times 11^{n+1} - 11^n \times 3}$

13. Solve modulus and find the interval of x for $|x^2 - 5x + 6|$.

14. If $\frac{x+y}{2} = \frac{y+z}{3} = \frac{z+x}{4}$, then find $x : y : z$.

15. $\sqrt{\left(\frac{5}{7}\right)^{6x-4}} = \frac{2401}{875}$, find value of x

16. If the sum of the first 20 terms of the series
    $\log_{7^{1/2}} x + \log_{7^{1/3}} x + \log_{7^{1/4}} x + \ldots$ is 460, then x is equal to:

LONG ANSWER TYPE QUESTIONS

17. Solve $\frac{2x-1}{3} \ge \frac{3x-2}{4} - \frac{2-x}{5}$

18. Evaluate the following
    $$\frac{1}{2}\log 9 + \frac{1}{4}\log 81 + 2\log 6 - \log 12$$

19. Simplify $\left(6a^{-2}bc^{-3} / 4ab^{-3}c^2\right) \div \left(5a^{-3}b^2c^{-1} / 3ab^{-2}c^3\right)$

20. Solve for x, $|x - 1| - |x - 2| = 10$

21. Solve for x : $\log_2 x - 3\log_{1/2} x = 6$

CASE STUDY BASED QUESTIONS

Case Study-I

Diwali is the biggest festival in India. People give gifts to their loved ones and wish them "Happy Diwali". People give different types of gifts to their loved ones but most of them prefer to give sweets in gifts. Therefore, all sweet shops remain very busy on the Diwali festival in India..

One shopkeeper A prepares 396 Gulab Jamuns and 342 Ras-Gullas. He packs them in combination. Each container consists of either Gulab Jamuns or Ras-Gullas but has an equal number of pieces.

22. Find the number of pieces, Shopkeeper A should put in each box so that number of boxes is the least.
    (*a*) 12 (*b*) 18 (*c*) 20 (*d*) 25

23. There is another sweets shopkeeper B beside shopkeeper A. Shopkeeper B prepares 350 Gulab Jamuns and 400 Ras-Gullas. His packing follows the same rules as of shopkeeper A. Find the number of pieces shopkeeper B should put in each box so that number of boxes is the least.
    (*a*) 20 (*b*) 18 (*c*) 50 (*d*) 5

Case Study-II

In a small island community, the population is growing exponentially. The current population is 10,000, and it is projected to double every 15 years. Additionally, the island's resources can support a maximum of 30,000 people.

24. Calculate the year in which the population is expected to reach its maximum capacity.
    (*a*) 25 years (*b*) 26 years
    (*c*) 27 years (*d*) 23 years

25. Assuming no intervention, calculate the year in which the population will exceed the island's resource capacity.
    (*a*) 24 years (*b*) 23 years
    (*c*) 22 years (*d*) 21 years

Credit - Akshat

ASIC CONCEPTS AND NUMBER SYSTEM

1. Let $x \in Q$, $y \in Q^c$, Which of the following statement is always WRONG?
   (*a*) $xy \in Q^c$
   (*b*) $y/x \in Q$, whenever defined
   (*c*) $\sqrt{2}x + y \in Q$
   (*d*) $x/y \in Q^c$, whenever defined

2. If x and y are two rational numbers such that $(x+y)+(x-2y)\sqrt{2} = 2x - y + (x - y - 1)\sqrt{6}$, then:
   (*a*) $x = 1, y = 1$
   (*b*) $x = 2, y = 1$
   (*c*) $x = 5, y = 1$
   (*d*) x and y can take infinitely many values

3. Which of the following statement is incorrect:
   (*a*) rational number + rational number = rational number
   (*b*) irrational number + rational number = irrational number
   (*c*) integer + rational number = rational number
   (*d*) irrational number + irrational number = Irrational number

4. The number of real roots of the equation $(x-1)^2 + (x-2)^2 + (x-3)^2 = 0$ is:
   (*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

5. If $x - a$ is a factor of $x^3 - a^2x + x + 2$, then 'a' is equal to
   (*a*) 0 (*b*) 2 (*c*) -2 (*d*) 1

6. Every irrational number can be expressed on the number line. This statement is
   (*a*) Always true
   (*b*) Never true
   (*c*) True subject to some condition
   (*d*) None of these

7. The multiplication of a rational number 'x' and an irrational number 'y' is
   (*a*) Always rational
   (*b*) Rational except when $y = \pi$
   (*c*) Always irrational
   (*d*) Irrational except when $x = 0$

8. If x, y are integral solutions of $2x^2 - 3xy - 2y^2 = 7$, then value of $|x + y|$ is
   (*a*) 2 (*b*) 4
   (*c*) 6 (*d*) 2 or 4 or 6

9. If a, b, c are real, then $a(a-b) + b(b-c) + c(c-a) = 0$, only if
   (*a*) $a + b + c = 0$
   (*b*) $a = b = c$
   (*c*) $a = b$ or $b = c$ or $c = a$
   (*d*) $a - b - c = 0$

10. If $2x^3 - 5x^2 + x + 2 = (x-2)(ax^2 - bx - 1)$, then a & b are respectively
    (*a*) 2, 1 (*b*) 2, – 1
    (*c*) 1, 2 (*d*) –1, 1/2

11. The value of $[e] - [-\pi]$ is, where [.] denotes greatest integer function.
    (*a*) 5 (*b*) 6
    (*c*) 7 (*d*) 8

12. If $L = \frac{1}{\sqrt{7}-\sqrt{8}} + \frac{1}{\sqrt{7}-\sqrt{6}} + \frac{1}{3-\sqrt{8}} + \frac{1}{\sqrt{5}+2} + \frac{1}{\sqrt{5}-\sqrt{6}} = 1 + 2\sqrt{a} + 2\sqrt{b}$, then $a \times b$ is equal to
    (*a*) 30 (*b*) 45
    (*c*) 8 (*d*) 0

13. If a, b, c are real and distinct numbers, then the value of $\frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)}$ is
    (*a*) 1 (*b*) $a\,b\,c$
    (*c*) 2 (*d*) 3

14. The remainder obtained when the polynomial $1 + x + x^3 + x^9 + x^{27} + x^{81} + x^{243}$ is divided by $x - 1$ is
    (*a*) 3 (*b*) 5
    (*c*) 7 (*d*) 11

LOGARITHM AND ITS PRINCIPLE PROPERTIES

15. $\frac{1}{1+\log_b a+\log_b c} + \frac{1}{1+\log_c a+\log_c b} + \frac{1}{1+\log_a b+\log_a c}$ has the value equal to
    (*a*) abc (*b*) $\frac{1}{abc}$
    (*c*) 0 (*d*) 1

16. $\log_7 \log_7 \sqrt{7(\sqrt{7\sqrt{7}})} =$
    (*a*) $3 \log_2 7$ (*b*) $1 - 3 \log_3 7$
    (*c*) $1 - 3\log_7 2$ (*d*) $1 - 10 \log_2 7$

17. $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ca}} abc} + \frac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to
    (*a*) 1/2 (*b*) 1
    (*c*) 2 (*d*) 4

18. If $\log_x \log_{18}(\sqrt{2}+\sqrt{8}) = \frac{1}{3}$. Then the value of $1000\,x$ is equal to
    (*a*) 8 (*b*) 1/8
    (*c*) 1/125 (*d*) 125

19. Number of real solutions of the equation

$$\sqrt{\log_{10}(-x)} = \log_{10}\sqrt{x^2} \text{ is:}$$

(*a*) none (*b*) exactly 1

(*c*) exactly 2 (*d*) 4

20. Greatest integer less than or equal to the number $\log_2 15.\log_{1/6} 2.\log_3 1/6$ is

(*a*) 4 (*b*) 3 (*c*) 2 (*d*) 1

21. The ratio $\dfrac{2^{\log_{2^{1/4}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1}$ simplifies to

(*a*) $a^2 - a - 1$ (*b*) $a^2 + a - 1$

(*c*) $a^2 - a + 1$ (*d*) $a^2 + a + 1$

22. If $3^{2\log_3 x} - 2x - 3 = 0$, then the number of values of 'x' satisfying the equation is

(*a*) zero (*b*) 1

(*c*) 2 (*d*) More than 2

23. The number $\log_2 7$ is

(*a*) an integer (*b*) a rational number

(*c*) an irrational number (*d*) a prime number

24. Anti logarithm of 0.75 to the base 16 has the value equal to

(*a*) 4 (*b*) 6 (*c*) 8 (*d*) 12

25. The sum of all the solutions to the equation $2\log x - \log(2x - 75) = 2$:

(*a*) 30 (*b*) 350 (*c*) 75 (*d*) 200

INEQUALITIES

26. If the solution set of the inequality $\log_{\sqrt{0.9}} \log_5 \left(\sqrt{x^2+5} + x\right) > 0$ contains 'n' integral values, then n equals to

(*a*) 7 (*b*) 8 (*c*) 6 (*d*) 10

27. If $\log_{0.5} \log_5 (x^2 - 4) > \log_{0.5} 1$, then '$x$' lies in the interval

(*a*) $\left(-3, -\sqrt{5}\right) \cup \left(\sqrt{5}, 3\right)$ (*b*) $\left(-3, -\sqrt{5}\right) \cup \left(\sqrt{5}, 3\sqrt{5}\right)$

(*c*) $\left(\sqrt{5}, 3\sqrt{5}\right)$ (*d*) ϕ

28. Solution set of the inequality $2 - \log_2(x^2 + 3x) \geq 0$ is:

(*a*) $[-4, 1]$

(*b*) $[-4, -3) \cup (0, 1]$

(*c*) $(-\infty, -3) \cup (1, \infty)$

(*d*) $(-\infty, -4) \cup [1, \infty)$

MODULUS FUNCTION

29. Solutions of $|4x + 3| + |3x - 4| = 12$ are

(*a*) $x = -\dfrac{7}{3}, \dfrac{3}{7}$ (*b*) $x = -\dfrac{5}{2}, \dfrac{2}{5}$

(*c*) $x = -\dfrac{11}{7}, \dfrac{13}{7}$ (*d*) $x = -\dfrac{3}{7}, \dfrac{7}{5}$

30. If $|x^2 - 2x - 8| + |x^2 + x - 2| = 3\,|x + 2|$, then the set of all real values of x is

(*a*) $[1, 4] \cup \{-2\}$ (*b*) $[1, 4]$

(*c*) $[-2, 1] \cup [4, \infty)$ (*d*) $(-\infty, -2] \cup [1, 4]$

31. The complete set of real 'x' satisfying $||x - 1| - 1| \leq 1$ is:

(*a*) $[0, 2]$ (*b*) $[-1, 3]$

(*c*) $[-1, 1]$ (*d*) $[1, 3]$

32. The number of real roots of the equation $|x|^2 - 3|x| + 2 = 0$ is

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

33. Number of real solution (x) of the equation $|x-3|^{3x^2-10x+3} = 1$ is

(*a*) exactly four (*b*) exactly three

(*c*) exactly two (*d*) exactly one

MISCELLANEOUS

34. Simplify: $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

(*a*) 0 (*b*) 1

(*c*) 3 (*d*) 4

35. The expression $x^2 - y^2 - z^2 + 2yz + x + y - z$ has a factor

(*a*) $x + y + z + 1$ (*b*) $-x + y + z$

(*c*) $x + y - z + 1$ (*d*) $x - y + z + 1$

36. Solve the equation $\dfrac{3x^4 + x^2 - 2x - 3}{3x^4 - x^2 + 2x + 3} = \dfrac{5x^4 + 2x^2 - 7x + 3}{5x^4 - 2x^2 + 7x - 3}$

(*a*) $x = 5, 2$ (*b*) $x = 4, 1$

(*c*) $x = 3, 8$ (*d*) $x = 1, 5$

37. If x, y, z are positive real number and a, b, c are rational numbers, then the value of $\dfrac{1}{1 + x^{b-a} + x^{c-a}} + \dfrac{1}{1 + x^{a-b} + x^{c-b}} + \dfrac{1}{1 + x^{b-c} + x^{a-c}}$ is

(*a*) -1 (*b*) 1 (*c*) 0 (*d*) 2

38. If $a^x = \sqrt{b}, b^y = \sqrt[3]{c}$ and $c^z = \sqrt{a}$ then the value of xyz is

(*a*) $\dfrac{1}{2}$ (*b*) $\dfrac{1}{3}$ (*c*) $\dfrac{1}{6}$ (*d*) $\dfrac{1}{12}$

39. If $\dfrac{\log a}{b - c} = \dfrac{\log b}{c - a} = \dfrac{\log c}{a - b}$, then $a^a \cdot b^b \cdot c^c =$

(*a*) 3 (*b*) 1

(*c*) 4 (*d*) 2

40. The number of prime numbers satisfying the inequality $\dfrac{x^2 - 1}{2x + 5} < 3$ is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

Prabal (JEE Main Level)

1. If A & B are two rational numbers and AB, A + B and A – B are rational numbers, then A/B is
   (*a*) Always rational (*b*) Never rational
   (*c*) Rational when $B \neq 0$ (*d*) Rational when $A \neq 0$

2. If $x^{x\sqrt[3]{x}} = \left(x.\sqrt[3]{x}\right)^x$, then $x =$
   (*a*) 1 (*b*) –1
   (*c*) 0 (*d*) 2

3. The equation $4^{(x^2+2)} - 9.2^{(x^2+2)} + 8 = 0$ has the solution
   (*a*) $x = \pm 1$ (*b*) $x = 10$
   (*c*) $x = \pm\sqrt{2}$ (*d*) $x = \sqrt{3}$

4. Logarithm of $32\sqrt[5]{4}$ to the base $2\sqrt{2}$ is
   (*a*) 3.6 (*b*) 5
   (*c*) 5.6 (*d*) 10

5. If $x = \log_a(bc)$, $y = \log_b(ca)$, $z = \log_c(ab)$, then which of the following is equal to 1
   (*a*) $x + y + z$
   (*b*) $(1 + x)^{-1} + (1 + y)^{-1} + (1 + z)^{-1}$
   (*c*) xyz
   (*d*) $x + y - z$

6. The solution of the equation $\log_7 \log_5 \left(\sqrt{x^2+5+x}\right) = 0$.
   (*a*) $x = 2$ (*b*) $x = 3$
   (*c*) $x = 4$ (*d*) $x = -2$

7. The value of $(0.05)^{\log_{\sqrt{20}}(0.1+0.01+0.001+........)}$ is
   (*a*) 81 (*b*) $\frac{1}{81}$
   (*c*) 20 (*d*) $\frac{1}{20}$

8. The value of $\log_2 .\log_3 ....\log_{100} 100^{99^{98^{\cdot^{\cdot^{2^1}}}}}$ is
   (*a*) 0 (*b*) 1 (*c*) 2 (*d*) 100!

9. The number of solution of $\log_2(x + 5) = 6 - x$ is
   (*a*) 2 (*b*) 0 (*c*) 3 (*d*) 1

10. If $\log_{10}2 = 0.30103$, $\log_{10}3 = 0.47712$, the number of digits in $3^{12} \times 2^8$ is
    (*a*) 7 (*b*) 8 (*c*) 9 (*d*) 10

11. Exhaustive set of values of x satisfying $\log_{|x|}(x^2 + x + 1) \geq 0$ is
    (*a*) $(-1, 0)$ (*b*) $(-\infty, -1) \cup (1, \infty)$
    (*c*) $(-\infty, \infty) - \{-1, 0, 1\}$ (*d*) $(-\infty,-1)\cup(-1,0)\cup(1,\infty)$

12. The set of real values of x satisfying $\log_{1/2}\left(x^2 - 6x + 12\right) \geq -2$ is
    (*a*) $(-\infty, 2]$ (*b*) $[2, 4]$
    (*c*) $[4, +\infty)$ (*d*) $[3, 8]$

13. If $\log_{0.04}(x - 1) \geq \log_{0.2}(x - 1)$ then x belongs to the interval
    (*a*) $(1, 2]$ (*b*) $(-\infty, 2]$
    (*c*) $[2, \infty)$ (*d*) $(-\infty, 2)$

14. If $\log_{0.3}(x - 1) < \log_{0.09}(x - 1)$, then x lies in the interval
    (*a*) $(2, \infty)$ (*b*) $(-2, -1)$
    (*c*) $(1, 2)$ (*d*) $(-2, 2)$

15. The minimum value of $f(x) = |x - 1| + |x - 2| + |x - 3|$ is equal to
    (*a*) 1 (*b*) 2 (*c*) 3 (*d*) 0

16. The set of real value(s) of p for which the equation $|2x + 3| + |2x - 3| = px + 6$ has more than two solutions is:
    (*a*) $[0, 4)$ (*b*) $(-4, 4)$
    (*c*) $R - \{4, -4, 0\}$ (*d*) $\{0\}$

17. Let $3^a = 4$, $4^b = 5$, $5^c = 6$, $6^d = 7$, $7^e = 8$ and $8^f = 9$. The value of the product $(abcdef)$ is:
    (*a*) 1 (*b*) 2 (*c*) $\sqrt{6}$ (*d*) 3

18. There are two positive solutions to the equation $\log_{2x}2 + \log_4 2x = -\frac{3}{2}$. The product of these two solution is:
    (*a*) $\frac{1}{32}$ (*b*) $\frac{1}{8}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{21}$

19. Number of real value of x satisfying the equation $\log_2(2x^2 + \sqrt{2}) = \frac{\sqrt{x^2+1}}{x^2+2}$ is:
    (*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

20. Number of values of x satisfying the equation $x^4 = |x|^{\log_2(x^2+12)}$ is:
    (*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

21. The number of zeros after decimal before the start of any significant digit in the number $N = (0.15)^{20}$ are:
    (*a*) 15 (*b*) 16 (*c*) 17 (*d*) 18

22. If $n, k \in N$, then the smallest value of k such that $k + 2k + 3k + ... + 24k = n^3$ is
    (*a*) 100 (*b*) 90 (*c*) 120 (*d*) 60

23. If $a, b, c \in R$ and $a, b, c \neq 0$ such that $\frac{a}{b}+\frac{b}{c}+\frac{c}{a} = 6$ and $\frac{b}{a}+\frac{c}{b}+\frac{a}{c} = 8$ then $\frac{a^3}{b^3}+\frac{b^3}{c^3}+\frac{c^3}{a^3} - 3$ is equal to
    (*a*) 81 (*b*) 48 (*c*) 72 (*d*) 84

Credit - Akshat

24. If $3^x = 5^y$, then how many ordered pairs $(x, y), x, y \in R$ satisfy the given equation?

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) Infinite

25. The value of $x + y + z$ satisfying the system of equations

$\log_2 x + \log_4 y + \log_4 z = 2$ is

$\log_3 y + \log_9 z + \log_9 x = 2$

$\log_4 z + \log_{16} x + \log_{16} y = 2$

(*a*) $\frac{175}{12}$ (*b*) $\frac{349}{24}$ (*c*) $\frac{353}{24}$ (*d*) $\frac{112}{3}$

26. Which is the correct order for a given number α, $\alpha > 1$

(*a*) $\log_2 \alpha < \log_3 \alpha < \log_e \alpha > \log_{10} \alpha$

(*b*) $\log_{10} \alpha < \log_3 \alpha < \log_e \alpha > \log_2 \alpha$

(*c*) $\log_{10} \alpha < \log_e \alpha < \log_2 \alpha > \log_3 \alpha$

(*d*) $\log_e \alpha < \log_3 \alpha < \log_2 \alpha > \log_{10} \alpha$

27. The smallest integral value of x such that $\sqrt{x+2} - \sqrt{x-2} < \frac{1}{10}$ is

(*a*) 400 (*b*) 20 (*c*) 401 (*d*) 399

28. $10^{\log_r(\log_q(\log_p x))} = 1$ and $\log_q(\log_r(\log_p x)) = 0$ then 'p' equals

(*a*) $r^{q/r}$ (*b*) rq (*c*) 1 (*d*) $r^{r/q}$

29. Which one of the following is the smallest?

(*a*) $\log_{10}\pi$ (*b*) $\sqrt{\log_{10} \pi^2}$

(*c*) $\left(\frac{1}{\log_{10} \pi}\right)^3$ (*d*) $\left(\frac{1}{\log_{10} \sqrt{\pi}}\right)$

30. The set of all solutions of the inequality $(1/2)^{x^2-2x} < 1/4$ contains the set

(*a*) $(-\infty, 0)$ (*b*) $(-\infty, 1)$ (*c*) $(1, \infty)$ (*d*) $(3, \infty)$

31. The value of b satisfying the equation,

$\log_e 2 \cdot \log_b 625 = \log_{10} 16 \cdot \log_e 10$ is

(*a*) 5 (*b*) 7 (*c*) 9 (*d*) 10

32. The solution set of the system of equation

$\log_3 x + \log_3 y = 2 + \log_3 2$ and $\log_{27}(x + y) = \frac{2}{3}$ is:

(*a*) $\{6, 3\}$ (*b*) $\{9, 6\}$

(*c*) $\{6, 12\}$ (*d*) $\{12, 6\}$

33. Which of the following statements are true?

(*a*) $\log_2 3 < \log_{12} 10$ (*b*) $\log_6 5 < \log_7 8$

(*c*) $\log_3 26 > \log_2 9$ (*d*) $\log_{16} 15 > \log_{10} 11 > \log_7 6$

34. If $x + y = a$ and $x^2 + y^2 = b$, then the value of $(x^3 + y^3)$ is

(*a*) ab (*b*) $a^2 + b$

(*c*) $a + b^2$ (*d*) $\frac{3ab - a^3}{2}$

35. If $x + y + z = 0$, then a factor of the expression $(x + y)^3 + (y + z)^3 + (z + x)^3$ is

(*a*) $3(x + y)(y + z)(z + x)$ (*b*) $3xyz$

(*c*) $(x + y - z)$ (*d*) $(x - y + z)$

36. The number of real solution/s of the equation $9^{\log_3(\log_e x)} = \log_e x - (\log_e x)^2 + 1$ is:

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

37. The set of all the solutions of the inequality $\log_{1-x}(x - 2) \geq -1$ is

(*a*) $(-\infty, 0)$ (*b*) $(2, \infty)$

(*c*) $(-\infty, 1)$ (*d*) ϕ

38. The complete solution of $\frac{x^2 - 1}{x + 3} \geq 0$ & $x^2 - 5x + 2 \leq 0$ is:

(*a*) $x \in \left[\frac{5-\sqrt{17}}{2}, \frac{5+\sqrt{17}}{2}\right]$ (*b*) $x \in \left[1, \frac{5+\sqrt{17}}{2}\right]$

(*c*) $x \in (-3, -1]$ (*d*) $(-3, -1) \cup [1, \infty)$

39. The number of the integral solutions of $x^2 + 9 < (x + 3)^2 < 8x + 25$ is:

(*a*) 1 (*b*) 3 (*c*) 4 (*d*) 5

40. Number of non-negative integral values of x satisfying the inequality $\frac{2}{x^2 - x + 1} - \frac{1}{x + 1} - \frac{2x - 1}{x^3 + 1} \geq 0$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

41. The solution set of inequality $\log_{(3x^2+1)} 2 < \frac{1}{2}$

(*a*) $|x| > 1$ (*b*) $|x| < 1$

(*c*) ϕ (*d*) None of these

42. The solution set of $\log 4 + \left(1 + \frac{1}{2x}\right)\log 3 = \log\left((3)^{1/x} + 27\right)$

(*a*) $(1/4, 1/2)$ (*b*) $\{1/4, 1/2\}$

(*c*) $(1/4, 1/2]$ (*d*) None of these

43. The solution set of $\log_7\left(\frac{2x - 6}{2x - 1}\right) > 0$

(*a*) $\left(-\infty, \frac{1}{2}\right)$ (*b*) $\left(-\infty, \frac{1}{2}\right]$

(*c*) $\left(-\frac{1}{2}, \frac{1}{2}\right]$ (*d*) None of these

44. If $\log_{x-3}(2x - 3)$ is a meaningful quantity then find the interval in which x must lie.

(*a*) $x \in (3, 4] \cup (4, \infty)$ (*b*) $x \in [3, 4) \cup (4, \infty)$

(*c*) $x \in (3, 4) \cup (4, \infty)$ (*d*) None of these

45. $\frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3} =$

(*a*) $(a + b)(b + c)(c + a)$ (*b*) 1

(*c*) 3 (*d*) None of these

INTEGER TYPE QUESTIONS

46. Suppose $x, y, z > 0$ and different than one and $\ln x + \ln y + \ln z = 0$. If $\frac{1}{e^k} = x^{\frac{1}{\ln y} + \frac{1}{\ln z}} \cdot y^{\frac{1}{\ln z} + \frac{1}{\ln x}} \cdot z^{\frac{1}{\ln x} + \frac{1}{\ln y}}$. The $k =$

47. If $\log_2(\log_3(\log_4(x))) = 0$ and $\log_3(\log_4(\log_2(y))) = 0$ and $\log_4(\log_2(\log_3(z))) = 0$ then the sum of x, y and z is

48. Let $\log_2 x + \log_4 x + \log_8 x = \log_k x$ for all $x \in R^+$. If $k = \sqrt[b]{a}$ where $a, b \in N$ then find the smallest positive value of $(a+b)$.

49. Find the value of the expression
$$6\left(\frac{2}{\log_4(2000)^6} + \frac{3}{\log_5(2000)^6}\right).$$

50. If $N = 7^{\log_{49} 900}$, $A = 2^{\log_2 4} + 3^{\log_2 4} + 4^{\log_2 2} - 4^{\log_2 3}$
$D = (\log_5 49)(\log_7 125)$
Find $P = \log_{\left(A - \frac{N}{10}\right)} |N + A + D + 6| - \log_5 2$.

51. If $a + b + c = 1$, $a^2 + b^2 + c^2 = 9$, $a^3 + b^3 + c^3 = 1$, then find value of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$.

52. If $a + b + c = 10$ and $ab + bc + ac = 20$ then find the value of $a^3 + b^3 + c^3 - 3abc$

53. If $(a-b)^3 + (b-c)^3 + (c-a)^3 = p(a-b)(b-c)(c-a)$, then $p =$

54. The value of $\sqrt{5\sqrt{5\sqrt{5\sqrt{5\ldots\ldots\ldots}}}}$ is

55. If $x = \sqrt[3]{7+5\sqrt{2}} - \frac{1}{\sqrt[3]{7+5\sqrt{2}}}$, then the value of $x^3 + 3x - 14$ is equal to

56. If $\left[\left\{\left(\frac{1}{7^2}\right)^{-2}\right\}^{-1/3}\right]^{1/4} = 7^x$ then $-3x =$

57. Number of cyphers after decimal before a significant figure starts in $\left(\frac{5}{4}\right)^{-100}$ is equal to [Use: $\log_{10}2 = 0.3010$]

58. Number of real solution of $\log_5 [2 + \log_3 (x + 3)] = 0$ is

59. If $4^A + 9^B = 10^C$, where $A = \log_{16}4$, $B = \log_3 9$ & $C = \log_x 83$, then find x.

Parikshit (JEE Advanced Level)

SINGLE CORRECT TYPE QUESTIONS

1. The expression $\log_p \log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\ldots\ldots\ldots\sqrt[p]{p}}}}}_{n \text{ radical sign}}$, where $p \geq 2$, $p \in N$; $n \in N$ when simplifies is
(*a*) Independent of p
(*b*) Independent of p and of n
(*c*) Dependent on both p and n
(*d*) Positive

2. The solution set of the inequality $\log_{\sin\left(\frac{\pi}{3}\right)}(x^2 - 3x + 2) \geq 2$ is
(*a*) $\left(\frac{1}{2}, 2\right)$ (*b*) $\left(1, \frac{5}{2}\right)$
(*c*) $\left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$ (*d*) $(1, 2)$

3. If $\sqrt{\log_4\{\log_3\{\log_2(x^2 - 2x + a)\}\}}$ is defined $\forall\, x \in R$, then the set of values of 'a' is
(*a*) $[9, \infty)$ (*b*) $[10, \infty)$ (*c*) $[15, \infty)$ (*d*) $[2, \infty)$

4. Number of integers, which satisfy the inequality, $\frac{(16)^{1/x}}{(2^{x+3})} > 1$ is equal to:
(*a*) Infinite (*b*) Zero (*c*) 1 (*d*) 4

5. The solution set of inequality $\frac{(3^x - 4^x) \cdot \ln(x+2)}{x^2 - 3x - 4} \leq 0$ is
(*a*) $(-\infty, 0] \cup (4, \infty)$ (*b*) $(-2, 0] \cup (4, \infty)$
(*c*) $(-1, 0] \cup (4, \infty)$ (*d*) $(-2, -1) \cup (-1, 0] \cup (4, \infty)$

6. Number of values of x in the interval $(0, 5)$ satisfying the equation $\frac{\ln(\sqrt{\sqrt{x^2+1}+x}) + \ln(\sqrt{\sqrt{x^2+1}-x})}{\ln x} = x$, is
(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 0

7. If $A = \log_{\sqrt{5}}\left(\left(5^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2} \cdots 10 \text{ times}}$, then value of $\log_{\sqrt{3}}(1024A + 1)$, is equal to
(*a*) 1 (*b*) 3 (*c*) 5 (*d*) 2

8. The number of positive integers not satisfying the inequality $\log_2(4^x - 2.2^x + 17) > 5$.
(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 1

9. If a, b, c are positive real numbers such that
$a^{\log_3 7} = 27; b^{\log_7 11} = 49$ and $c^{\log_{11} 25} = \sqrt{11}$.
The value of $\left(a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}\right)$ equals
(*a*) 489 (*b*) 469 (*c*) 464 (*d*) 400

10. Find the value of x that satisfies the equation
$$\log\left(\frac{x^{1/x}}{x^{1/(x+1)}}\right) = \frac{1}{5050}$$
(*a*) 1 (*b*) 10 (*c*) 100 (*d*) 1000

11. Number of positive solution which satisfy the equation $\log_2 x \cdot \log_4 x \cdot \log_6 x = \log_2 x \cdot \log_4 x + \log_2 x \cdot \log_6 x + \log_4 x \cdot \log_6 x$?
(*a*) 0 (*b*) 1 (*c*) 2 (*d*) infinite

12. Sum of all the real solutions of the inequality

$$\frac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x^2-9)} \le 0 \text{ is}$$

(*a*) 5 (*b*) 4 (*c*) 8 (*d*) 0

13. The set of values of x satisfying simultaneously the inequalities $\dfrac{\sqrt{(x-8)(2-x)}}{\log_{0.1}\left(\dfrac{10}{7}(\log_2 5-1)\right)} \ge 0$ and $2^{x-3}-31>0$ is:

(*a*) singleton set

(*b*) an empty set

(*c*) an infinite set

(*d*) a set consisting of exactly two elements.

14. If $|x-3|^2-6|x-3|-23<4$, then

(*a*) $x \in (-12, 6)$

(*b*) 17 integers satisfy the inequality

(*c*) 11 non-negative integers satisfy the inequality

(*d*) 6 negative integers satisfy the inequality

MULTIPLE CORRECT TYPE QUESTIONS

15. The roots of the equation $|x| = 49^{\left(\frac{1}{2}+\log_{\frac{1}{7}}27+\log_{343}81\right)}$ include

(*a*) One positive number greater than 1 only

(*b*) Two real number

(*c*) Two irrational number

(*d*) One negative rational number

16. Which of the following is true?

(*a*) $(\log_{10}2)^2+1>\log_{10}4$

(*b*) $\log_{10}90>\log_5 50$

(*c*) $\log_4\log_3\log_2 16>\log_{16}4$

(*d*) $2(\log_{10}3)^2-3(\log_{10}2)^2>(\log_{10}2)\times(\log_{10}3)$

17. Indicate all correct alternatives, where base of the log is 2.

The equation $x^{\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}}=\sqrt{2}$ has:

(*a*) At least one real solution

(*b*) Exactly three real solutions

(*c*) Exactly one irrational solution

(*d*) Imaginary roots

18. The equation $x^{\left[(\log_3 x)^2-\frac{9}{2}\log_3 x+5\right]}=3\sqrt{3}$ has

(*a*) Exactly three real solution

(*b*) At least one real solution

(*c*) Exactly one irrational solution

(*d*) Complex roots

19. Solution set of the inequality

$$(\log_2 x)^4-\left(\log_{1/2}\frac{x^3}{8}\right)^2+9\log_2\left(\frac{32}{x^2}\right)<4\left(\log_{1/2}x\right)^2 \text{ is}$$

$(a, b)\cup(c, d)$ then the correct statement is

(*a*) $a=2b$ and $d=2c$

(*b*) $b=2a$ and $d=2c$

(*c*) $\log_c d=\log_b a$

(*d*) there are 4 integers in (c, d)

20. Choose the correct from the following

(*a*) $\dfrac{81^{\frac{1}{\log_5 9}}+3^{\frac{3}{\log_{\sqrt{6}}3}}}{409}\left((\sqrt{7})^{\frac{2}{\log_{25}7}}-(125)^{\log_{25}6}\right)=1$

(*b*) $5^{\log_{1/5}\frac{1}{2}}+\log_{\sqrt{2}}\left(\dfrac{4}{\sqrt{7}+\sqrt{3}}\right)+\log_{1/2}\left(\dfrac{1}{(10+2\sqrt{21})}\right)=6$

(*c*) $\sqrt{10^{2+\frac{1}{2}\log(16)}}=20$

(*d*) None of these

21. Choose the correct from the following

(*a*) $\log_2(\log_{1/2}(x))<2$, for all $x\in\left(\dfrac{1}{16}, 1\right)$

(*b*) $\log_{1/2}(\log_3(x))>3$, for all $x\in(1, 3^{1/8})$

(*c*) $(\log_2(x)-1)(\log_3(x)-2)\le 0$, for all $x\in[2, 9]$

(*d*) $(\log_2(x)-1)(\log_{1/2}(x)-2)\le 0$, for all $x\in\left(0, \dfrac{1}{4}\right]\cup[2,\infty)$

22. The solution set of the system of equations $\log_3 x+\log_3 y=2+\log_3 2$ and $\log_{27}(x+y)=\dfrac{2}{3}$ is

(*a*) (6, 3) (*b*) (3, 6) (*c*) (6, 12) (*d*) (12, 6)

23. Consider the quadratic equation, $(\log_{10}8)x^2-(\log_{10}5)x=2(\log_2 10)^{-1}-x$. Which of the following quantities are irrational.

(*a*) sum of the roots

(*b*) product of the roots

(*c*) sum of the coefficients

(*d*) discriminant

COMPREHENSION BASED QUESTIONS

Comprehension (Q. No. 24 to 26): Let α and β are the solutions of the equation $\left(\sqrt{x}\right)^{\log_5 x-1}=5$ where $\alpha\in I$ and $\beta\in Q$ Then [**Use:** $\log_{10}2=0.3010$, $\log_{10}3=0.4771$]

24. The number of significant digits before decimal in $(\alpha)^{10}$ is

(*a*) 13 (*b*) 14

(*c*) 15 (*d*) None of these

25. Number of zeros after decimal before a significant digit in $(\beta)^{10}$ is

(*a*) 5 (*b*) 7 (*c*) 8 (*d*) 6

26. The value of $(\beta)^{\log_{25} 9}$ is

(*a*) $\frac{1}{3}$ (*b*) 5 (*c*) $\frac{1}{5}$ (*d*) 9

MATCH THE COLUMN TYPE QUESTIONS

27. Match the Column:

	Column-I		Column-II
A.	The value(s) of x, which does not satisfy the equation $\log_2^2(x^2 - x) - 4\log_2(x-1)\log_2 x = 1$, is (are)	p.	2
B.	The value of x satisfying the equation $2^{\log_2 \text{[illegible]}} = 13$, is	q.	3
C.	The number $N = \left(\frac{1}{\log_2 \pi} + \frac{1}{\log_6 \pi}\right)$ is less than	r.	4
D.	Let $l = (\log_3 4 + \log_2 9)^2 - (\log_3 4 - \log_2 9)^2$ and $m = (0.8)\left(1 + 9^{\log_3 8}\right)^{\log_{65} 5}$ then $(l + m)$ is divisible by	s.	5
		t.	6

(*a*) A → r, t, s; B → q; C → r, s, q; D → q, r

(*b*) A → q, r, s, t; B → p; C → q, r; D → r, s

(*c*) A → q, r, s, t; B → p; C → q, r, s, t; D → p, r, s

(*d*) A → t; B → s; C → q, t; D → r, s

28. Match the column:

	Column-I		Column-II
A.	$\log_{\sin 30^\circ}(\cos 60^\circ) + 1$	p.	3
B.	$\log_{4/3}(1.\bar{3}) + 3$	q.	5
C.	$\log_{2-\sqrt{3}}(2+\sqrt{3}) + 6$	r.	4
D.	$\log_{\tan 20^\circ}\tan 70^\circ + 4$	s.	2
E.	$\log_{\cot 40^\circ}\tan 50^\circ$	t.	0
F.	$\log_{0.125}(8) + 8$	u.	–1
G.	$\log_{1.5}(0.\bar{6}) + 9$	v.	8
H.	$\log_{2.25}(0.\bar{4})$	w.	7
I.	$\log_{10}(0.\bar{9})$	x.	1

(*a*) A → q, B → p, C → s, D → v, E → u, F → u, G → q, H → w, I → x

(*b*) A → s, B → r, C → q, D → p, E → x, F → w, G → v, H → u, I → t

(*c*) A → s, B → v, C → t, D → p, E → t, F → u, G → w, H → x, I → w

(*d*) A → q, B → s, C → r, D → v, E → u, F → v, G → v, H → w, I → x

29. Match the following columns:

	Column-I		Column-II
A.	If $a = 3\left(\sqrt{8+2\sqrt{7}} - \sqrt{8-2\sqrt{7}}\right)$, $b = \sqrt{(42)(30)+36}$ then the value of $\log_a b$ is equal to	p.	–1
B.	If $a = \sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}$, $b = \sqrt{(42)(30)+36}$ then the value of $\log_a b$ is equal to	q.	1
C.	If $a = \sqrt{3+2\sqrt{2}}, b = \sqrt{3-2\sqrt{2}}$ then the value of $\log_a b$ is equal to	r.	2
D.	If $a = \sqrt{7+\sqrt{7^2-1}}, b = \sqrt{7-\sqrt{7^2-1}}$, then the value of $\log_a b$ is equal to	s.	$2 + 2\log_2 3$

(*a*) A → s, B → p, C → q, D → p

(*b*) A → r, B → p, C → r, D → p

(*c*) A → r, B → s, C → p, D → p

(*d*) A → p, B → q, C → p, D → r

30. Match the columns:

	Column-I		Column-II
A.	If $p = \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$ then $\log_{(5+2\sqrt{6})} p$ is	p.	0
B.	If $r = \frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ then $\log_{9+2\sqrt{15}}(1/r)$ is	q.	2
C.	If $t = \frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$ then $\log_{\sqrt{3}} t^2$ is	r.	–1
D.	If $k = \frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$ then $\log_e(k+1)$ is	s.	1

(*a*) A → t, B → s, C → r, D → q

(*b*) A → s, B → r, C → q, D → p

(*c*) A → r, B → p; C → r, D → s

(*d*) A → t, B → q, C → s, D → r

31. Let $f(x)=\dfrac{x^2-6x+5}{x^2-5x+6}$

Column I		Column II	
A.	If $-1<x<1$, then $f(x)$ satisfies	p.	$0<f(x)<1$
B.	If $1<x<2$, then $f(x)$ satisfies	q.	$f(x)<0$
C.	If $3<x<5$, then $f(x)$ satisfies	r.	$f(x)>0$
D.	If $x>5$, then $f(x)$ satisfies	s.	$f(x)<1$

(*a*) A → p, B → q, C → q, D → s

(*b*) A → q, B → s, C → r, D → q

(*c*) A → s, B → r; C → q, D → s

(*d*) A → p, B → q, C → s, D → r

32. Match the values of x given in Column-II satisfying the exponential equation given in Column-I (Do not verify). Remember that for $a>0$, the terms a^x is always greater than zero $\forall\, x \in R$.

Column-I		Column-II	
A.	$5^x-24=\dfrac{25}{5^x}$	p.	-3
B.	$(2^{x+1})(5^x)=200$	q.	-2
C.	$4^{2/x}-5(4^{1/x})+4=0$	r.	-1
D.	$2^{2x+1}-33(2^{x-1})+4=0$	s.	0
E.	$\dfrac{2^{x-1}\cdot 4^{x+1}}{8^{x-1}}=16$	t.	1
F.	$3^{2x+1}+10.(3^x)+3=0$	u.	2
G.	$64(9^x)-84(12^x)+27(16^x)=0$	v.	3
H.	$5^{2x}-7^x-5^{2x}(35)+7^x(35)=0$	w.	None

(*a*) A → u, B → u, C → t, D → q, v, E → p,q,r,s,t,u,v F → w, G → t,u, H → s

(*b*) A → q, B → t, C → u, D → v, E →t,u,v F → w, G → t,H → q

(*c*) A → p, B → u, C → s, D → q, E → p,q, F → s, G → u, H → q

(*d*) A → q, B → s, C → r, D → s, E →q,r, F → q, G → u, H → q

INTEGER TYPE QUESTIONS

33. Find the number of integral solution of the equation $\log_{\sqrt{x}}(x+|x-2|)=\log_x(5x-6+5|x-2|)$.

34. If a, b are co-prime numbers and satisfying

$$(2+\sqrt{3})^{\frac{1}{\log_a(2-\sqrt{3})}+\frac{1}{\log_b\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)}}=\frac{1}{12},$$ then $(a+b)$ is equal to

35. The sum of all integral values of x satisfying the equation $2\log_8(2x)+\log_8(x^2-2x+1)=\dfrac{4}{3}$ is.

36. If the complete solution set of the inequality $(\log_{10}x)^2 \geq \log_{10}x+2$ is $(0, a] \cup \left[\dfrac{1}{a^2},\infty\right)$ then find the value of $10a$.

37. If complete solution set of inequality $\log_{1/2}(x+5)^2 > \log_{1/2}(3x-1)^2$ is $(-\infty, p)\cup(q, r)\cup(s,\infty)$ then find $3\left(\dfrac{p^2+q^2+r^2}{s^2}\right)$

38. Solve the equation $x^{0.5\log_{\sqrt{x}}(x^2-x)}=3^{\log_9 4}$.

39. If the solution set of $(0.3)^{\log_{\frac{1}{3}}\log_2\frac{3x+6}{x^2+2}}>1$ is $\left(\dfrac{-1}{\alpha},\alpha\right)$ then α =

40. If the solution set of $\log_{0.5}\left(\log_6\dfrac{x^2+x}{x+4}\right)<0$ is $(\alpha, \beta)\cup(-2\alpha,\infty)$ then $-\alpha+\beta=$

41. If the solution set of $\log_3\dfrac{|x^2-4x|+3}{x^2+|x-5|}\geq 0$ is $\left(-\infty,-\dfrac{\alpha}{\beta}\right]\cup\left[\dfrac{1}{\alpha},\alpha\right]$, then $\alpha\beta=$

42. For the equation
$(0.4)^{\log^2 x+1}=(6.25)^{2-p\log x}$
(base 10)
If $p=2$, let number of real roots be m,
If $p=3$, let number of real roots be n,
Then $m+n=$

43. If p is the smallest value of x satisfying the equation $2^x+\dfrac{15}{2^x}=8$ then the value of 4^p is equal to

44. Positive numbers x, y and z satisfy $xyz=10^{81}$ and $(\log_{10}x)(\log_{10}yz)+(\log_{10}y)(\log_{10}z)=468$.
Find the value of
$(\log_{10}x)^2+(\log_{10}y)^2+(\log_{10}z)^2$

45. If (x_1, y_1) and (x_2, y_2) are the solution of the system of equation
$\log_{225}(x)+\log_{64}(y)=4$
$\log_x(225)-\log_y(64)=1$,
then find the value of $\log_{30}(x_1y_1x_2y_2)$.

46. Suppose n be an integer greater than 1. Let $a_n=\dfrac{1}{\log_n 2002}$. Suppose $b=a_2+a_3+a_4+a_5$ and $c=a_{10}+a_{11}+a_{12}+a_{13}+a_{14}$. Then find the value of $(c-b)$.

47. If $\log_b a.\log_c a+\log_a b.\log_c b+\log_a c.\log_b c=3$ (where a, b, c are different positive real numbers ≠ 1), then find the value of $a\,b\,c$.

48. If the product of alll solutions of the equation $\dfrac{(2009)x}{2010}=(2009)^{\log_x(2010)}$ can be expressed in the lowest form as $\dfrac{m}{n}$ then the value of $(m-n)$ is

49. The complete solution set of the inequality $\frac{1}{\log_4 \frac{x+1}{x+2}} < \frac{1}{\log_4(x+3)}$, is $(-a, \infty)$, then determine 'a'.

50. Consider the system of equations

$$\log_{10}(2000xy) - \log_{10}x \cdot \log_{10}y = 4$$
$$\log_{10}(2yz) - \log_{10}y \cdot \log_{10}z = 1$$

and $\log_{10}(zx) - \log_{10}z \cdot \log_{10}x = 0$,

$x, y, z \le 90$ then $x + y + z =$

51. Number of solutions of the equation:

$\log^2(4-x) + \log(4-x) \cdot \log\left(x+\frac{1}{2}\right) - 2\log^2\left(x+\frac{1}{2}\right)$ are

52. If x and y satisfying both the equations

$\log_3 x + \log_2 y = 2; 3^x - 2^y = 23$

then sum of all the values of x and y is

53. If the product of the positive roots of the equation $\sqrt{2005}(x)^{\log_{2005} x} = x^2$ is p^q then $p + q =$

54. The solution set of

$\log_3(\sqrt{x} + |\sqrt{x} - 1|) = \log_9(4\sqrt{x} - 3 + 4|\sqrt{x} - 1|)$ is

$[\alpha, \beta] \cup \{\gamma\}$, then $\gamma^{\alpha+\beta} =$

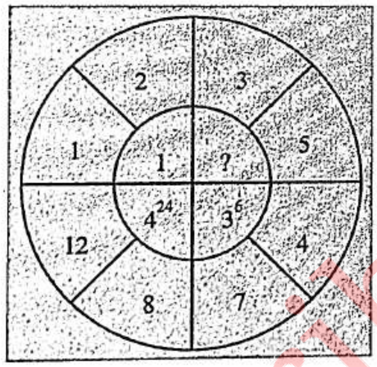
PYQ's (Past Year Questions)

INEQUALITIES

1. Let the point $(p, p+1)$ lie inside the region $E = \left\{(x, y): 3 - x \le y \le \sqrt{9 - x^2}, 0 \le x \le 3\right\}$. If the set of all values of p is the interval (a, b), then $b^2 + b - a^2$ is equal to __________ **[6 April, 2023 (Shift-I)]**

2. The missing value in the following figure is

[18 Mar, 2021 (Shift-I)]



Use the logic which gives answer in single digit.

3. The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is **[10 April, 2019 (Shift-II)]**

(*a*) 3 (*b*) 2 (*c*) 4 (*d*) 1

LOGARITHM

4. The number of integral solutions x of

$\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \ge 0$ is **[11 April, 2023 (Shift-I)]**

(*a*) 6 (*b*) 8

(*c*) 5 (*d*) 7

5. If the sum of all the roots of the equation $e^{2x} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$ is $\log_e p$, then p is equal to __________. **[27 June, 2022 (Shift-I)]**

6. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is **[26 Feb, 2021 (Shift-I)]**

7. If for $x \in \left(0, \frac{\pi}{2}\right)$ $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1), n > 0$, then the value of n is equal to: **[16 March, 2021 (Shift-I)]**

(*a*) 16 (*b*) 12 (*c*) 9 (*d*) 20

8. The inverse of $y = 5^{\log x}$ is: **[17 March, 2021 (Shift-I)]**

(*a*) $x = y^{\log 5}$ (*b*) $x = y^{\frac{1}{\log 5}}$ (*c*) $x = e^{\log_5 y}$ (*d*) $x = 5^{\frac{1}{\log y}}$

9. The sum of the roots of the equation, **[31 Aug, 2021 [Shift-II]**

$x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$, is:

(*a*) $\log_2 12$ (*b*) $\log_2 13$

(*c*) $\log_2 11$ (*d*) $\log_2 14$

10. The number of solutions of the equation $\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x+1)^2 - 4 = 0, x > 0$, is. **[20 July, 2021 (Shift-II)]**

11. The number of distinct solutions of the equation, $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$ in the interval $[0, 2\pi]$, is __________ **[9 Jan, 2020 (Shift-I)]**

12. Let m be the minimum possible value of $\log_3(3^{y_1} + 3^{y_2} + 3^{y_3})$, where y_1, y_2, y_3 are real numbers for which $y_1 + y_2 + y_3 = 9$. Let M be the maximum possible value of $(\log_3 x_1 + \log_3 x_2 + \log_3 x_3)$, where x_1, x_2, x_3 are positive real numbers for which $x_1 + x_2 + x_3 = 9$. Then the value of $\log_2(m^3) + \log_3(M^2)$ is _____. **[JEE Adv, 2020]**

PW Challengers

1. If $\log_4(x+2y)+\log_4(x-2y)=1$, then the minimum value of $|x|-|y|$ is ____.

2. Let a,b,c,d be positive integers and $\log_a b=\frac{3}{2}, \log_c d=\frac{5}{4}$. If $a-c=9$, then $b-d=$

3. Let $x \in N$ such that $2^{1+[\log_2(x-2)]}-x=20$. ([·] is G.I.F.) The smallest value of x, is

4. If $\sqrt{4+\sqrt{8-\sqrt{32+\sqrt{768}}}}=a\sqrt{2}\cos\left(\frac{11\pi}{b}\right)$, where a and b are natural numbers then find $a+b$.

5. Let $r_1, r_2, r_3 \ldots r_n$ be n positive integers, not necessarily distinct, such that $(x+r_1)(x+r_2)(x+r_1)\ldots(x+r_n)$ $=x^n+56x^{n-1}\ldots+2009$ then the value of n is equal to

6. If $(a+1)(b+1)(c+1)(d+1)=1$
$(a+2)(b+2)(c+2)(d+2)=2$
$(a+3)(b+3)(c+3)(d+3)=3$
$(a+4)(b+4)(c+4)(d+4)=4$
Then the value of $(a+5)(b+5)(c+5)(d+5)$ is equal to.

7. Find sum of all possible natural numbers 'n' for which $\frac{5n^2-7n+84}{n}$ is divisible by 5.

8. The value of

$$\left[2008+\log_{\left(\frac{6561}{256}\right)}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots}}}\right)\right]$$ is

(where [·] is G.I.F.)

9. Let a, b and c be distinct non zero real numbers such that $\frac{1-a^3}{a}=\frac{1-b^3}{b}=\frac{1-c^3}{c}$. The value of $10(a^3+b^3+c^3)$, is

10. Match the Column:

	Column-I		Column-II
A.	Number of integral pair of the form (x, y) satisfying $\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{y}}=\frac{1}{\sqrt{20}}$ is/are equal to	p.	16
B.	Number of positive integral solutions of the equation $3x+5y=1008$ is/are equal to	q.	2
C.	Number of integers n such that $\sqrt{\frac{3n-5}{n+1}}$ is also an integer, is/are equal to	r.	0
D.	Number of integers n (positive, negative or 0) such that n^2+73 is divisible by $(n+73)$, is/are equal to	s.	67
		t.	3

(a) A → t; B → s; C → q; D → p
(b) A → r; B → p; C → q; D → s
(c) A → q; B → p; C → r; D → s
(d) A → s; B → p; C → q; D → p

11. $\sqrt[3]{20+14\sqrt{2}}+\sqrt[3]{20-14\sqrt{2}}=a$ then find the absolute value of $a-2023$.

12. If the sum of all real numbers x and y such that the following system of inequalities holds:

$$\begin{cases} 4^{-x}+27^{-y}=\frac{5}{6} \\ \log_{27}y-\log_4 x\geq\frac{1}{6} \\ 27^y-4^x\leq 1 \end{cases}$$ is k then find 6 k.

13. Solve the inequality $|\log_2 x-3|+|2^x-8|\geq 9$.
(a) $x\in(0,1]\cup[4,\infty]$
(b) $x\in(0,2]\cup[4,\infty]$
(c) $x\in(0,1]\cup[3,\infty]$
(d) $x\in(0,1]\cup[2,\infty]$

14. Solve the inequality
$$\log_2(x^{12}+3x^{10}+5x^8+3x^6+1)<1+\log_2(x^4+1)$$
(a) $x\in\left(-\sqrt{\frac{-1+\sqrt{6}}{2}},\sqrt{\frac{-1+\sqrt{5}}{2}}\right)$
(b) $x\in\left(-\sqrt{\frac{-1+\sqrt{5}}{2}},\sqrt{\frac{-1+\sqrt{5}}{2}}\right)$
(c) $x\in\left(-\sqrt{\frac{-1+\sqrt{6}}{2}},\sqrt{\frac{-1+\sqrt{6}}{2}}\right)$
(d) $x\in\left(-\sqrt{\frac{-1+\sqrt{7}}{2}},\sqrt{\frac{-1+\sqrt{7}}{2}}\right)$

15. For what values of a, the inequality (for x)
$$\log_{\frac{1}{a}}\left(\sqrt{x^2+ax+5}+1\right)\cdot\log_5\left(x^2+ax+6\right)+\log_a 3\geq 0$$
has exactly one solution?

Credit - Akshat

ANSWER KEY

CONCEPT APPLICATION

1. (*a*) **2.** (*a*) **3.** (*d*) **4.** (*c*) **5.** (*d*) **6.** (*i*) 14 (*ii*) 52 (*iii*) 194 **9.** [0] **10.** [–224]

11. $x^6 - y^6$ **12.** $p = 3/2, q = 1, r = 4/3$ **13.** (*d*) **14.** (*c*) **15.** (*b*) **16.** [60] **17.** [99] **18.** [4]

19. (*d*) **20.** $[2ab]$ **21.** ϕ **22.** $x = \pm\sqrt{\frac{3}{5}}$ **23.** (*c*) **24.** $x \in (-\infty, -3) \cup [-2, 0] \cup [6, \infty)$

25. $x \in (-6, 0] \cup [2, 3] \cup (6, \infty) \cup \{4\}$ **26.** [6] **27.** (*d*) **28.** [1, –1] **29.** $[x \in (\sin 4, \sin 3) \cup [\sin 1, \sin 2]]$

30. $x \in [1, 2]$ **31.** {9} **32.** $x \in (2, 3]$

33. $\left(\frac{1}{5}, \frac{2}{5}\right)$ **34.** (243) **35.** [–5] **36.** [18]

37. [–1] **38.** (*i*) $(1, \infty)$ (*ii*) $[1, \infty)$ (*iii*) $(0, 1)$ (*iv*) $(0, 1]$ (*v*) $(0, 1)$ (*vi*) $(0, 1]$ (*vii*) $(1, \infty)$ (*viii*) $[1, \infty)$ (*ix*) $(3, \infty)$ (*x*) $[5/2, \infty)$

39. (*i*) $x \in \left(\frac{3}{2}, \frac{19}{2}\right)$ (*ii*) $x \in \left(\frac{2}{3}, \frac{17}{24}\right]$ (*iii*) $x \in (4^{16}, \infty)$ (*iv*) $x \in \left(0, \frac{1}{2}\right)$ **40.** {3, –3} **41.** {–2, 2} **43.** $\frac{12 - 4a}{3 + a}$

44. (*i*) $\left\{2^{\pm\sqrt{2}}\right\}$ (*ii*) $x = a^{-\log_5 2}$ (*iii*) {1/32, 2} (*iv*) {1} (*v*) {1}

47. [–1] **48.** [44] **49.** (*d*) **50.** (*b*, *c*)

51. (*i*) {–1, 5} (*ii*) {–3, –1, 7, 9} (*iii*) (14, –4, 0, 10, 2, 8) **52.** (–6, 8) **53.** (*d*) **54.** $(0, 2) \cup (4, \infty)$ **55.** $x \geq 1$

56. (*i*) $x \in (-\infty, 1) \cup (1, \infty)$ (*ii*) $x \in (-\infty, -5] \cup [5, \infty)$ (*iii*) $x \in (-7, 7)$ (*iv*) $x \in [-10, 10]$ (*v*) $x \in R$ (*vi*) $x \in \phi$ (*vii*) $x \in R$
(*viii*) $x \in R$ (*ix*) $x \in \phi$

57. (*i*) $x \in (-\infty, 0) \cup (2, \infty)$ (*ii*) $1 < x < 3$ (*iii*) $x \in (-2, -1) \cup (0, 1)$ (*iv*) $x \in [-1, 0] \cup [1, 2]$ (*v*) $-4/3 \leq x \leq 2$ (*vi*) $x \in \phi$

58. (*i*) $-1 \leq x \leq 5$ (*ii*) $x \in (-\infty, -3] \cup [9, \infty) \cup [-1, 7]$ (*iii*) $x \in [-4, 0] \cup [2, 8] \cup [10, 14]$

59. (*i*) $x \in \left[\frac{1}{5}, \frac{1}{4}\right] \cup \left[\frac{3}{4}, \frac{4}{5}\right]$ (*ii*) $x \in \left[\frac{2}{3}, 2\right]$ (*iii*) $x \in \left[\frac{2}{15}, \frac{2}{5}\right] \cup \left[\frac{6}{5}, \frac{22}{15}\right]$ (*iv*) $x \in (-3, -2) \cup (2, 3)$

60. (*i*) $x \in \{-11, -5, -1\}$ (*ii*) $x \in \{-8, -6, -2, -4, 0, 2, 4, 6, 8, 10, 12, 14, 16, 18\}$

61. (*i*) $x \in [1, 13]$ (*ii*) $x \in [-8, -6] \cup [-4, 0] \cup [2, 4] \cup [6, 8] \cup [10, 14] \cup [16, 18]$

62. (*i*) $x \in (-2, -1] \cup [1, 2)$ (*ii*) $x \in (-\infty, -3] \cup (-2, -1] \cup [3, \infty) \cup [1, 2)$ (*iii*) $x \in (-\infty, -3] \cup [3, \infty)$
(*iv*) $x \in (-\infty, -4] \cup [-1, 5] \cup [6, \infty)$ (*v*) $x \in (-\infty, -4] \cup [6, \infty] \cup \{2\}$

63. (*i*) $x \in [1, \infty]$ (*ii*) $x \in (0, \infty) \cup \{-1\}$ (*iii*) $x \in (-1, 0) \cup (0, 3)$ (*iv*) $x \in \phi$ (*v*) $x \in (2, 6)$

64. $[-6, \infty)$

SCHOOL LEVEL PROBLEMS

1. (*a*) **2.** (*c*) **3.** (*d*) **4.** (*d*) **5.** (*c*) **6.** (*d*) **7.** (*b*) **8.** (*a*) **9.** (*b*) **22.** (*b*)
23. (*a*) **24.** (*c*) **25.** (*c*)

PRARAMBH (TOPICWISE)

1. (*b*) **2.** (*b*) **3.** (*d*) **4.** (*a*) **5.** (*c*) **6.** (*a*) **7.** (*d*) **8.** (*b*) **9.** (*b*) **10.** (*a*)
11. (*b*) **12.** (*d*) **13.** (*d*) **14.** (*c*) **15.** (*d*) **16.** (*c*) **17.** (*b*) **18.** (*d*) **19.** (*c*) **20.** (*c*)
21. (*d*) **22.** (*b*) **23.** (*c*) **24.** (*c*) **25.** (*d*) **26.** (*b*) **27.** (*a*) **28.** (*b*) **29.** (*c*) **30.** (*a*)
31. (*b*) **32.** (*d*) **33.** (*b*) **34.** (*a*) **35.** (*d*) **36.** (*c*) **37.** (*b*) **38.** (*d*) **39.** (*b*) **40.** (*d*)

PRABAL (JEE MAIN LEVEL)

1. (*c*) **2.** (*a*) **3.** (*a*) **4.** (*a*) **5.** (*b*) **6.** (*c*) **7.** (*a*) **8.** (*b*) **9.** (*d*) **10.** (*c*)
11. (*d*) **12.** (*b*) **13.** (*c*) **14.** (*a*) **15.** (*b*) **16.** (*d*) **17.** (*b*) **18.** (*a*) **19.** (*b*) **20.** (*d*)
21. (*b*) **22.** (*b*) **23.** (*c*) **24.** (*d*) **25.** (*c*) **26.** (*c*) **27.** (*c*) **28.** (*a*) **29.** (*a*) **30.** (*d*)
31. (*a*) **32.** (*a*) **33.** (*b*) **34.** (*d*) **35.** (*a*) **36.** (*b*) **37.** (*d*) **38.** (*b*) **39.** (*d*) **40.** (*d*)
41. (*a*) **42.** (*b*) **43.** (*a*) **44.** (*c*) **45.** (*a*) **46.** [3] **47.** [89] **48.** [75] **49.** [1] **50.** [2]
51. [1] **52.** [400] **53.** [3] **54.** [5] **55.** [0] **56.** [1] **57.** [9] **58.** [1] **59.** [10]

PARIKSHIT (JEE ADVANCED LEVEL)

1. (*a*) **2.** (*c*) **3.** (*a*) **4.** (*a*) **5.** (*d*) **6.** (*d*) **7.** (*d*) **8.** (*a*) **9.** (*b*) **10.** (*c*)
11. (*c*) **12.** (*d*) **13.** (*a*) **14.** (*b*) **15.** (*b, d*) **16.** (*a, d*) **17.** (*a, b, c*) **18.** (*a,b,c,d*) **19.** (*b, c*) **20.** (*a, b, c*)
21. (*a,b,c,d*) **22.** (*a, b*) **23.** (*c, d*) **24.** (*b*) **25.** (*b*) **26.** (*a*) **27.** (*c*) **28.** (*b*) **29.** (*c*) **30.** (*b*)
31. (*a*) **32.** (*a*) **33.** [1] **34.** [7] **35.** [2] **36.** [10] **37.** [17] **38.** [2] **39.** [2] **40.** [1]
41. [6] **42.** [2] **43.** [9] **44.** [5625] **45.** [12] **46.** [1] **47.** [1] **48.** [1] **49.** [1] **50.** [7]
51. [3] **52.** [5] **53.** [2007] **54.** [4]

PYQ's (PAST YEAR QUESTIONS)

1. [3] **2.** [4] **3.** (*d*) **4.** (*a*) **5.** [45] **6.** [1] **7.** (*b*) **8.** (*c*) **9.** (*c*) **10.** [1]
11. [8] **12.** [8]

PW CHALLENGERS

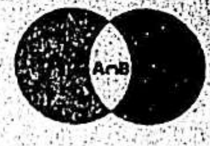
1. [$\sqrt{3}$] **2.** [93] **3.** [44] **4.** [50] **5.** [4] **6.** [29] **7.** [63] **8.** [2007] **9.** [30] **10.** (*a*)
11. [2019] **12.** [5] **13.** (*a*) **14.** (*b*) **15.** [2]

> *"Tension is who you think you should be. Relaxation is who you are."*
>
> **– Chinese Proverb**

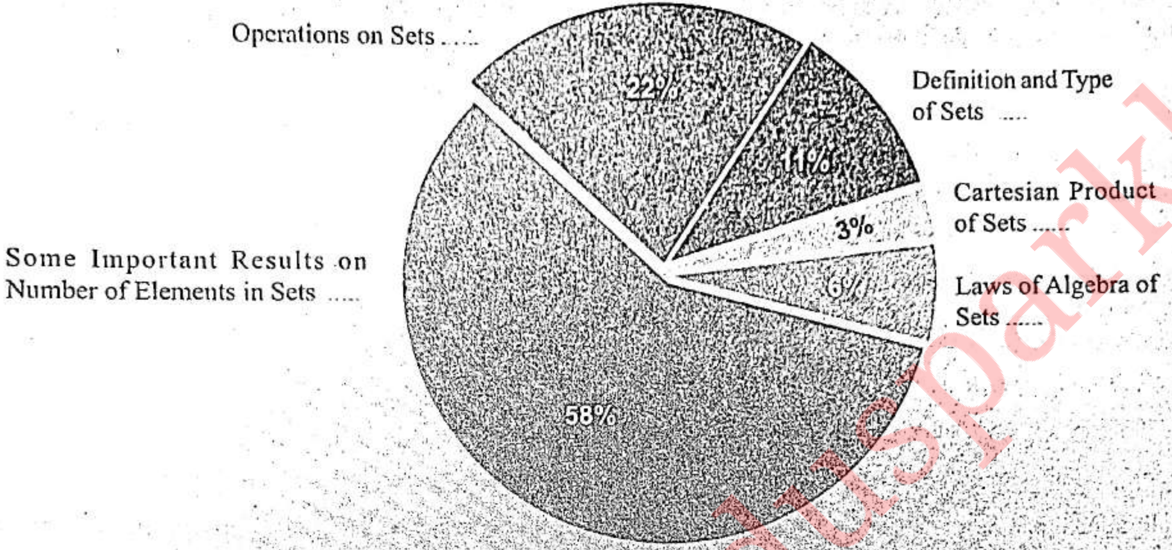
CHAPTER

2

Sets



Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)



"How's the Josh?" for these Topics: *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

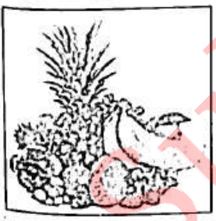
INTRODUCTION

The concept of set serves as a fundamental part of the present day mathematics. It permeates virtually every branch of mathematics. Yet, most mathematics students receive only a cursory overview of the theory of sets.

DEFINITION

A set is a well defined collection of objects.

By well defined we mean there should be no ambiguity regarding the inclusion and exclusion of the objects.

		
Set of Fruits in a Basket	Set of Coin in a Bank	Set of Books in a Library

Collection	Set
1. Consonant in English Alphabet	Set
2. Difficult topics in Mathematics	Not a Set
3. Collection of past Presidents of India	Set
4. Group of Intelligent Students in JEE Batch	Not a Set

Note:

1. Generally, If you can see an adjective like good, difficult intelligent, brave in a sentence then it does not describe a set.
2. Sets are usually denoted by capital letters $A, B, \ldots, X, Y, Z$. Collection of objects or things in a set called as elements

   The elements of the set are denoted by small letters.
3. In a set, the order in which elements are written makes no difference.
4. In a set, the repetition of elements not allowed.

REPRESENTATION OF SET

There are two methods for representing a set.

(*i*) **Tabulation method or Roster form**

All the elements belonging to the set are written in curly brackets and separated by commas

If A is the set of days of a week, then

A = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

(*ii*) **Set Builder Method or Set rule method**

In this method, we use the definition, which is satisfied by all the elements of set.

In above example set A may be written as

$A = \{x : x \text{ is a day of week}\}$

NOTATIONS OF SET OF DIFFERENT NUMBERS

(*i*) Set of all natural numbers $N = \{1, 2, 3, \ldots\}$

(*ii*) Set of all integers Z or $I = \{0, \pm1, \pm2, \ldots\}$

(*iii*) Set of non zero integers Z_0 or $I_0 = \{\pm1, \pm2, \pm3, \ldots\}$

(*iv*) Set of all rational numbers

$Q = \{x : x = p/q$, where p and q relatively prime integers and $q \neq 0\}$

(*v*) Set of real numbers is denoted by R

(*vi*) Set of complex numbers is denoted by C

CARDINAL NUMBER

The number of distinct elements in a set A is denoted by $n(A)$ and it is known as cardinal number of the set A.

- Two finite sets A and B are equivalent if their cardinal number are same.

Example:

$A = \{x \in Z \text{ and } x^2 - 5x + 6 = 0\} \Rightarrow A = \{2, 3\}$

$\therefore n(A) = 2$

TYPES OF SETS

(*i*) **Null set or Void set or Empty set:**

A set having no element is called as null set or empty set or void set. It is denoted by ϕ or { }. The null set is the subset of every set.

Example:

Set of even prime numbers less than 2.

Set of natural numbers strictly lying between 5 and 6.

$A = \{x : x \in N, 5 < x < 6\} = \phi$

(*ii*) **Finite Set**

A set which is empty or consists of a definite number elements

Example:

Set of all natural numbers less than 6 = {1, 2, 3, 4, 5}

(*iii*) **Infinite sets**

A set whose elements can not be counted.

Example

Set of real numbers and natural numbers.

(*iv*) **Singleton Set**

A set having one and only one element is called singleton s or unit set.

Example:

Set of all positive integral roots of the equation

$x^2 - 2x - 15 = 0.$

$\Rightarrow (x + 3)(x - 5) = 0$

$\Rightarrow x = -3$ or $x = 5$ i.e., only one positive integral root is 5.

$C = \{x \in R : x - 5 = 0\}$

$\Rightarrow x = 5 \Rightarrow C = \{5\}$ i.e., only one element.

Set of all positive integral roots = {5}.

Train Your Brain

Example 1: Solve $3x^2 - 12x = 0$.

When

(*i*) $x \in N$

(*ii*) $x \in Z$

Sol. $3x^2 - 12x = 0 \Leftrightarrow 3x(x - 4) = 0 \Leftrightarrow x = 0$ or $x = 4$

(*i*) when $x \in N \Rightarrow x = 4$

(*ii*) when $x \in Z \Rightarrow x = 0$ or $x = 4$

Example 2: If $Q = \left\{x : x = \dfrac{1}{y}, \text{where } y \in N\right\}$, then

(*a*) $0 \in Q$

(*b*) $1 \in Q$

(*c*) $2 \in Q$

(*d*) $\dfrac{2}{3} \in Q$

Sol. (*b*) Here $\dfrac{1}{y} \neq 0, \dfrac{1}{y} \neq 2, \dfrac{1}{y} \neq \dfrac{2}{3}$, $[\because y \in N]$

$\therefore \dfrac{1}{y}$ can be 1, $[\because y$ can be 1].

Concept Application

1. A = $\{x : x \neq x\}$ represents
   - (*a*) $\{0\}$
   - (*b*) $\{\ \}$
   - (*c*) $\{1\}$
   - (*d*) $\{x\}$
2. The set $\{x : x$ is a positive integer less than $6\}$ in roster form is
   - (*a*) $\{1, 2, 3, 4, 5\}$
   - (*b*) $\{1, 2, 3, 4, 5, 6\}$
   - (*c*) $\{2, 4, 6\}$
   - (*d*) $\{1, 3, 5\}$
3. Which of the following sets is a finite set?
   - (*a*) $A = \{x\,;\, x \in Z$ and $x^2 - 5x + 6 = 0\}$
   - (*b*) $B = \{x\,;\, x \in Z$ and x^2 is even$\}$
   - (*c*) $D = \{x\,;\, x \in Z$ and $x > -10\}$
   - (*d*) All of these

EQUAL AND EQUIVALENT SETS

- Two sets A and B are said to be Equivalent sets if their cardinalities are same i.e., $n(A) = n(B)$.
- Two finite sets A and B are said to be Equal sets if their cardinalities are same, and the members in both the sets are the same, i.e., A = B.
- If A and B are equal sets, we denote it by $A = B$.
- If A and B are unequal sets, we denote it by $A \neq B$.

SUBSET AND SUPERSET

If A and B are two sets such that every element of A is also an element of B, then A is a subset of B and B is superset of A. We write $A \subseteq B$.

Note:

- Every set is a subset of itself i.e. $A \subseteq A$ for all A.
- Empty set ϕ is subset of every set
- If $A \subseteq B$ and $B \subseteq A$ then A and B are said to be equal sets, i.e. $A = B$.
- If $A \subseteq B$ and $A \neq B$, there A is called as Proper subset of B and denoted by $A \subset B$.
- If a set A has n elements, then the number of subsets of $A = 2^n$.

POWER SET

Power set of a set A is the collection of all subsets of A and is denoted by $P(A)$.

Let A be a finite set containing m elements i.e., $n(A) = m$, then

- The number of elements in the power set of A, $n(P(A)) = 2^m$.
- The number of non-void/non-empty subsets of $A = (2^m) - 1$.
- The number of proper subsets of $A = 2^m - 1$.
- The number of non-void proper subsets of $A = 2^m - 2$.

Example:

The number of elements in the power set of set $A = \{1, 2\}$ is 2^2.

UNIVERSAL SET

The universal set is the superset for all the sets under the consideration.

The set of complex numbers is the universal set for all possible sets related to numbers.

Train Your Brain

Example 3: The number of non-empty subsets of the set $\{1, 2, 3, 4\}$ is

- (*a*) 15
- (*b*) 14
- (*c*) 16
- (*d*) 17

Sol. (*a*) The number of non- empty subsets

$= 2^n - 1 = 2^4 - 1 = 16 - 1 = 15.$

Example 4: Consider the following sets.

$A = \{0\}$

$B = \{x : x > 15$ and $x < 5\}$,

$C = \{x : x - 5 = 0\}$,

$D = \{x : x^2 = 25\}$,

$E = \{x : x$ is an integral positive root of the equation $x^2 - 2x - 15 = 0\}$

Choose the pair of equal sets

- (*a*) A and B
- (*b*) C and D
- (*c*) C and E
- (*d*) B and C

Sol. (*c*) Since, $0 \in A$ and 0 does not belong to any of the sets B, C, D and E, it follows that $A \neq B$, $A \neq C$, $A \neq D$, $A \neq E$. Since, $B = \phi$, but none of the other sets are empty. Therefore $B \neq C$, $B \neq D$ and $B \neq E$. Also, $C = \{5\}$ but $-5 \in D$, hence $C \neq D$.

Since, $E = \{5\}$, $C = E$. Further, $D = \{-5, 5\}$ so C & E are equal sets.

Example 5: If $A = \{x, y\}$ then the power set of A is

- (*a*) $\{x^y, y^x\}$
- (*b*) $\{\phi, x, y\}$
- (*c*) $\{\phi, \{x\}, \{2y\}\}$
- (*d*) $\{\phi, \{x\}\ \{y\}, \{x, y\}\}$

Sol. (*d*) The collection of all the subsets of the set A is called the power set of A. It is denoted by $P(A)$

Given $A = \{x, y\}$; $P(A) = \{\phi, \{x\}, \{y\}, \{x, y\}\}$

Concept Application

4. Consider the following sets.
   I. $A = \{1, 2, 3\}$
   II. $B = \{x \in R : x^2 - 2x + 1 = 0\}$
   III. $C = \{1, 2, 2, 3\}$
   IV. $D = \{x \in R : x^3 - 6x^2 + 11x - 6 = 0\}$
   Which of the following sets are equal?
   (*a*) $A = B = C$ (*b*) $A = C = D$
   (*c*) $A = B = D$ (*d*) $B = C = D$
5. The number of the proper subsets of $\{a, b, c\}$ is:
   (*a*) 3 (*b*) 8 (*c*) 6 (*d*) 7
6. Given the sets
   $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$. Which of the following sets may be considered as universal set for all the three sets A, B and C?
   (*a*) $\{0, 1, 2, 3, 4, 5, 6\}$
   (*b*) ϕ
   (*c*) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
   (*d*) $\{1, 2, 3, 4, 5, 6, 7, 8\}$
7. The cardinality of the set $P\{P[P(\phi)]\}$ is
   (*a*) 0 (*b*) 1 (*c*) 2 (*d*) 4

INTERVALS AS SUBSETS OF R

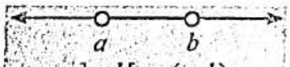
Four type of intervals can be defined as *a* subsets of R.

Let $a, b \in R$, such that $a < b$

1. **Open Interval**

   (a, b) or] a, b [$= \{x : a < x < b\}$

   = Set of all real numbers between a and b, not including a and b both.

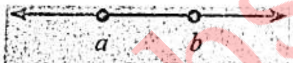


$]a, b[$ or (a, b)

2. **Closed Interval**

   $[a, b] = \{x : a \le x \le b\}$

   = Set of all real numbers between a and b as well as including a and b both.

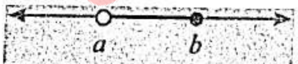


$[a, b]$

3. **Open-closed Interval (semi closed or semi open interval)**

   $(a, b]$ or $]\, a, b] = \{x : a < x \le b\}$

   = Set of all real numbers between a and b, a is not included but b is included.

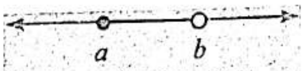


$(a, b]$ or $]\, a, b]$

4. **Closed-open interval (semi closed or semi open interval)**

   $[a, b)$ or $[a, b[= \{x : a \le x < b\}$

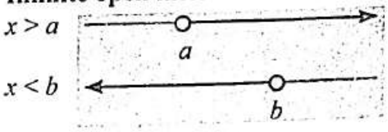
   = Set of all real numbers between a and b including a but excluding b.



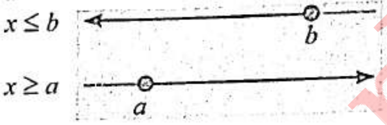
$[a, b)$ or $[a, b[$

Some More Representations on Number Line

Infinite open interval



Infinite close interval



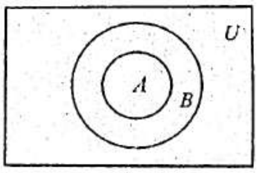
$(0, \infty) = R^+$

$(-\infty, 0) = R^-$

$(-\infty, \infty) = R$

VENN-DIAGRAMS

Venn – diagram is a systematic representation of sets in pictorial form. A set is represented by circle inside the universal set which is represented by rectangular region.



$A \subset B$

OPERATIONS ON SETS

(*a*) **Union of sets**

If A and B are two sets, then the union of two sets is denoted by $A \cup B$ (read as "A union B") and defined as

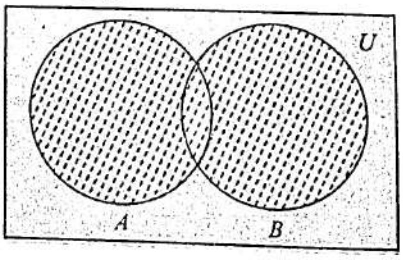
$A \cup B = \{x : x \in A \text{ or } x \in B\}$

Union is also known as join or "logical sum" of A and B.

Example:

$A = \{1, 2, 3\}$, $B = \{1, 3, 5, 7\}$ then $A \cup B = \{1, 2, 3, 5, 7\}$

Case-I: If A and B are not equal sets but they have some elements in common.



$A \cup B$

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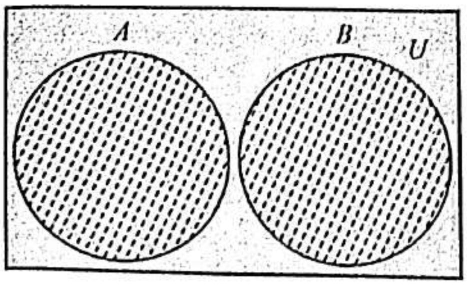
Case-II: If A and B have no elements in common.



$A \cup B$

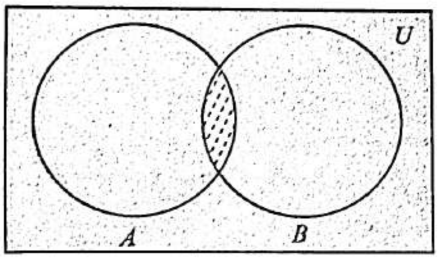
(*b*) **Intersection of sets**

The intersection of sets A and B is denoted by $A \cap B$ (read as "A intersection B") and defined as $A \cap B = \{x : x \in A \text{ and } x \in B\}$

Example:

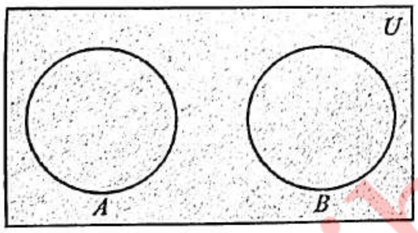
$A = \{1, 2, 3, 5, 6, 9\}$, $B = \{2, 3, 4, 5, 7, 8, 10\}$ then $A \cap B = \{2, 3, 5\}$

Case-I: If A and B are not equal sets but they have some elements in common.



$A \cap B$

Case-II: If A and B have no elements in common.



Note:

Disjoint sets

Two sets are said to be disjoint sets if they have no elements in common, that is $A \cap B = \phi$

Example: $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{7, 8, 9, 10, 11\}$ then A and B are disjoint sets

(*c*) **Difference of sets**

The difference of A and B, i.e.

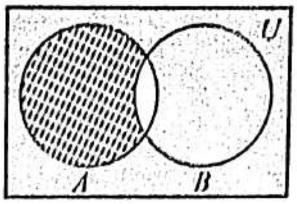
$A - B$ = {all those elements of A which do not belong to B}

$A - B = \{x : x \in A \text{ and } x \notin B\}$

Similarly, $B - A$ = {all those elements of B that do not belong to A}

$B - A = \{x : x \in B \text{ and } x \notin A\}$

Example: $A = \{2, 3, 4, 5, 6, 7\}$, $B = \{3, 5, 7, 9, 11, 13\}$ then $A - B = \{2, 4, 6\}$



$A - B$

Properties

If A and B are any two sets, then

(*i*) $A - B = A \cap B'$

(*ii*) $B - A = B \cap A'$

(*iii*) $A - B = A \Leftrightarrow A \cap B = \phi$

(*iv*) $(A - B) \cup B = A \cup B$

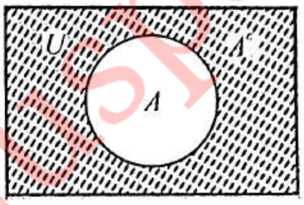
(*v*) $(A - B) \cap B = \phi$

(*vi*) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

(*d*) **Complement of Set**

Let U be the universal set and A be a subset of U, then complement of set A represented as A' or A^c is the set of all elements of U which do not belong to set A.

Thus $A' = \{x : x \in U \text{ and } x \notin A\}$ OR $A' = \{x : x \notin A\}$



A'

Example: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{1, 3, 5, 7, 9\}$ then $A^c = \{2, 4, 6, 8\}$

Properties:

- $A \cap A^c = \phi$ (A and A^c are disjoint set)
- $A \cup A^c = U$
- $(A^c)^c = A$
- $U^c = \phi$
- $\phi^c = U$

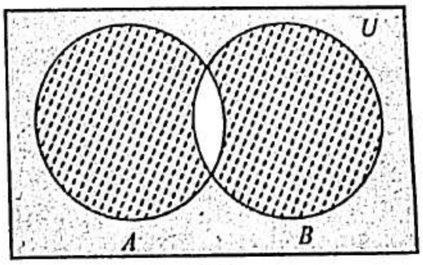
ADVANCED LEARNING

(*e*) **Symmetric difference of two sets**

The symmetric difference of sets A and B is the set $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$

Thus, $A \Delta B = \{x : x \in (A - B) \cup (B - A)\} = \{x : x \notin A \cap B\}$

Example: $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $B = \{1, 3, 5, 6, 7, 8, 9\}$ then $A \Delta B = \{2, 4, 9\}$



$A \Delta B$

Credit - Akshat

Train Your Brain

Example 6: If $aN = \{ax : x \in N\}$ describe the set $3N \cap 7N$.

Sol. $3N = \{3x : x \in N\} = \{3, 6, 9, \dots\}$

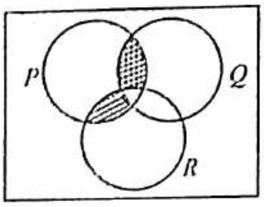
$7N = \{7x : x \in N\} = \{7, 14, 21, 28, \dots\}$

$\therefore 3N \cap 7N = \{x : x$ is a multiple of 3 and x is a multiple of 7$\}$

$= \{x : x$ is a multiple of 21$\}$

$= \{21, 42, 63, \dots\} = 21N$

Example 7: What does the shaded portion of the Venn diagram given below represent?



(*a*) $(P \cap Q) \cap (P \cap R)$

(*b*) $((P \cap Q) - R) \cup ((P \cap R) - Q)$

(*c*) $((P \cup Q) - R) \cap ((P \cap R) - Q)$

(*d*) $((P \cap Q) \cup R) \cap ((P \cup R) - R)$

Sol. (*b*) In the given Venn diagram, shaded area between sets P and Q is $(P \cap Q) - R$ and shaded area between P and R is $(P \cap R) - Q$. So, both the shaded area is union of these two area and is represented by $((P \cap Q) - R) \cup ((P \cap R) - Q)$.

Example 8: If $A = \{x \in R : 0 < x < 3\}$ and $B = \{x \in R : 1 \le x \le 5\}$ then $A \Delta B$ is

(*a*) $\{x \in R : 0 < x < 1\}$

(*b*) $\{x \in R : 3 \le x \le 5\}$

(*c*) $\{x \in R : 0 < x < 1$ or $3 \le x \le 5\}$

(*d*) ϕ

Sol. (*c*) From the given we have in interval notation $A = (0, 3)$ and $B = [1, 5]$

Clearly $A - B = (0, 1) = \{x \in R : 0 < x < 1\}$ and $B - A = [3, 5] = \{x \in R : 3 \le x \le 5\}$

$\therefore A \Delta B = (A - B) \cup (B - A) = (0, 1) \cup [3, 5]$

$= \{x \in R : 0 < x < 1$ or $3 \le x \le 5\}$

Concept Application

8. Let $A = \{(n, 2n) : n \in N\}$ and $B = \{(2n, 3n) : n \in N\}$. What is $A \cap B$ equal to?

(*a*) $\{(n, 6n) : n \in N\}$ (*b*) $\{(2n, 6n) : n \in N\}$

(*c*) $\{(n, 3n) : n \in N\}$ (*d*) ϕ

9. Let X = {Ram, Geeta, Akbar} be the set of students of Class XI, who are in school hockey team and Y = {Geeta, David, Ashok} be the set of students from class XI, who are in the school football team, Then $X \cap Y$ is

(*a*) {Ram, Geeta} (*b*) {Ram}

(*c*) {Geeta} (*d*) {None of these}

10. Let $A = \{3, 6, 9, 12, 15, 18, 21\}$

$B = \{4, 8, 12, 16, 20\}$

$C = \{2, 4, 6, 8, 10, 12, 14, 16\}$

and $D = \{5, 10, 15, 20\}$

(I) $(A \cup B) - C = \{3, 9, 18, 21\}$

(II) $(A - B) \cap (C - D) = \{6\}$

(III) $(A \cap B \cap C) \cup D = \{12, 5, 15, 10, 20\}$

Which of the following is correct?

(*a*) Only I and II

(*b*) Only II and III

(*c*) Only III and I

(*d*) None of these

SOME IMPORTANT RESULTS ON NUMBER OF ELEMENTS IN SETS

(*i*) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(*ii*) $n(A \cup B) = n(A) + n(B) \Leftrightarrow A$ and B are disjoint sets.

(*iii*) $n(A - B) = n(A) - n(A \cap B)$

(*iv*) $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$

(*v*) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C$ $- n(A \cap C) + n(A \cap B \cap C)$

(*vi*) No. of elements in exactly two of the sets A, B, C $= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(*vii*) No. of elements in exactly one of the sets A, B, C $= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

(*viii*) $n(A' \cup B') = n(U) - n(A \cap B)$

(*ix*) $n(A' \cap B') = n(U) - n(A \cup B)$

LAWS OF ALGEBRA OF SETS

(*a*) **Idempotent laws**

(*i*) $A \cup A = A$

(*ii*) $A \cap A = A$

(*b*) **Identity laws**

(*i*) $A \cup \phi = A$

(*ii*) $A \cap U = A$

i.e. ϕ and U are identity elements for union and intersectio respectively.

(c) **Commutative laws**

(i) $A \cup B = B \cup A$

(ii) $A \cap B = B \cap A$

i.e. union and intersection are commutative

(d) **Associative laws**

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $A \cap (B \cap C) = (A \cap B) \cap C$

i.e. union and intersection are associative

(e) **Distributive laws**

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

i.e. union and intersection are distributive over intersection and union respectively.

(f) **De–Morgan's laws**

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

(iii) $(A')' = A$

CARTESIAN PRODUCT

ADVANCED LEARNING

The Cartesian product of two sets A and B is the set $\{(a, b): a \in A$ and $b \in B\}$ and is denoted by $A \times B$. If A has m elements and B has n elements, then $A \times B$ has mn elements

Example: $A = \{1, 2, 3\}$ and $B = \{a, b\}$ then $A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

$n(A \times B) = 3 \times 2 = 6$

Properties

For three sets A, B, and C

(i) $n(A \times B) = n(A) \times n(B)$

(ii) $A \times B = \phi$, if either A or B is an empty set

(iii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(iv) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(v) $A \times (B - C) = (A \times B) - (A \times C)$

(vi) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

(vii) $A \times B = B \times A \Leftrightarrow A = B$

Train Your Brain

Example 9: If A and B be two sets containing 3 and 6 elements respectively, what can be the minimum number of elements in $A \cup B$? Find also, the maximum number of elements in $A \cup B$.

Sol. We have $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

This shows that $n(A \cup B)$ is minimum or maximum according as $n(A \cap B)$ is maximum or minimum respectively.

Case-I

When $n(A \cap B)$ is minimum, i.e., $n(A \cap B) = 0$

This is possible only when $A \cap B = \phi$. In this case,

$n(A \cup B) = n(A) + n(B) - 0 = n(A) + n(B) = 3 + 6 = 9$

So maximum number of elements in $A \cup B$ is 9.

Case-II

When $n(A \cap B)$ is maximum.

This is possible only when $A \subseteq B$.

In this case $n(A \cap B) = 3$.

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B) = (3 + 6 - 3) = 6$

So, minimum number of elements in $A \cup B$ is 6.

Example 10: If A, B and C are three sets and U is the universal set such that $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Find $n(A' \cap B')$.

Sol. We have $A' \cap B' = (A \cup B)'$

$\therefore n(A' \cap B') = n((A \cup B)') = n(U) - n(A \cup B)$

$= n(U) - [n(A) + n(B) - n(A \cap B)]$

$= 700 - (200 + 300 - 100) = 300$

Example 11: Prove that for non empty sets A and B.

$A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

Sol. Consider

$(A \cup B) - (A \cap B) = (A \cup B) \cap (A \cap B)'$

$= (A \cup B) \cap (A' \cup B')$

$= [(A \cup B) \cap A'] \cup [(A \cup B) \cap B']$

$= [(A \cap A') \cup (B \cap A')] \cup [(A \cap B') \cup (B \cap B')]$

$= [\phi \cup (B \cap A')] \cup [(A \cap B') \cup \phi]$

$= (B \cap A') \cup (A \cap B')$

$= (B - A) \cup (A - B)$ ...(1)

Now R.H.S. $= (A - B) \cup (B - A) \cup (A \cap B)$

$= [(A \cup B) - (A \cap B)] \cup (A \cap B)$ using (1)

$= [(A \cup B) \cap (A \cap B)' \cup (A \cap B)$

$= [(A \cup B) \cup (A \cap B)] \cap [(A \cap B)' \cup (A \cap B)]$

$= (A \cup B) \cap U$, where U is universal set.

$= A \cup B =$ L.H.S.

Example 12: In a certain city, only two newspapers A and B are published. It is known that 25% of the city population reads A and 20% reads B, while 8% reads both A and B. It is also known that 30% of those who read A but not B, look into advertisements and 40% of those who read B but not A, look into advertisements, while 50% of those who read both A and B, look into advertisements. What % of the population read an advertisement?

Sol. Let L = Set of people who read paper A

M = Set of people who read paper B

Then $n(L) = 25$, $n(M) = 20$, $n(L \cap M) = 8$

$n(L - M) = n(L) - n(L \cap M) = 25 - 8 = 17$

$n(M - L) = n(M) - n(L \cap M) = 20 - 8 = 12$

$\therefore$ % of people reading an advertisement

$= (30\% \text{ of } 17) + (40\% \text{ of } 12) + (50\% \text{ of } 8)$

$$= \frac{51}{10} + \frac{24}{5} + 4 = 13.9\%$$

Concept Application

11. A market research group conducted a survey of 1000 consumers and reported that 720 consumers liked product A and 450 consumers liked product B. What is the least number that must have liked both products?

(*a*) 170 (*b*) 280

(*c*) 220 (*d*) None of these

12. A set A has 3 elements and another set B has 6 elements. Then

(*a*) $3 \le n(A \cup B) \le 6$

(*b*) $3 \le n(A \cup B) \le 9$

(*c*) $6 \le n(A \cup B) \le 9$

(*d*) $0 \le n(A \cup B) \le 9$

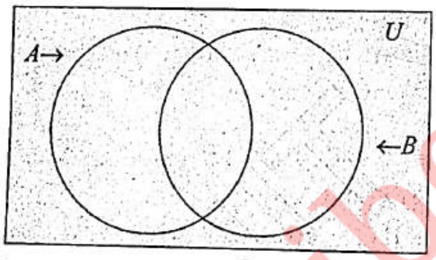
13. At a certain conference of 100 people, there are 29 Indian women and 23 Indian men of these Indian people 4 are doctors and 24 are either men or doctors. There are no foreign doctors. How many foreigners and women doctors are attending the conference?

(*a*) 48, 1 (*b*) 34, 3 (*c*) 46, 4 (*d*) 42, 2

14. The number of elements in the set $\{(a, b) : 2a^2 + 3b^2 = 35, a, b \in Z\}$ where Z is the set of all integers, is

(*a*) 2 (*b*) 4 (*c*) 8 (*d*) 12

15. Find P, Q, R and S in given Venn diagram in terms of A, B where A and B are non empty sets



16. The universal set U and the sets O, P and S are given by

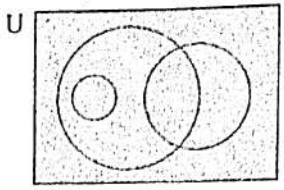
$U = \{x : x$ is an integer such that $3 \le x \le 100\}$,

$O = \{x : x$ is an odd number$\}$,

$P = \{x : x$ is a prime number$\}$,

$S = \{x : x$ is a perfect square$\}$.

In the Venn diagram below, each of the sets O, P and S is represented by a circle.

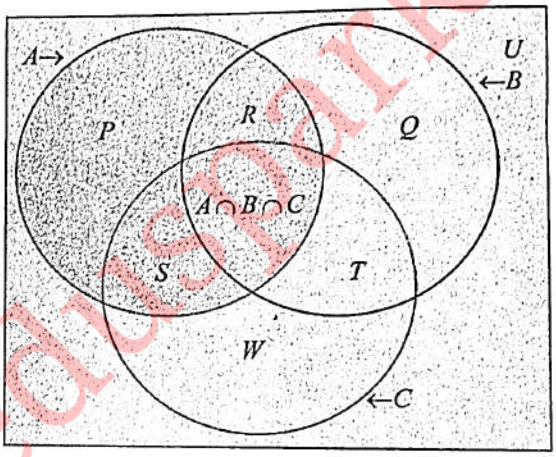


(*i*) Copy the Venn diagram and label each circle with the appropriate letter.

(*ii*) Place each of the numbers 34, 35, 36 and 37 in the appropriate part of your diagram.

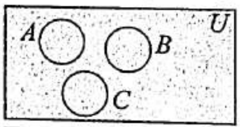
(*iii*) State the value of $n(O \cap S)$ and of $n(O \cup S)$.

17. Find P, Q, R, S, T, U and V in given Venn diagram in terms of A, B, C where A, B and C:

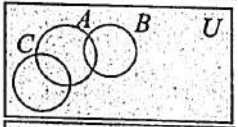


18. Shade $(A \cup B) \cap (A \cup C)$ in the following diagrams.

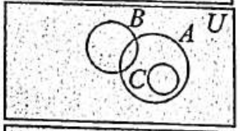
(*a*)



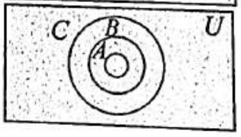
(*b*)



(*c*)



(*d*)



Aarambh (Solved Examples)

1. Among employee of a company taking vacations last years, 90% took vacations in the summer, 65% in the winter, 10% in the spring, 7% in the autumn, 55% in winter and summer, 8% in the spring and summer, 6% in the autumn and summer, 4% in winter and spring, 4% in winter and autumn, 3% in the spring and autumn, 3% in the summer, winter and spring 3% in the summer, winter and autumn, 2% in the summer, autumn and spring, and 2% in the winter, spring and autumn. Percentage of employee that took vacations during every season

   (*a*) 4 (*b*) 3

   (*c*) 2 (*d*) 8

Sol. Suppose that number of employee taking vacations is 100.

S_u – set of employee taking leave in summer

S_w – set of employee taking leave in Winter

S_p – set of employee taking leave in Spring

S_a – set of employee taking leave in Autumn

$n(S_u) = 90,\ n(S_w) = 65,\ n(S_p) = 10,\ n(S_a) = 7$

$n(S_w \cap S_u) = 55,\ n(S_p \cap S_u) = 8,\ n(S_a \cap S_u) = 6$

$n(S_w \cap S_p) = 4,\ n(S_w \cap S_a) = 4,\ n(S_p \cap S_a) = 3$

$n(S_u \cap S_a) = 3,\ n(S_u \cap S_w \cap S_a) = 3$

$n(S_u \cap S_w \cap S_p) = 3,\ n(S_u \cap S_a \cap S_p) = 2$

$n(S_w \cap S_p \cap S_a) = 2$

$\Rightarrow n(S_u \cap S_p \cap S_w \cap S_a)$

$= n(S_u) + n(S_p) + n(S_w) + n(S_a) - n(S_u \cap S_p) - n(S_p \cap S_w) - n(S_w \cap S_a) - n(S_u \cap S_a) - n(S_u \cap S_w) - n(S_p \cap S_a) + n(S_u \cap S_p \cap S_w) + n(S_u \cap S_w \cap S_a) + n(S_w \cap S_a \cap S_u) + n(S_u \cap S_p \cap S_a) - n(S_p \cup S_u \cup S_a \cup S_w)$

$= 90 + 65 + 10 + 7 - 55 - 8 - 6 - 4 - 4 - 3 + 3 + 3 + 2 + 2 - 100 = 2$

Therefore, option (*c*) is the correct answer.

2. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5\}$ then the number of elements in $(A \cup B) \times (A \cap B) \times (A \Delta B)$ is

   (*a*) 5 (*b*) 30

   (*c*) 10 (*d*) 4

Sol. $A \cup B = \{1, 2, 3, 4, 5\},\ n(A \cup B) = 5$

$A \cap B = \{3, 4\},\ n(A \cap B) = 2$

$\therefore A \Delta B = (A - B) \cup (B - A) = \{1, 2\} \cup \{5\} = \{1, 2, 5\}$

$n(A \Delta B) = 3$. Hence $n((A \cup B) \times (A \cap B) \times (A \Delta B))$

$= 5 \times 2 \times 3 = 30$.

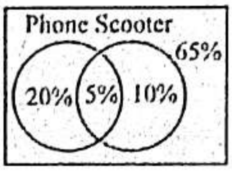
Therefore, option (*b*) is the correct answer.

3. In a certain town 25% families own a cell phone, 15% families own a scooter and 65% families own neither a cell phone nor a scooter. If 1500 families own both a cell phone and a scooter, then total number of families in the town is

   (*a*) 10000 (*b*) 20000

   (*c*) 30000 (*d*) 40000

Sol. Let the total population of town be x.



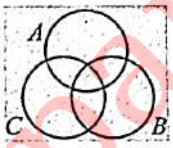
$$\therefore \frac{25x}{100} + \frac{15x}{100} - 1500 + \frac{65x}{100} = x$$

$$\Rightarrow \frac{105x}{100} - x = 1500 \quad \Rightarrow \frac{5x}{100} = 1500$$

$\Rightarrow x = 30000$

Therefore, option (*c*) is the correct answer.

4. The shaded region in the given figure represents



   (*a*) $A \cap (B \cup C)$ (*b*) $A \cup (B \cap C)$

   (*c*) $A \cap (B - C)$ (*d*) $A - (B \cup C)$

Sol. Shaded region contain elements of A not in B and not in C hence it is $A - (B \cup C)$

Therefore, option (*d*) is the correct answer.

5. If $A = \{x : x^2 - 5x + 6 = 0\}$, $B = \{2, 4\}$, $C = \{4, 5\}$, then $A \times (B \cap C)$ is

   (*a*) $\{(2, 4), (3, 4)\}$

   (*b*) $\{(4, 2), (4, 3)\}$

   (*c*) $\{(2, 4), (3, 4), (4, 4)\}$

   (*d*) $\{(2, 2), (3, 3), (4, 4), (5, 5)\}$

Sol. Clearly, $A = \{2, 3\}$, $B = \{2, 4\}$, $C = \{4, 5\}$

$\Rightarrow B \cap C = \{4\}$

$\therefore A \times (B \cap C) = \{(2, 4), (3, 4)\}$

Therefore, option (*a*) is the correct answer.

6. The set $(B \cup C) \cap (A \cap B' \cap C')' \cap C'$ is equal to

   (*a*) $B \cap C'$ (*b*) $A \cap C$

   (*c*) $B' \cap C'$ (*d*) None of these

Sol. $(B \cup C) \cap (A \cap B' \cap C')' \cap C'$

$= (B \cup C) \cap (A' \cup B \cup C) \cap C'$

$= (B \cup C) \cap C' = (B \cup C) \cap C'$

$= (B \cap C') \cup \phi = B \cap C'$

Therefore, option (*a*) is the correct answer.

7. Let $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6\}$. Then the number of se
   C such that $A \cap B \subseteq C \subseteq A \cup B$ is

   (*a*) 6 (*b*) 9

   (*c*) 8 (*d*) 10

Sol. Here, $A \cap B = \{2, 4\}$

and $A \cup B = \{1, 2, 3, 4, 6\}$

$\because A \cap B \subseteq C \subseteq A \cup B$

$\therefore C$ can be $\{2, 4\}, \{1, 2, 4\}, \{3, 2, 4\}, \{6, 2, 4\}, \{1, 6, 2, 4\}, \{6, 3, 2, 4\}, \{1, 3, 2, 4\}, \{1, 2, 3, 4, 6\}$

Thus, number of set C which satisfy the given condition is 8.

Therefore, option (*c*) is the correct answer.

8. If $A = \phi$, $P(A)$ denotes power set of A, then number of elements in $P(P(P(P(P(A)))))$ is

(*a*) 1 (*b*) 2^4

(*c*) 2^5 (*d*) 2^{16}

Sol. $n(P(A)) = 2^0 = 1$

$\Rightarrow n(P(P(A))) = 2^1 = 2$

$\Rightarrow n(P(P(P(A)))) = 2^2 = 4$

$\Rightarrow n(P(P(P(P(A))))) = 2^4 = 16$

$\Rightarrow n(P(P(P(P(P(A)))))) = 2^{16}$

Therefore, option (*d*) is the correct answer.

9. If A and B be two sets such that $n(A) = 15$, $n(B) = 25$, then number of possible values of $n(A \Delta B)$ (symmetric difference of A and B) is

(*a*) 30 (*b*) 16 (*c*) 26 (*d*) 40

Sol. $n(A \Delta B) = n(A \cup B) - n(A \cap B)$ for maximum $n(A \Delta B)$, $n(A \cup B)$ should be maximum and $n(A \cap B)$ is minimum

For $n(A \cap B)$ to be minimum, $A \cap B = \phi$

$\Rightarrow n(A \cup B) = 25 + 15 = 40$

$n(A \Delta B) = 40$

For $n(A \cap B)$ to be maximum, $A \subset B$

$n(A \cap B) = 15$

$\Rightarrow n(A \cup B) = 25$

$n(A \Delta B) = 25 - 15 = 10$

$\Rightarrow$ Range of $(A \Delta B) = \{10, 12, 14, 16, \ldots\ldots, 40\}$

Therefore, option (*b*) is the correct answer.

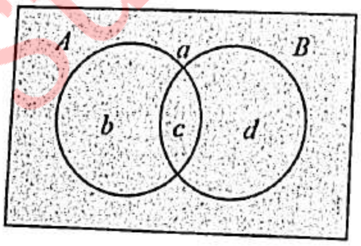
10. $|X|$ represent number of elements in region X. Now the following conditions are given

$|U| = 14$, $|(A - B)^c| = 12$, $|A \cup B| = 9$ and $|A \Delta B| = 7$, where A and B are two subsets of the universal set U and A^c represents complement of set A, then

(*a*) $|A| = 2$ (*b*) $|B| = 5$

(*c*) $|A| = 4$ (*d*) $|B| = 7$

Sol.



$a + b + c + d = 14$ ...(*i*)

$a + c + d = 12$ ...(*ii*)

$b + c + d = 9$ ...(*iii*)

$b + d = 7$

$b = 2, a = 5, d = 5, c = 2$

$|A| = b + c = 4$

$|B| = d + c = 7$

Therefore, option (*c, d*) is the correct answers.

11. For any three sets. P, Q and R, S is an element of $(P \Delta Q) \Delta R$, then S may belong to

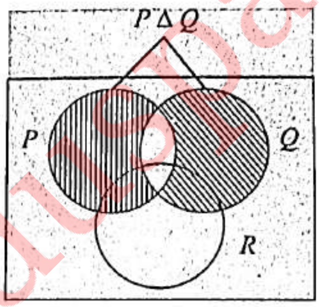
(*a*) Exactly one of P, Q and R, if P, Q and R disjoint sets

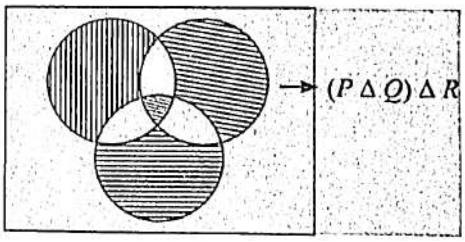
(*b*) At least one of P, Q and R, but not in all three of them at the same time

(*c*) Exactly two of P, Q and R

(*d*) Exactly one of P, Q and R or in all the three of them

Sol.





Therefore, option (*a, d*) is the correct answers.

12. Suppose $A_1, A_2, \ldots, A_{30}$ are thirty sets each having 5 elements and $B_1, B_2, \ldots, B_n$ are n sets each having 3 elements. Let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ and each elements of S belongs to exactly 10 of A_i's and exactly 9 of B_j's. The value of n is equal to

Sol. Number of elements in

$A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_{30}$ is 30×5 but each element is used 10 times, so

$$n(S) = \frac{30 \times 5}{10} = 15 \quad \ldots(i$$

Similarly, number of elements in $B_1 \cup B_2 \ldots \cup B_n$ is 3 n bu each element is repeated 9 times, so

$$n(S) = \frac{3n}{9}$$

$\Rightarrow 15 = \dfrac{3n}{9}$ [from Eq. (*i*)]

$\Rightarrow n = 45$

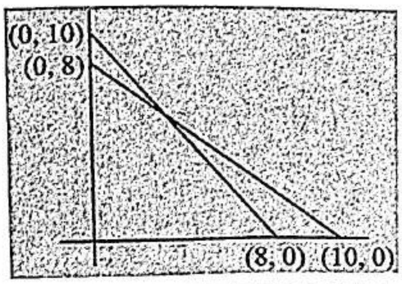
Therefore, 45 is the correct answer.

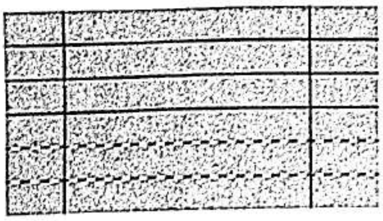
13. $A = \{(x, y) : x, y \in I, x \geq 0, y \geq 0 \text{ and } 4x + 5y \leq 40\}$

$B = \{(x, y) : x, y \in I, x \geq 0, y \geq 0 \text{ and } 5x + 4y \leq 40\}$

where I denotes the set of integers, then $n(A \cap B) =$

Sol.





$$\text{Total} = \frac{9 \times 10}{2} = 45$$

Therefore, 45 is the correct answer.

14. In a survey it was found that 21 persons liked product P_1, 26 liked product P_2 and 29 liked product P_3. If 14 persons liked products P_1 and P_2; 12 persons liked product P_3 and P_1; 14 persons liked products P_2 and P_3, and 8 liked all the three products. Find how many liked product P_3 only.

Sol. Let $n(P_1)$ be a number of people liking product P_1.

Let $n(P_2)$ be a number of people liking product P_2.

Let $n(P_3)$ be a number of people liking product P_3.

Then, according to the question:

$n(P_1) = 21$, $n(P_2) = 26$,

$n(P_3) = 29$, $n(P_1 \cap P_2) = 14$

$n(P_1 \cap P_3) = 12$,

$n(P_2 \cap P_3) = 14$,

$n(P_1 \cap P_2 \cap P_3) = 8$

$\therefore$ Number of people liking product P_3 only:

$= 29 - (4 + 8 + 6)$

$= 29 - 18 = 11$

Therefore, 11 is the correct answer.

> *"Sometimes the most productive thing you can do is relax."*
>
> **– Mark Black**

School Level Problems

SINGLE CORRECT TYPE QUESTIONS

1. Write solution set of equation $x^2 - 3x + 2 = 0$ in roster form
   (*a*) $\{1, 3\}$ (*b*) $\{2, 4\}$
   (*c*) $\{1, 4\}$ (*d*) $\{1, 2\}$

2. Which one of the following is the correct representation of set $A = \{2, 4, 8, 16, ...$ in set builder form?
   (*a*) $\{x : x = 2n \text{ where } n \in N\}$
   (*b*) $\{x : x = 2^n \text{ where } n \in N\}$
   (*c*) $\{x : x = 4n \text{ where } n \in N\}$
   (*d*) $\{x : x = 2n + 4 \text{ where } n \in N$

3. Two finite sets have M and N elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of M and N are respectively.
   (*a*) 6, 3 (*b*) 8, 5
   (*c*) 4, 1 (*d*) none of these

4. Every set is of itself
   (*a*) Proper subset
   (*b*) Improper subset
   (*c*) Compliment
   (*d*) None of the above

5. If $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then
   (*a*) $B = C$
   (*b*) $A = B$
   (*c*) $A = C$
   (*d*) None of these

6. A survey shows that 63% of the americans like cheese whereas 76% like apples. If X% of the americans like both cheese and apples, then we have
   (*a*) $39 \le x \le 63$ (*b*) $x \le 63$
   (*c*) $x \le 39$ (*d*) none of these.

7. What is the interval of $f(x) = (x - 1)(x - 2)(x - 3).(x^3 + 6x^2 + 11x + 6)$ where $f(x)$ is positive?
   (*a*) $(-\infty, -3) \cup (3, \infty)$
   (*b*) $(3, -2) \cup (1, 1) \cup (2, 3)$
   (*c*) $(-\infty, -3) \cup (-2, -1) \cup (1, 2) \cup (3, \infty)$
   (*d*) $(-\infty, \infty)$

8. In a family of 10 members, 7 of them like tea or coffee, 4 of them like tea and 5 of them like coffee. How many of them like only tea?
   (*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

9. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 5\}$, $B = \{6, 7\}$ then $A \cap B'$ is
   (*a*) B' (*b*) A (*c*) A' (*d*) B

VERY SHORT ANSWER TYPE QUESTIONS

10. Write the following sets in roster form:
    (i) $A = \{x : x \text{ is an integer and } -3 \le x < 7\}$
    (ii) $B = \{x : x \text{ is a natural number less than } 6\}$

11. Given that $N = \{1, 2, 3, ... 100\}$, then
    (i) Write the subset A of N, whose elements are odd numbers.
    (ii) Write the subset B of N, whose elements are represented by $x + 2$, where $x \in N$.

12. Let $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{2, 4, 6\}$, $B = \{3, 5\}$ and $C = \{1, 2, 4, 7\}$, find $A' \cup (B \cap C')$

13. In a class of 60 students, 23 play hockey, 15 play basketball, 20 play cricket and 7 play hockey and basketball, 5 play cricket and basketball, 4 play hockey and cricket, 15 do not play any of the three games. Find
    How many play hockey but not cricket

14. Let $U = \{x : x \in N, x \le 9\}$; $A = \{x$: is an even number, $0 < x < 10\}$; $B = \{2, 3, 5, 7\}$. Write the set $(A \cup B)'$.

15. Given $L = \{1, 2, 3, 4\}$, $M = \{3, 4, 5, 6\}$ and $N = \{1, 3, 5\}$
    Verify that $L - (M \cup N) = (L - M) \cap (L - N)$

16. In a class of 60 students, 25 students play cricket and 20 students play tennis, and 10 students play both the games. Find the number of students who play neither.

LONG ANSWER TYPE QUESTIONS

17. Let A, B and C be sets. Then show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

18. Out of 100 students; 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science; 4 in English and Science; 4 in all the three. Find how many passed
    (i) in English and Mathematics but not in Science
    (ii) in Mathematics and Science but not in English
    (iii) in Mathematics only
    (iv) in more than one subject only

19. If A and B are subsets of the universal set U, then show that
    (*i*) $A \subset A \cup B$
    (*ii*) $A \subset B \Leftrightarrow A \cup B = B$
    (*iii*) $(A \cap B) \subset A$

20. In a town of 10,000 families, it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C. 5% families buy A and B, 3%

buy B and C and 4% buy A and C. If 2% families buy all the three papers. Find the no. of families which buy

(i) A only

(ii) B only

(iii) none of A, B, and C.

21. If $P(A) = P(B)$, Show that $A = B$

CASE STUDY BASED QUESTIONS

CASE STUDY-I

The school organised a farewell party for 100 students and school management decided three types of drinks will be distributed in farewell party i.e. Milk (M), Coffee (C) and Tea (T).

Organiser reported that 10 students had all the three drinks M, C, T. 20 students had M and C; 30 students had C and T; 25 students had M and T. 12 students had M only; 5 students had C only; 8 students had T only.

Based on the above information, answer the following questions.

22. The number of students who did not take any drink, is

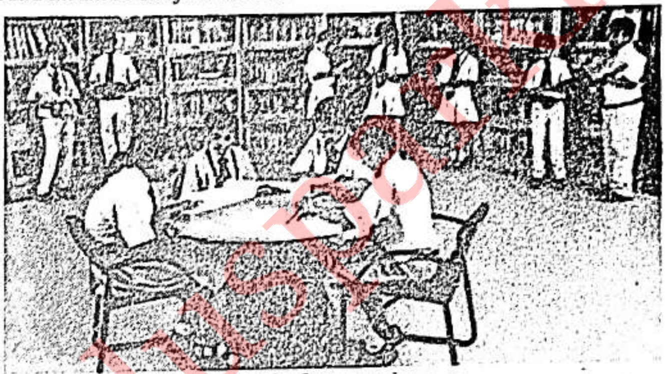
(*a*) 20 (*b*) 30 (*c*) 10 (*d*) 25

23. The number of students who prefer Milk and Coffee but not tea is

(*a*) 12 (*b*) 10 (*c*) 15 (*d*) 20

CASE STUDY-II

In a library, 25 students read physics, chemistry and mathematics books. It was found that 15 students read mathematics, 12 students read physics while 11 students read chemistry. 5 students read both mathematics and chemistry, 9 students read physics and mathematics. 4 students read physics and chemistry and 3 students read all three subject books.



Based on the above information, answer the following questions.

24. The number of students who reading only chemistry is

(*a*) 5 (*b*) 4 (*c*) 2 (*d*) 1

25. The number of students who reading only, one of the subjects is

(*a*) 5 (*b*) 8 (*c*) 11 (*d*) 6

Prarambh (Topicwise)

DEFINITION AND TYPE OF SETS

1. The set of intelligent students in a class is

(*a*) A null set

(*b*) A singleton set

(*c*) A finite set

(*d*) Not a well defined collection

2. Which of the following is the empty set

(*a*) $\{x : x$ is a real number and $x^2 - 1 = 0\}$

(*b*) $\{x : x$ is a real number and $x^2 + 1 = 0\}$

(*c*) $\{x : x$ is a real number and $x^2 - 9 = 0\}$

(*d*) $\{x : x$ is a real number and $x^2 = x + 2\}$

3. The set $A = \{x : x \in \mathrm{R}, x^2 = 16$ and $2x = 6\}$ equals

(*a*) ϕ (*b*) $\{14,3,4\}$

(*c*) $\{3\}$ (*d*) $\{4\}$

4. If a set A has n elements, then the total number of subsets of A is

(*a*) n (*b*) n^2 (*c*) 2^n (*d*) $2n$

5. The number of proper subsets of the set $\{1, 2, 3\}$ is

(*a*) 8 (*b*) 7 (*c*) 6 (*d*) 5

6. If $X = \{8^n - 7n - 1 : n \in N\}$ and $Y = \{49(n - 1) : n \in N\}$, then

(*a*) $X \subseteq Y$ (*b*) $Y \subseteq X$

(*c*) $X = Y$ (*d*) None of these

OPERATIONS ON SETS

7. Given the sets $A = \{1, 2, 3\}$, $B = \{3, 4\}$, $C = \{4, 5, 6\}$, then $A \cup (B \cap C)$ is

(*a*) $\{3\}$ (*b*) $\{1, 2, 3, 4\}$

(*c*) $\{1, 2, 4, 5\}$ (*d*) $\{1, 2, 3, 4, 5, 6\}$

8. If $A \subseteq B$, then $A \cup B$ is equal to

(*a*) A (*b*) $B \cap A$

(*c*) B (*d*) None of these

9. If A and B are any two sets, then $A \cup (A \cap B)$ is equal to

(*a*) A (*b*) B

(*c*) A^c (*d*) B^c

10. If A and B are two given sets, then $A \cap (A \cap B)^c$ is equal to

(*a*) A (*b*) B

(*c*) ϕ (*d*) $A \cap B^c$

11. If $N_a = \{an : n \in N\}$ then $N_3 \cap N_4 =$

(*a*) N_7 (*b*) N_{12}

(*c*) N_3 (*d*) N_4

12. If $aN = \{ax : x \in N\}$ and $bN \cap cN = dN$, where $b, c \in N$ are relatively prime, then

(*a*) $d = bc$ (*b*) $c = bd$

(*c*) $b = cd$ (*d*) None of these

13. If the sets A and B are defined as

$A = \{(x, y) : y = \frac{1}{x},\ 0 \neq x \in R\}$

$B = \{(x, y) : y = -x, x \in R\}$, then

(*a*) $A \cap B = A$ (*b*) $A \cap B = B$

(*c*) $A \cap B = \phi$ (*d*) None of these

14. Let $A = [x : x \in R, |x| < 1]$; $B = [x : x \in R, |x - 1| \geq 1]$ and $A \cup B = R - D$, then the set D is

(*a*) $[x : 1 < x \leq 2]$ (*b*) $[x : 1 \leq x < 2]$

(*c*) $[x : 1 \leq x \leq 2]$ (*d*) None of these

15. If the sets A and B are defined as

$A = \{(x, y) : y = e^x, x \in R\}$

$B = \{x, y\} : y = x, x \in R$, then

(*a*) $B \subseteq A$ (*b*) $A \subseteq B$

(*c*) $A \cap B = \phi$ (*d*) $A \cup B = A$

16. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n - 1) : n \in N\}$ then $X \cup Y$ is equal to

(*a*) X (*b*) Y

(*c*) N (*d*) None of these

SOME IMPORTANT RESULTS ON NUMBER OF ELEMENTS IN SETS

17. Sets A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A \cup B$

(*a*) 3 (*b*) 6

(*c*) 9 (*d*) 18

18. If A and B are two sets such that $n(A) = 70$, $n(B) = 60$, and $n(A \cup B) = 110$, then $n(A \cap B)$ is equal to

(*a*) 240 (*b*) 50

(*c*) 40 (*d*) 20

19. Let $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$ then $n(A^c \cap B^c)$

(*a*) 400 (*b*) 600 (*c*) 300 (*d*) 200

20. If $A = [(x, y) : x^2 + y^2 = 25]$ and $B = [(x, y) : x^2 + 9y^2 = 144]$, then $A \cap B$ contains

(*a*) One point

(*b*) Three points

(*c*) Two points

(*d*) Four points

21. In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons travelling by car or bus is

(*a*) 80 percent (*b*) 40 percent

(*c*) 60 percent (*d*) 70 percent

22. In a class of 55 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics, 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects. The number of students who have taken exactly one subject is

(*a*) 6 (*b*) 9 (*c*) 7 (*d*) 22

LAWS OF ALGEBRA OF SETS

23. If A, B and C are any three sets, then $A \times (B \cap C)$ is equal to

(*a*) $(A \times B) \cup (A \times C)$ (*b*) $(A \times B) \cap (A \times C)$

(*c*) $(A \cup B) \times (A \cup C)$ (*d*) $(A \cap B) \times (A \cap C)$

24. If A, B and C are any three sets, then $A \times (B \cup C)$ is equal to

(*a*) $(A \times B) \cup (A \times C)$ (*b*) $(A \cup B) \times (A \cup C)$

(*c*) $(A \times B) \cap (A \times C)$ (*d*) None of these

25. If A, B and C are any three sets, then $A - (B \cup C)$ is equal to

(*a*) $(A - B) \cup (A - C)$ (*b*) $(A - B) \cap (A - C)$

(*c*) $(A - B) \cup C$ (*d*) $(A - B) \cap C$

26. If $A = [x : x$ is a multiple of $3]$ and $B = [x : x$ is a multiple of $5]$, then $A - B$ is ($\bar{A}$ means complement of A)

(*a*) $\bar{A} \cap B$ (*b*) $A \cap \bar{B}$

(*c*) $\bar{A} \cap \bar{B}$ (*d*) $\overline{A \cap B}$

27. If A, B and C are non-empty sets, then $(A - B) \cup (B - A)$ equals

(*a*) $(A \cup B) - B$ (*b*) $A - (A \cap B)$

(*c*) $(A \cup B) - (A \cap B)$ (*d*) $(A \cap B) \cup (A \cup B)$

CARTESIAN PRODUCT OF SETS

28. If $A = \{0, 1\}$, and $B = \{1, 0\}$, then $A \times B$ is equal to

(*a*) $\{0, 1, 1, 0\}$ (*b*) $\{(0, 1), (1, 0)\}$

(*c*) $\{0, 0\}$ (*d*) $\{(0,1),(0,0),(1,1),(1,0)\}$

29. If $A = \{2, 4, 5\}$, $B = \{7, 8, 9\}$ then $n(A \times B)$ is equal to
(*a*) 6 (*b*) 9 (*c*) 3 (*d*) 0

30. If the set A has p elements, B has q elements, then the number of elements in $A \times B$ is
(*a*) $p + q$ (*b*) $p + q + 1$
(*c*) pq (*d*) p^2

31. If $A = \{a, b\}$, $B = \{c, d\}$, $C = \{d, e\}$ then $\{(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)\}$ is equal to
(*a*) $A \cap (B \cup C)$ (*b*) $A \cup (B \cap C)$
(*c*) $A \times (B \cup C)$ (*d*) $A \times (B \cap C)$

32. If $A = \{x : x^2 - 5x + 6 = 0\}$, $B = \{2, 4\}$, $C = \{4, 5\}$ then $A \times (B \cap C)$ is
(*a*) $\{(2, 4), (3, 4)\}$
(*b*) $\{(4, 2), (4, 3)\}$
(*c*) $\{(2, 4), (3, 4), (4, 4)\}$
(*d*) $\{(2,2), (3,3), (4,4), (5,5)\}$

33. If P, Q and R are subsets of a set A, then $R \times (P^c \cup Q^c)^c =$
(*a*) $(R \times P) \cap (R \times Q)^c$ (*b*) $(R \times Q) \cap (R \times P)$
(*c*) $(R \times P) \cup (R \times Q)$ (*d*) None of these

34. Let $S = \{x \in R : (x-3)^2 + (x-2)^2 + 5(x-4)^2 = 0\}$. Then S is not
(*a*) Singleton set (*b*) Empty set
(*c*) Void set (*d*) null set

35. Let $S = \{x \in R : 4^x + 2^{x+1} - 8 = 0\}$. Then $x =$
(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

36. The set $(A \cap B')' \cup (B \cap C)$ is equal to
(*a*) $A' \cup B \cup C$ (*b*) $A' \cup B$
(*c*) $A' \cap C'$ (*d*) $A' \cap B$

37. If A and B are two given sets, then $A \cap (A \cap B)^c$ is equal to
(*a*) A (*b*) B (*c*) ϕ (*d*) $A \cap B^c$

38. Let $A = \{(x, y) : x \in R, y \in R, x^3 + y^3 = 1]\}$, $B = \{(x, y) : x \in R, y \in R, x - y = 1\}$ and $C = \{(x, y) : x \in R, y \in R, x + y = 1\}$. If $A \cap B$ contains 'p' elements and $A \cap C$ contains 'q' elements then find $(q - p)$.
(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

39. In a examination of a certain class, at least 70% of the students failed in Physics, at least 72% failed in Chemistry, at least 80% failed in Mathematics and at least 85% failed in English. How many at least must have failed in all the four subjects?
(*a*) 9%
(*b*) 7%
(*c*) 15%
(*d*) Cannot be determined due to insufficient data

40. **Statement-I:** $N \cup (B \cap Z) = (N \cup B) \cap Z$ for any subset B of R, where N is the set of positive integers, Z is the set of integers, R is the set of real numbers.
Statement-II: Let $A = \{n \in N : 1 \le n \le 24$, n is a multiple of $3\}$. There exists no subset B of N such that the number of elemets in A is equal to the number of elements in B.
Which of the above statements is/are correct?
(*a*) I only (*b*) II only
(*c*) Both I and II (*d*) Neither I nor II

41. In year 10 of a Kuala Lumpur International School, there are 198 students.
C = {students who like chilli}
D = {students who like durian}
90 like durian and 130 like chilli. The number who like both is x and the number who like neither is $212 - 2x$. Find the value of x.
(*a*) 76 (*b*) 78
(*c*) 80 (*d*) None of these

Prabal (JEE Main Level)

SINGLE CORRECT TYPE QUESTIONS

1. Let $A = \{x : x \in R, x \ge 2\}$ and $B = \{x : x \in R, x < 4\}$. Then $A \cap B =$
(*a*) $\{x : x \in R, 2 < x < 4\}$, (*b*) $\{x : x \in R, 2 \le x < 4\}$
(*c*) B (*d*) A

2. If $A = \{\phi, \{(\phi)\}\}$, then the power set $P(A)$ of A is
(*a*) A (*b*) $\{\phi, \{\phi\}, A\}$
(*c*) $\{\phi, \{\phi\}, \{\{\phi\}\}, A\}$ (*d*) $\{\phi, \{\phi\}, \{\{\phi\}\}\}$

3. If $A = \{2, 3\}$ and $B = \{x \mid x \in N$ and $x < 3\}$, then $A \times B$ is
(*a*) $\{(2, 1), (2, 2), (3, 1), (3, 2)\}$
(*b*) $\{(1, 2), (1, 3), (2, 2), (2, 3)\}$
(*c*) $\{(1, 2), (2, 2), (3, 3), (3, 2)\}$
(*d*) $\{(1, 1), (2, 2), (3, 3), (3, 2)\}$

4. Let U be the universal set containing 700 elements. If A, B are subsets of U such that $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$. Then $n(A' \cap B') =$
(*a*) 400 (*b*) 600 (*c*) 300 (*d*) 200

5. Let $R: x \to y$ be a relation on N defined by $x + 2y = 8$. Then the domain of R is
(*a*) $\{2, 4, 8\}$ (*b*) $\{2, 4, 6, 8\}$
(*c*) $\{2, 4, 6\}$ (*d*) $\{1, 2, 3, 4\}$

6. If $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{x \in N : 30 < x^2 < 70\}$, $B = \{x : x$ is a prime number less than $10\}$, then which of the following is incorrect?
(*a*) $A \cup B = \{2, 3, 5, 6, 7, 8\}$
(*b*) $A \cap B = \{7, 8\}$
(*c*) $A - B = \{6, 8\}$
(*d*) $A \Delta B = \{2, 3, 5, 6, 8\}$

7. Let X be the universal set for sets A and B, if $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$, then $n(A' \cap B')$ is equal to 300 provided $n(X)$ is equal to

(*a*) 600
(*b*) 700
(*c*) 800
(*d*) 900

8. If two sets A and B are having 80 elements in common, then the number of elements common to each of the sets $A \times B$ and $B \times A$ are

(*a*) 2^{80}
(*b*) 80^2
(*c*) 81
(*d*) 79

9. If A and B be two universal sets and $A \cup B \cup C = U$. Then, $((A - B) \cup (B - C) \cup (C - A))'$ is equal to

(*a*) $A \cup B \cup C$
(*b*) $A \cup (B \cap C)$
(*c*) $A \cap B \cap C$
(*d*) $A \cap (B \cup C)$

10. The set $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$ is equal to

(*a*) $B \cup C'$
(*b*) $A \cap C$
(*c*) $B' \cap C'$
(*d*) None of these

11. If there are three athelitic teams in a school 21 are in the basketball team,26 in hockey team and 29 in the football team. 14 play hockey and basketball, 15 play hockey and football, 12 play football and basketball and 8 play all the games. Then total number of members are

(*a*) 42
(*b*) 43
(*c*) 45
(*d*) None of these

12. If $P(A)$ denotes the power set of A and A is the void set, then what is number of elements in $P\{P\{P\{P(A)\}\}\}$?

(*a*) 0
(*b*) 1
(*c*) 4
(*d*) 16

13. There are 4 prime numbers between n and $2n$. Then, possible values of n is

(*a*) $n = 4$
(*b*) $n = 13$
(*c*) $n = 10$
(*d*) $n = 50$

14. Let $S = \{x \in R : 2^{333x-2} + 2^{111x+1} = 2^{222x+2} + 1\}$. Then Sum of all elements of S is

(*a*) $\frac{111}{2}$
(*b*) $\frac{2}{111}$
(*c*) 2
(*d*) 111

15. Let F_1 be the set of all parallelograms, F_2 the set of all rectangles, F_3 the set of all rhombuses, F_4 the set of all squares and F_5 the set of trapeziums in a plane. Then F_1 may be equal to

(*a*) $F_2 \cap F_4$
(*b*) $F_3 \cap F_4$
(*c*) $F_2 \cap F_3$
(*d*) $F_2 \cup F_3 \cup F_4 \cup F_1$

16. Let $A = \{(x, y) : a^x = a^y; a > 0 \text{ and } a \neq 1; a, x, y \in R\}$
$B = \{(x, y); xy = 1; x, y \in R_0\}$
Choose the correct statements amongst the following.

(*a*) $A \cap B = B$
(*b*) $A \cap B = A$
(*c*) $n(B) > n(A)$
(*d*) A and B are non-comparable

17. Let A_1, A_2 and A_3 be subsets of a set X. Which one of th following is correct?

(*a*) $A_1 \cup A_2 \cup A_3$ is the smallest subset of X containin elements of each of A_1, A_2 and A_3.
(*b*) $A_1 \cup A_2 \cup A_3$ is the smallest subset of X containin either A_1 or $(A_2 \cup A_3)$ but not both.
(*c*) The smallest subset of X containing $A_1 \cup A_2$ and A equals the smallest subset of X containing both A_1, an $A_2 \cup A_3$ only if $A_2 = A_3$.
(*d*) $A_1 \cup A_2 \cup A_3$ is the largest subset of X containin elements of each of A_1, A_2 and A_3.

INTEGER TYPE QUESTIONS

18. Universal set, $U = \{x : x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$

and $A = \{x : x^2 - 5x + 6 = 0\}$

$B = \{x : x^2 - 3x + 2 = 0\}$

Then $n(A \cap B)'$ is equal to

19. If $n(A \cap B) = 5$, $n(A \cap C) = 7$ and $n(A \cap B \cap C) = 3$, the the minimum possible value of $n(B \cap C)$ is

20. If $A = \{1,2,3,4\}$, then the number of subsets of set containing element 3, is

21. If $A = \left\{n : \frac{n^3 + 5n^2 + 2}{n} \text{ is an integer and } n \text{ itself is an integer}\right\}$ then the number of elements in the set A, is

22. From 50 students taking examinations in Mathematic Physics and Chemistry, 37 passed Mathematics, 24 Physi and 43 Chemistry. At most 19 passed Mathematics a Physics, at most 29 passed Mathematics and Chemistr and at most 20 passed Physics and Chemistry. Th largest possible number that could have passed all thr examinations is

Credit - Akshat

Parikshit (JEE Advanced Level)

SINGLE CORRECT TYPE QUESTIONS

1. Let $A = \{n \in N$: H.C.F. $(n, 45) = 1\}$ and let $B = \{2k : k \in \{1, 2, ..., 100\}\}$. Then the sum of all the elements of $A \cap B$ is given by
(*a*) 5265 (*b*) 4372 (*c*) 5264 (*d*) 5371

2. Let $A = \{n \in N \mid n^2 \leq n + 10{,}000\}$, $B = \{3k + 1 \mid k \in N\}$ and $C = \{2k \mid k \in N\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to ________.
(*a*) 831 (*b*) 832 (*c*) 732 (*d*) 730

3. Let Z be the set of all integers,
$A = \{(x, y) \in Z \times Z : (x - 2)^2 + y^2 \leq 4\}$
$B = \{(x, y) \in Z \times Z : x^2 + y^2 \leq 4\}$ and
$C = \{(x, y) \in Z \times Z : (x - 2)^2 + (y - 2)^2 \leq 4\}$
If the total number of relations from $A \cap B$ to $A \cap C$ is 2^p, then the value of p is ________.
(*a*) 16 (*b*) 49 (*c*) 25 (*d*) 9

4. If $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ are two sets. Then,
(*a*) $P \subset Q$ and $Q - P \neq \phi$ (*b*) $Q \not\subset P$
(*c*) $P \not\subset Q$ (*d*) $P = Q$

5. If $A = \{x \in c; x^2 = 1\}$ and $B = \{x \in c; x^4 = 1\}$, then $A \Delta B =$
(*a*) $\{-1, 1\}$; (*b*) $\{-i, i\}$;
(*c*) $\{-1, 1, i, -i\}$ (*d*) none

6. Let $A = \{n \in N: n$ is 3-digit number$\}$, $B = \{9k + 2: k \in N\}$ and $C = \{9k + l: k \in N\}$ for some $l (0 < l < 9)$. If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then l is equal to
(*a*) 5 (*b*) 7 (*c*) 9 (*d*) 20

7. There is a group of 265 person who like either singing or dancing or painting. In this group, 200 like singing, 110 like dancing and 55 like painting. If 60 persons like both singing and dancing, 30 like both singing and painting and 10 like all three activities, then the number of person who like only dancing and painting
(*a*) 10 (*b*) 20 (*c*) 30 (*d*) 40

8. In a group of 50 people, two tests were conducted, one for diabetes and one for blood pressure. 30 people were diagnosed with diabetes and 40 people were diagonsed with high blood pressure. What is the minimum number of people who were having diabetes and high blood pressure?
(*a*) 0 (*b*) 10 (*c*) 20 (*d*) 30

9. Out of 800 boys in a school, 224 played Cricket, 240 played Hockey and 336 played Basketball. Of the total, 64 played both Basketball and Hockey; 80 played Cricket and Basketball and 40 played Cricket and Hockey, 24 played all the three games. The number of boys who did not play any game is
(*a*) 128 (*b*) 216 (*c*) 240 (*d*) 160

10. If $A = \{(x, y) : y = e^{-x}\}$ and $B = \{(x, y) : y = -x\}$. Then,
(*a*) $A \cap B = \phi$ (*b*) $A \subset B$
(*c*) $B \subset A$ (*d*) $A \cap B = \{0, 1\}$

11. If $A_1, A_2, A_3, ...A_{100}$ are sets such that $n(A_i) = i + 2$; $A_1 \subset A_2 \subset A_3 ... \subset A_{100}$ and $\bigcap_{i=3}^{100} A_i = A_n$ then $n(A)$ is equal to
(*a*) 3 (*b*) 4 (*c*) 5 (*d*) 16

12. Let R be set of points inside a rectangle of sides a and b $(a, b > 1)$ with two sides along the positive direction of x-axis and y-axis and C be the set of points inside a unit circle central at origin, then
(*a*) $R = \{(x, y): 0 \leq x \leq a, 0 \leq y \leq b\}$;
(*b*) $R = \{(x, y): 0 < x < a, 0 < y < b\}$;
(*c*) $R \cup C = \text{R}$;
(*d*) None of these

13. For any three sets A_1, A_2, A_3. Let $B_1 = A_1$, $B_2 = A_2 - A_1$ and $B_3 = A_3 - (A_1 \cup A_2)$, then which of the following statement is always true.
(*a*) $A_1 \cup A_2 \cup A_3 \supset B_1 \cup B_2 \cup B_3$
(*b*) $A_1 \cup A_2 \cup A_3 = B_1 \cup B_2 \cup B_3$
(*c*) $A_1 \cup A_2 \cup A_3 \subset B_1 \cup B_2 \cup B_3$
(*d*) None of the above

14. If $A = \left\{x : \cos x > \frac{-1}{2}, 0 \leq x \leq \pi\right\}$ and $B = \left\{x : \sin x > \frac{1}{2}, \frac{\pi}{3} \leq x < \pi\right\}$ then
(*a*) $A \cap B = \left\{x : \frac{\pi}{3} \leq x \leq \frac{2\pi}{3}\right\}$
(*b*) $A \cap B = \left\{x : \frac{-\pi}{3} \leq x \leq \frac{2\pi}{3}\right\}$
(*c*) $A \cup B = \left\{x : \frac{-5\pi}{6} \leq x \leq \frac{5\pi}{6}\right\}$
(*d*) $A \cup B = \left\{x : 0 \leq x \leq \frac{\pi}{6}\right\}$

15. If $A = \{(x, y) : y = e^{2x}, x \in R\}$ and $B = \{(x, y) : y = e^{-2x}, x \in R\}$, then $A \cap B$ contains
(*a*) No points (*b*) One point
(*c*) Two points (*d*) Three points

16. If $A = \left\{(x, y) : y = \frac{4}{x}, 0 \neq x \in R\right\}$ and $B = \{(x, y) : y = x, x > 0, x \in R\}$
(*a*) $A \cap B = \phi$
(*b*) $A \cap B$ is singleton set
(*c*) $A \cap B$ has infinite elements
(*d*) $A \cap B$ has two elements

17. Let $A_1, A_2, ... A_m$ be m sets such that $O(A_i) = p \ \forall \ i = 1, 2, ..., m$ and $B_1, B_2, ..., B_n$ be n sets such that $O(B_j) = q \ \forall \ j = 1, 2, ..., n$. If $\bigcup_{i=1}^{m} A_i = \bigcup_{j=1}^{n} B_j = S$ and each element of S belongs

to exactly α number of A_i's and β number of B_j's, where $O(A_i)$ denotes number of elements in O_i, then,

(*a*) $pm = nq$ (*b*) $\alpha pm = \beta nq$

(*c*) $\beta pm = \alpha nq$ (*d*) $(pm)^{\beta} = (nq)^{\alpha}$

18. In an office, every employee likes at least one of tea, coffee and milk. The number of employees who like only tea, only coffee, only milk and all the three are all equal. The number of employees who like only tea and coffee, only coffee and milk and only tea and milk are equal and each is equal to the number of employees who like all the three. Then a possible value of the number of emplyees in the office is

(*a*) 65 (*b*) 90 (*c*) 77 (*d*) 85

19. If $A = \left\{ p : p = \frac{(n+2)(2n^5 + 3n^4 + 4n^3 + 5n^2 + 6)}{n^2 + 2n}, n, p \in Z^+ \right\}$ then the number of elements in the set A, is

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 6

20. Let $S = \{1, 2, 3, 4\}$. The total number of unordered pairs of disjoint subsets of S is equal to

(*a*) 25 (*b*) 34 (*c*) 42 (*d*) 41

21. An investigator interviewed 100 students to determine the performance of three drinks milk, coffee and tea. The investigator reported that 10 students take all three drinks milk, coffee and tea; 20 students take milk and coffee, 30 students take coffee and tea, 25 students take milk and tea, 12 students take milk only, 5 students take coffee only and 8 students take tea only. Then, the number of students who did not take any of the three drinks, is

(*a*) 10 (*b*) 20 (*c*) 25 (*d*) 30

22. If $A = \{\theta : 2\cos^2\theta + \sin\theta \le 2\}$ and $B = \left\{\theta : \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}\right\}$ then $A \cap B$ is equal to

(*a*) $\left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6}\right\}$

(*b*) $\left\{\theta : \pi \le \theta \le \frac{3\pi}{2}\right\}$

(*c*) $\left\{\theta : \frac{\pi}{2} \le \theta \le \frac{5\pi}{6} \text{ or } \theta \le \frac{3\pi}{2}\right\}$

(*d*) None of these

23. Consider all $2^n - 1$ non-empty subsets of the set $\{1, 2, 3, \ldots, n\}$. For every such subset, we find the product of the reciprocals of each of its elements. Then the sum of all these products is

(*a*) n (*b*) n + 1

(*c*) $\frac{1}{n^n}$ (*d*) none of these

24. Set A has 6 elements and its power set has n elements. Let m be sum of distinct cardinal numbers of elements of power set of A, then the value of $n - m$ is

(*a*) 43 (*b*) 40 (*c*) 35 (*d*) 50

25. If α, β, γ are number of elements in sets A, B and respectively and sum of number of all subsets of A, B and is 28, then maximum number of ordered triplets (α, β, γ) i

(*a*) 1 (*b*) 2 (*c*) 4 (*d*) 6

MULTIPLE CORRECT TYPE QUESTIONS

26. The number of elements in the set $\{n \in N: 10 \le n \le 100$ an $3^n - 3$ is a multiple of 7$\}$ is k, then k is divisible by

(*a*) 3 (*b*) 5 (*c*) 15 (*d*) 21

27. Let A = {1,2,3,4,5,6,7}. Define B = {T ⊆ A: either 1 ∉ or 2 ∈ T} and C = {T ⊆ A: T the sum of all the elements T is a prime number.} If the number of elements in the s B ∪ C is α, then sum of digits of α is divisible by

(*a*) 2 (*b*) 4 (*c*) 8 (*d*) 9

28. If the difference between the number of subset of two se A and B is 120 then correct statement is

(*a*) Sum of number of elements of two sets is 10

(*b*) Difference of number of elements of two sets is 4

(*c*) One of power set of two sets has less than 8 elements

(*d*) Product of cardinalities of two sets is 21

29. A class has 175 students. The following table shows th number of students studying one or more of the followin subjects in this case

Subjects	No. of students
Mathematics	100
Physics	70
Chemistry	46
Mathematics and Physics	30
Mathematics and Chemistry	28
Physics and Chemistry	23
Mathematics, Physics and Chemistry	18

(*a*) The number of students who are not offered any these three subjects = 22

(*b*) Number of students studying Mathematics only = 60

(*c*) Number of students studying Physics only = 35

(*d*) Number of students studying Chemistry only = 13

30. If $U = R$ and let $A = \{x \in R: 0 < x < 2\}$, $B = \{x \in R: 1 < x \le$ which of the following is true?

(*a*) $A \cap B = \{x \in R: 1 < x < 2\}$

(*b*) $A' = \{x \in R: x \le 0 \text{ or } x \ge 2\}$

(*c*) $A \cup B = \{x \in R: 0 \le x \le 3\}$

(*d*) None of these

31. Let $S = \{(n, b, c)\} \in N \times N \times N: a + b + c = 21, a \le b \le$ and $T = \{(a, b, c)\} \in N \times N \times N$: a, b, c are in A.P}, whe N is the set of all natural numbers. Then find the divisors number of elements in the set $S \cap T$.

(*a*) 1 (*b*) 6 (*c*) 7 (*d*) 8

COMPREHENSION BASED QUESTIONS

Comprehension (Q. No. 32 to 33):

n a survey of 100 students, the number of students studying the various languages is found as: English only 18; English but not Hindi 23; English and Sanskrit 8; Sanskrit and Hindi 8; English 26; Sanskrit 48 and no language 24. Find

32. How many students are studying Hindi,

(*a*) 18 (*b*) 9

(*c*) 4 (*d*) 15

33. How many students are studying English and Hindi both.

(*a*) 3 (*b*) 6

(*c*) 9 (*d*) 2

MATCH THE COLUMN TYPE QUESTIONS

34. Match the following sets for all sets A, B and C

	Column-I		Column-II
A.	$((A' \cup B')' - A)'$	p.	$A - B$
B.	$[B' \cup (B' - A)]'$	q.	U
C.	$(A - B) - (B - C)$	r.	B
D.	$(A - B) \cap (C - B)$	s.	$(A \cap C) - B$

(*a*) A → p, B → q, C → r, D → s

(*b*) A → q, B → r, C → p, D → s

(*c*) A → q, B → r, C → s, D → p

(*d*) A → p, B → r, C → q, D → s

35. Match the set P in Column-I with its super set Q in Column-II

	Column-I		Column-II
A.	$[3^{2n} - 8n - 1 : n \in N]$	p.	$\{49(n-1) : n \in N\}$
B.	$\{2^{3n} - 1 : n \in N\}$	q.	$\{64(n-1) : n \in N\}$
C.	$\{3^{2n} - 1 : n \in N\}$	r.	$\{7n : n \in N\}$
D.	$\{2^{3n} - 7n - 1 : n \in N\}$	s.	$\{8n : n \in N\}$

(*a*) A → p, B → q, C → r, D → s

(*b*) A → q, B → r, C → p, D → s

(*c*) A → q, B → r, C → s, D → p

(*d*) A → p, B → r, C → q, D → s

INTEGER TYPE QUESTIONS

36. Find the number of ways to choose an ordered pair (a, b) of numbers from the set $\{1, 2, ..., 10\}$ such that $|a - b| \le 5$.

37. In a class of 80 students numbered 1 to 80, all odd numbered students opt of Cricket, students whose numbers are divisible by 5 opt for Football and those whose numbers are divisible by 7 opt for Hockey. If the number of students who do not opt any of the three games is n, then $\frac{n}{4}$ is equal to

38. A survey shows that 61%, 46% and 29% of the people watched "3 idiots", "Rajneeti" and "Avatar" respectively. 25% people watched exactly two of the three movies and 3% watched none. What percentage of people watched all the three movies?

39. If $n(A) = 4$ and $n(B) = 7$, then the difference between maximum and minimum value of $n(A \cup B)$ is

40. Let $S = \{x \in R : |x - 2|^{10x^2 - 1} = |x - 2|^{3x}\}$. Then $n(S)$ is

41. Let $S = \{x \in R : 5^x \sqrt[x]{8^{x-1}} = 500\}$. Then $n(S)$ is

42. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, and 3 read all the three newspapers. Find the number of people who read at least one of the newspapers.

43. Times, Mirror and Sun are three newspapers

(*i*) All readers of the Times read the Sun

(*ii*) Every person either reads the Sun or does not read the Mirror.

(*iii*) 11 people read the Sun but do not read the Mirror

(*iv*) 8 people read either the Times or the Mirror but not both.

(*v*) 10 people read the Sun and either read the Mirror or do not read the Times

(*vi*) 14 people either read the Sun and not the Mirror or read both the Sun and Times

(*vii*) 9 people neither read the Times nor the Mirror

Find the number of people who read Times and Mirror both

44. Let $S = \{x \in R : 5^x + \frac{125}{5^x} = 30\}$. Then sum of all possible value of x equal to.

45. Let $T = \left\{x \mid \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}\right\}$. Then number of elements in T equal to.

46. Given that $n(U) = 40$, $n(A) = 28$, $n(B) = 25$, $n(A \cap B) = x$ and $n(A' \cap B') = y$. Find the sum of greatest and least values of x and y both.

47. Let $S = \{1, 2, 3 ..., 50\}$. The number of non-empty subsets A of S such that product of elements in A is even, is $2^m(2^n - 1)$, then the value of $(m + n)$ is equal to ....

48. The maximum number of sets obtainable from A and B applying union and difference operation is ....

49. In a factory 70% of the workers like oranges and 64% like apples. If x% like both oranges and apples, then the minimum value of x is ....

50. Suppose $A_1, A_2, ..., A_6$ are six sets each with 4 elements and $B_1, B_2, ..., B_n$ are n sets each two elements such that $A_1 \cup A_2 \cup ... \cup A_6 = B_1 \cup B_2 \cup ... \cup B_n = S$ (say). Given that each element of S belongs to exactly 4 of the A_i's and exactly 3 of the B_j's then find the value of n.

PYQ's (Past Year Questions)

1. Let A and B be two finite sets with m and n elements respectively. The total number of subsets of the set A is 56 more than the total number of subsets of B. Then the distance of the point $P(m, n)$ from the point $Q(-2, -3)$ is **[27 Jan, 2024 (Shift-II)]**

   (a) 10 (b) 6
   (c) 4 (d) 8

2. Let the set $C = \{(x, y) \mid x^2 - 2^y = 2023, x, y \in \mathbb{N}\}$. Then $\sum_{(x,y)\in C} (x + y)$ is equal to **[29 Jan, 2024 (Shift-II)]**

3. A group of 40 students appeared in an examination of 3 subjects - Mathematics, Physics & Chemistry. It was found that all students passed in at least one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, at most 11 students passed in both Mathematics and Physics, at most 15 students passed in both Physics and Chemistry, at most 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is **[30 Jan, 2024 (Shift-I)]**

4. Let S be the set of positive integral values of a for which $\dfrac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32} < 0, \forall x \in R$.

   Then, the number of elements in S is: **[31 Jan, 2024 (Shift-I)]**

   (a) 1 (b) 0
   (c) ∞ (d) 3

5. Let $S = \left\{x \in R : (\sqrt{3} + \sqrt{2})^x + (\sqrt{3} - \sqrt{2})^x = 10\right\}$. Then the number of elements in S is: **[01 Feb, 2024 (Shift-I)]**

   (a) 4 (b) 0
   (c) 2 (d) 1

6. The number of elements in the set $S = \{(x, y, z) : x, y, z \in Z, x + 2y + 3z = 42, x, y, z \geq 0\}$ equals __________. **[01 Feb, 2024 (Shift-I)]**

7. In a survey of 220 students of a higher secondary school, it was found that at least 125 and at most 130 students studied Mathematics; at least 85 and at most 95 studied Physics; at least 75 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students who studied all the three subjects. Then $m + n$ is equal to ________ **[04 April, 2024 (Shift-I)]**

8. If $S = \{a \in R : |2a - 1| = 3[a] + 2\{a\}\}$ where $[t]$ denotes th greatest integer less than or equal to t and $\{t\}$ represents th fractional part of t, then $72\sum_{a\in S} a$ is equal to ________. **[05 April, 2024 (Shift-I**

9. Let the set $S = \{2, 4, 8, 16, \ldots, 512\}$ be partitioned into 3 sets A, B, C with equal number of elements such that $A \cup B \cup C =$ and $A \cap B = B \cap C = A \cap C = \phi$. The maximum number such possible partitions of S is equal to : **[05 April, 2024 (Shift-II**

   (a) 1680 (b) 1520 (c) 1710 (d) 1640

10. Let $A = \{n \in [100, 700] \cap N : n$ is neither a multiple of nor a multiple of $4\}$. Then the number of elements in A is **[06 April, 2024 (Shift-I**

    (a) 300 (b) 280 (c) 310 (d) 290

11. If the set $R = \{(a, b) ; a + 5b = 42, a, b \in \mathbb{N}\}$ has m elemen and $\sum_{n=1}^{m}\left(1 + i^{n!}\right) = x + iy$, where $i = \sqrt{-1}$, then the value $m + x + y$ s: **[08 April, 2024 (Shift-I**

    (a) 8 (b) 12 (c) 4 (d) 5

12. An organization awarded 48 medals in event 'A', 25 event 'B' and 18 in event 'C'. If these medals went total 60 men and only five men got medals in all the thr events, then, how many received medals in exactly two three events? **[11 April, 2023 (Shift-I**

    (a) 10 (b) 9 (c) 21 (d) 15

13. The number of elements in the set $\{n \in \mathrm{N} : 10 \leq n$ 100 and $3^n - 3$ is a multiple of $7\}$ is ________.

    [15 April, 2023 (Shift-I

14. The number of elements in the set $\{n \in \mathbb{Z} : |n^2 - 10n + 19| <$ is __________. **[10 April, 2023 (Shift-**

15. Let $S = \left\{x \in [-6, 3] - \{-2, 2\} : \dfrac{|x + 3| - 1}{|x| - 2} \geq 0\right\}$ and

    $T = \{x \in Z : x^2 - 7|x| + 9 \leq 0\}$. Then the number of eleme of $S \cap T$ is **[28 July, 2022 (Shift-I**

    (a) 7 (b) 5 (c) 4 (d) 3

16. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Define $B = \{T \subseteq A :$ either 1 or $2 \in T\}$ and $C = \{T \subseteq A :$ T is the sum of all the eleme of T is a prime number$\}$. Then the number of elements the set $B \cup C$ is ________ **[25 July, 2022 (Shift-I**

17. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{3, 6, 7, 9\}$. Tl the number of elements in the set $\{C \subseteq A : C \cap B \neq$ is __________ **[26 July, 2022 (Shift-I**

Credit - Akshat

18. The sum of all the elements of the set $\{\alpha \in \{1, 2, ..., 100\} : \text{HCF}(\alpha, 24) = 1\}$ is **[24 June, 2022 (Shift-II)]**

19. Let $A = \{n \in N : \text{H.C.F.}(n, 45) = 1\}$ and Let $B = \{2k : k \in \{1, 2, ..., 100\}\}$. Then the sum of all the elements of $A \cap B$ is_______. **[26 June, 2022 (Shift-I)]**

20. Let $S = \{4, 6, 9\}$ and $T = \{9, 10, 11, ..., 1000\}$. If $A = \{a_1 + a_2 + ... + a_k : k \in N, a_1, a_2, a_3, ..., a_k \in S\}$, then the sum of all the elements in the set $T - A$ is equal to **[29 July, 2022 (Shift-I)]**

21. Let $S = \{(x, y) \in N \times N : 9(x-3)^2 + 16(y-4)^2 \leq 144\}$ and $T = \{(x, y) \in R \times R : (x-7)^2 + (y-4)^2 \leq 36\}$. Then $n(S \cap T)$ is equal to **[29 July, 2022 (Shift-II)]**

22. If $A = \{x \in R; |x - 2| > 1\}$, $B = \{x \in R : \sqrt{x^2 - 3} > 1\}$, $C = \{x \in R; |x - 4| \geq 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^C \cap Z$ is ______. **[27 Aug, 2021 (Shift-I)]**

23. The number of elements in the set $\{x \in R : (|x| - 3)\,|x + 4| = 6\}$ is equal to [16 March, 2021 (Shift-I)]

(*a*) 3 (*b*) 2 (*c*) 4 (*d*) 1

24. Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K can not belong to the set: **[26 Aug, 2021 (Shift-I)]**

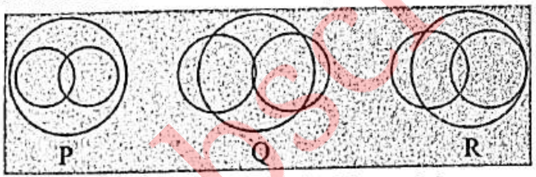
(*a*) {80, 83, 86, 89}

(*b*) {84, 87, 90, 93}

(*c*) {84, 86, 88, 90}

(*d*) {79, 81, 83, 85}

25. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement? **[17 March, 2021 (Shift-I)]**



(*a*) Q and R (*b*) None of these

(c) P and R (*d*) P and Q

26. Let $S = \{1, 2, 3, 4, 5, 6, 9\}$. Then the number of elements in the set $T = \{A \subseteq S : A \neq \phi$ and the sum of all the elements of A is not a multiple of 3$\}$ is **[27 Aug, 2021 (Shift-II)]**

27. The number of elements in the set $\{n \in \{1, 2, 3, ...100\} \mid (11)^n > (10)^n + (9)^n\}$ is ______ **[22 July, 2021 (Shift-II)]**

28. Let $A = \{n \in N : n$ is a 3-digit number$\}$ $B = \{9k + 2 : k \in N\}$ and $C = \{9k + \ell : k \in N\}$ for some $\ell (0 < \ell < 9)$. If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then ℓ is equal to__________. **[24 Feb, 2021 (Shift-I)]**

29. Let $A = \{n \in N \mid n^2 \leq n + 10{,}000\}$, $B = \{3k + 1 \mid k \in N\}$ and $C = \{2k \mid k \in N\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to ____. **[27 July, 2021 (Shift-II)]**

30. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be: **[5 Sep, 2020 (Shift-I)]**

(*a*) 63 (*b*) 36

(*c*) 38 (*d*) 54

31. If $A = \{x \in R : |x| < 2\}$ and $B = \{x \in R : |x - 2| \geq 3\}$: then: **[9 Jan, 2020 (Shift-II)]**

(*a*) $B - A = R - (-2, 5)$ (*b*) $A \cap B = (-2, -1)$

(*c*) $A - B = [-1, 2)$ (*d*) $A \cup B = R - (2, 5)$

32. Let $\bigcup_{i=1}^{50} X_i = \bigcup_{i=1}^{n} Y_i = T$ where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i's and exactly 6 of sets Y_i's, then n is equal to **[4 Sep, 2020 (Shift-II)]**

(*a*) 50 (*b*) 15

(*c*) 30 (*d*) 45

33. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be: **[4 Sep, 2020 (Shift-I)]**

(*a*) 37 (*b*) 55

(*c*) 29 (*d*) 65

34. Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than total number of subsets of B, then the value of $m \cdot n$ is ________. **[6 Sep, 2020 (Shift-I)]**

35. Let $X = \{n \in N : 1 \leq n \leq 50\}$. If $A = \{n \in X : n$ is a multiple of 2$\}$; $B = \{n \in X : n$ is a multiple of 7$\}$, then the number of elements in the smallest subset of X containing both A and B is ____________. **[7 Jan, 2020 (Shift-II)]**

36. In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is: **[10 Jan, 2019 (Shift-I)]**

(*a*) 102 (*b*) 42

(c) 1 (*d*) 38

37. Let A, B and C be sets such that $\phi \neq A \cap B \subseteq C$. Then which of the following statements is not true? **[12 April, 2019 (Shift-II)]**

(*a*) If $(A-C) \subseteq B$ then $A \subseteq B$

(*b*) $(C \cup A) \cap (C \cup B) = C$

(*c*) If $(A-B) \subseteq C$, then $A \subseteq C$

(*d*) $B \cap C \neq \phi$

38. Let Z be the set of integers. If $A = \{X \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$ and $B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set $A \times B$, is: **[12 Jan, 2019 (Shift-II)]**

(*a*) 2^{15} (*b*) 2^{18} (*c*) 2^{12} (*d*) 2^{10}

39. Let $S = \{1, 2, 3, \ldots\ldots, 100\}$. The number of non-empty subsets A of S such that the product of elements in A is even is: **[12 Jan, 2019 (Shift-I)]**

(*a*) $2^{100} - 1$ (*b*) $2^{50}(2^{50} - 1)$

(*c*) $2^{50} - 1$ (*d*) $2^{50} + 1$

PW Challengers

1. Let X be a set of 56 elements. Find the least positive integer n such that for any 15 subsets of X, if the union of every 7 sets of these subsets contains at least n elements, then there exist 3 of the 15 subsets whose intersection is nonempty.

2. Let n be a positive integer such that $1 \leq n \leq 1000$. Let M_n be the number of integers in the set

$X_n = \left\{\sqrt{4n+1}, \sqrt{4n+2}, \ldots\ldots, \sqrt{4n+1000}\right\}$. Let $a = \max\{M_n : 1 \leq n \leq 1000\}$, and $b = \min\{M_n : 1 \leq n \leq 1000\}$. Find $a - b$.

3. Consider a set $S = \left\{a_1^{2025}, a_2^{2025}, a_3^{2025}, \ldots\ldots, a_n^{2025}\right\}$ and Say S_i denote the subsets of $S\ \forall i$, then, the total number of unordered pairs of dispoint subset S is equal to, (where a_i's are distinct $\forall i = 1, \ldots, n$)

(*a*) $\dfrac{3^n+1}{2}$ (*b*) $\dfrac{3^{n+1}}{2}$

(*c*) $\dfrac{3^n-1}{2}+2025$ (*d*) $\dfrac{3^{n-1}}{2}$

4. Consider the set X_1, X_2, X_3 such that $X_2, X_3, \subseteq X_1$, and X_1 contain 6626 elements, $n(X_2 \cup X_3) = 793$.

If $n\left(X_2' \cap X_3'\right) = \alpha_1^3 + \alpha_2^3 + \alpha_3^3 + \alpha_4^3 = \beta_1^3 + \beta_2^3 + \beta_3^3 + \beta_4^3$ for some positive integers $\alpha_1 < \alpha_3 < \beta_1 \leq \beta_3 < \beta_2 < \alpha_2 < \beta_4 < \alpha_4$

Then $\dfrac{\left(\alpha_3^2 + \alpha_4\right) - \left(\alpha_1^2 + \alpha_2\right)}{(\beta_2 - \beta_1)(\beta_4 - \beta_3)} =$

(*a*) $\dfrac{6}{7}$ (*b*) $\dfrac{8}{7}$ (*c*) $\dfrac{7}{6}$ (*d*) $\dfrac{4}{7}$

5. If $A_1 \times A_2 \times A_3 \times A_4$
$= \{(1, 1, 1, 1), (2, 4, 8, 16), (3, 9, 27, 81), \ldots\}$
and $\forall a_i \in A_1, \forall b_i \in A_2, \forall c_i \in A_3, \forall d_i \in A_4$

Define $\alpha = 7\sum_{i=1}^{10} a_i$; $\beta = 2\sum_{i=1}^{10} b_i$

$\eta = 8\sum_{i=1}^{10} c_i$; $\delta = \sum_{i=1}^{10} d_i$

Then, $\alpha + \beta + \eta - \delta =$

6. A, B, C be three sets such that $n(A) = 2$, $n(B) = 3$, $n(C) = 4$ If $P(X)$ denotes power set of X,

$K = \dfrac{n(P(P(C)))}{n(P(P(A))) \times n(P(P(B)))}$. Sum of digits of K is____.

7. Let U be set with number of elements in it is 2009. A i a subset of U with $n(A) = 1681$ and out of these 168 elements, exactly 1075 elements belong to a subset B o U. If $n(A - B) = m^2 + p_1 p_2 p_3$ for some positive intege m and distinct primes $p_1 < p_2 < p_3$ then for least m fin $7\dfrac{p_1 p_3}{p_2}$.

8. Consider $A_1, A_2, \ldots, A_{2025}$ be 2025 sets, such that $A_1 \subset A$ $\subset A_3, \ldots \subset A_{2025}$, then choose the correct statement fro following:

(*a*) If $n(A_i) = i + 1$, then $\bigcup_{i=1}^{2025} A_i$ contains 2026 elements

(*b*) If $n(A_i) = i + 1$, then $\bigcup_{i=1}^{2025} A_i$ contains 2025 elements

(*c*) If $n(A_i) = i + 2$, and $\bigcup_{i=3}^{2025} A_i = A$ then $n(A) = 5!$

(*d*) If $n(A_i) = i + 2$, and $\bigcup_{i=3}^{2025} A_i = A$ then $n(A) = 6!$

9. $A = \{x : x \in N$, G.C.D. $(x, 36) = 1, x < 36\}$, $B = \{y : y \in$ G.C.D. $(y, 40) = 1, y < 40\}$; (G.C.D. stands for greate common divisors)

Column-I		**Column-II**	
A.	$n(A \cap B)$	p.	10
B.	$C = \{x : x \in A \cup B, x$ is prime $\}$, $n(C)=$	q.	9
C.	$n(A \Delta B)$	r.	21
D.	$n((A - B) \times (B - A))$	s.	11

10. Consider the set X, such that

$$X = \left\{ x : x = \frac{(n^2)!\left((n!)^2 + 2026n!\right) + (n!)^{n+1}(14n^5 + 30n^4 + 10n^3 + 123n^2 + 36n + 24)(n! + 2026)}{\left((n!)^2 + 2026n!\right)(n!)^{n+1}}, x \in Z \right\}$$

Then the number of elements in set A = ?

11. Find the maximum number of pair wise disjoint sets of the form $S(a, b) = \{n^2 + an + b : n \in Z\}$ with $a, b \in Z$.

12. Let $S = \{1,2,3....n\}$ and $f(n)$ be the number of those subsets of S which do not contain consecutive elements of S then

(a) $f(n) = \frac{n(n-1)(n-2)}{6}$

(b) $f(n) = 2f(n-1)$

(c) $f(n) = f(n-1) + f(n-2)$

(d) $f_4 = 8$

13. Let $a_1, a_2, a_3, ... a_{23}$ be 23 integers (not necessarily distinct). Then, the prime number which always divides the sum of elements of atleast one subset of these numbers can be

(a) 23 (b) 29 (c) 31 (d) 37

14. If the collection of all these three elements subsets drawn from the set $\{1, 2, 3, \ldots, 300\}$. Find the number of these subsets for which the sum of the three elements is a multiple of 3.

15. How many 3-element subset of the set $\{1, 2, 3, \ldots, 20\}$ are there such that the product of the three number in the subset is divided by 4?

"SCENT RELAXATION" DURING STUDY

- Use calming scents like lavender, eucalyptus, or chamomile while studying or before bed. These scents can help reduce anxiety and promote relaxation.
- Scents are linked to emotions and can help create a peaceful atmosphere conducive to relaxation.
- Light a lavender-scented candle or use a diffuser in your study space to create a calming environment while you prepare for exams.

ANSWER KEY

CONCEPT APPLICATION

1. (*b*) 2. (*a*) 3. (*a*) 4. (*b*) 5. (*d*) 6. (*c*) 7. (*d*) 8. (*d*) 9. (*c*)
10. (*b*) 11. (*a*) 12. (*c*) 13. (*a*) 14. (*c*) 15. P → $A \cap \overline{B}$, Q → $A \cap B$, R → $B \cap \overline{A}$ and S → $\overline{A} \cap \overline{B}$
16. (*iii*) 4,54

SCHOOL LEVEL PROBLEMS

1. (*d*) 2. (*b*) 3. (*a*) 4. (*b*) 5. (*a*) 6. (*a*) 7. (*c*) 8. (*a*) 9. (*b*) 22. (*a*)
23. (*b*) 24. (*a*) 25. (*c*)

PRARAMBH (TOPICWISE)

1. (*d*) 2. (*b*) 3. (*a*) 4. (*c*) 5. (*b*) 6. (*a*) 7. (*b*) 8. (*c*) 9. (*a*) 10. (*d*)
11. (*b*) 12. (*a*) 13. (*c*) 14. (*b*) 15. (*c*) 16. (*b*) 17. (*b*) 18. (*d*) 19. (*c*) 20. (*d*)
21. (*c*) 22. (*d*) 23. (*b*) 24. (*a*) 25. (*b*) 26. (*b*) 27. (*c*) 28. (*d*) 29. (*b*) 30. (*c*)
31. (*c*) 32. (*a*) 33. (*b*) 34. (*a*) 35. (*b*) 36. (*b*) 37. (*d*) 38. (*b*) 39. (*b*) 40. (*a*)
41. (*b*)

PRABAL (JEE MAIN LEVEL)

1. (*b*) 2. (*c*) 3. (*a*) 4. (*c*) 5. (*c*) 6. (*d*) 7. (*b*) 8. (*b*) 9. (*c*) 10. (*a*)
11. (*b*) 12. (*d*) 13. (*c*) 14. (*b*) 15. (*d*) 16. (*d*) 17. (*d*) 18. [3] 19. [3] 20. [8]
21. [4] 22. [14]

PARIKSHIT (JEE ADVANCED LEVEL)

1. (*c*) 2. (*b*) 3. (*c*) 4. (*d*) 5. (*b*) 6. (*a*) 7. (*a*) 8. (*c*) 9. (*d*) 10. (*a*)
11. (*c*) 12. (*b*) 13. (*a*) 14. (*a*) 15. (*b*) 16. (*b*) 17. (*c*) 18. (*c*) 19. (*c*) 20. (*d*)
21. (*b*) 22. (*d*) 23. (*a*) 24. (*a*) 25. (*d*) 26. (*a,b,c*) 27. (*a,b,c*) 28. (*a,b,d*) 29. (*a,b,c,d*) 30. (*a,b*)
31. (*a,c*) 32. (*a*) 33. (*a*) 34. (*b*) 35. (*c*) 36. [80] 37. [7] 38. [7] 39. [4] 40. [5]
41. [1] 42. [52] 43. [3] 44. [3] 45. [1] 46. [50] 47. [50] 48. [8] 49. [34] 50. [9]

PYQ's (PAST YEAR QUESTIONS)

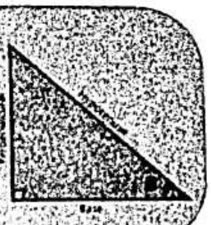
1. (*a*) 2. [46] 3. [10] 4. (*b*) 5. (*c*) 6. [169] 7. [45] 8. [18] 9. (*a*) 10. (*a*)
11. (*b*) 12. (*c*) 13. [15] 14. [6] 15. (*d*) 16. [107] 17. [112] 18. [1633] 19. [5264] 20. [11]
21. [27] 22. [256] 23. (*b*) 24. (*d*) 25. (*b*) 26. [80] 27. [96] 28. [5] 29. [832] 30. (*b*)
31. (*a*) 32. (*c*) 33. (*b*) 34. [28] 35. [29] 36. (*d*) 37. (*a*) 38. (*a*) 39. (*b*)

PW CHALLENGERS

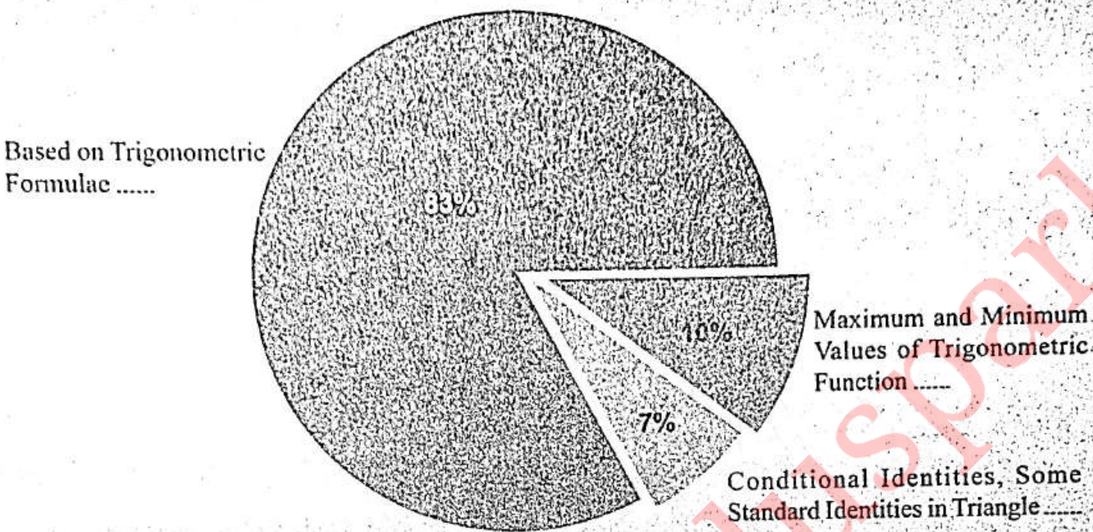
1. [41] 2. [22] 3. (*a*) 4. (*c*) 5. [22] 6. [7] 7. [86] 8. (*a*)
9. A → q, B → s, C → p, D → r 10. [3] 11. [2] 12. (*c,d*) 13. (*a*) 14. [1495100] 15. [795]

CHAPTER 3

Trigonometric Functions



Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)



"How's the Josh?" for these Topics: *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

NTRODUCTION

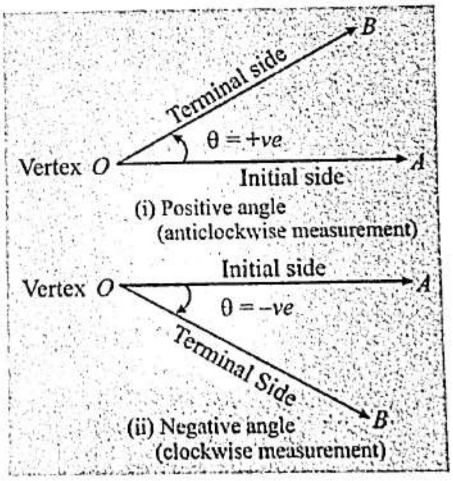
The word trigonometry is derived from three greek words.

Tri → three
Gon → sides
Metron → Measure

In the ancient time trigonometry defines relations between elements of a triangle. In a triangle there are six basic elements, three sides and three angles. Any three line segments will form a triangle if they satisfy three triangular inequalities i.e. the sum of any two line segments is greater than third side. In Euclidean geometry the sum of three angles of a triangle is 180°. These requirements impose limitations on the manner in which the relations between the elements are defined.

ANGLE

Angle is a measure of rotation of a given ray about its initial point. The original ray is called the initial side and the final position of the ray after rotation is called the terminal side of the angle. The point of rotation is called the vertex. If the direction of rotation is anticlockwise, the angle is said to be positive and if the direction of rotation is clockwise, then the angle is negative.



(i) Positive angle (anticlockwise measurement)

(ii) Negative angle (clockwise measurement)

SYSTEMS FOR MEASUREMENT OF ANGLES

An angle can be measured in the following systems.

1. **Sexagesimal System (British System):** In this system $\frac{1}{360}$ of a complete circular turn is called a degree (°), $\frac{1}{60}$ of a

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degree is called a minute (′) and $\frac{1}{60}$ of a minute is called a second (″).

One right angle = 90°, $1° = 60'$, $1' = 60''$

ADVANCED LEARNING

2. **Centesimal System (French System):** In this system $\frac{1}{400}$ of a complete circular turn is called a grade (g), $\frac{1}{100}$ of a grade is called a minute (′) and $\frac{1}{100}$ of a minute is called a second (″).

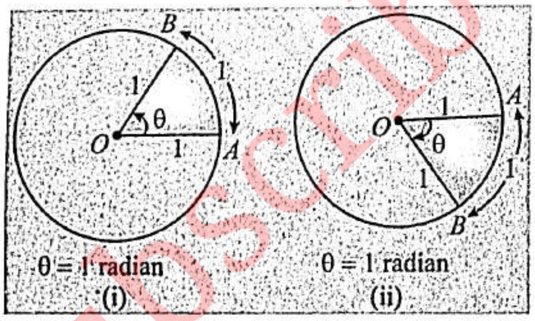
   $\therefore$ One right angle = 100^g; $1^g = 100'$; $1' = 100''$

   Note: The minutes and seconds in the Sexagesimal system are different with the minutes and seconds respectively in the Centesimal System. Symbols in both systems are also different.

3. **Circular System (Radian Measurement):** The angle subtended by an arc of a circle whose length is equal to the radius of the circle at the centre of the circle is called a radian. In this system the unit of measurement is radian (ᶜ) As the circumference of a circle of radius 1 unit is 2π, therefore one complete revolution of the initial side subtends an angle of 2π radian.

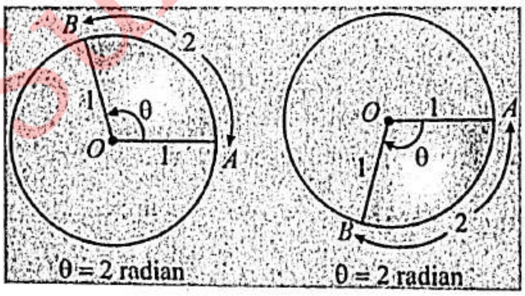
More generally, in a circle of radius r, an arc of length r will subtend an angle of 1 radian. It is well-known that equal arcs of a circle subtend equal angle at the centre. Since in a circle of radius r, an arc of length r subtends an angle whose measure is 1 radian, an arc of length ℓ will subtend an angle whose measure is $\frac{\ell}{r}$ radian. Thus, if in a circle of radius r, arc of length ℓ subtends an angle θ radian at the centre, we have

$\theta = \frac{\ell}{r}$ or $\ell = r\theta$.



θ = 1 radian
(i)

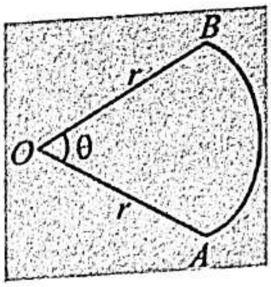
θ = 1 radian
(ii)



θ = 2 radian

θ = 2 radian

Area of Circular Sector



Area = $\frac{1}{2}r^2\theta$ sq. units

Note: θ radian is written as θ^c or can be written simply as θ. Wh the unit of angle is not mentioned. it must be taken as radians.

e.g. θ = 15 implies 15 radian

Relation between radian, degree and grade:

$\frac{\pi}{2}$ radian = 90° = 100^g

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Example 1: Find the radian measure corresponding to −37° 30′.

Sol. $60' = 1°$

$$\therefore 30' = \left(\frac{1}{2}\right)^\circ$$

$$\therefore -37°\,30' = -37\frac{1}{2}^\circ = -\frac{75}{2}^\circ$$

$360° = 2\pi$ radians

$$\therefore \quad \frac{-75^\circ}{2} = -\frac{2\pi}{360}\times\frac{75}{2} \text{ radians} = \frac{-5\pi}{24} \text{ radians}$$

Example 2: The minute hand of a clock is 10 cm long. How far does the tip of the hand move in 20 minutes?

Sol. The minute hand moves through 120° in 20 minutes or moves through $\frac{2\pi}{3}$ radians.

Since the length of the minute hand is 10 cm, the distance moved by the tip of the hand is given by the formula

$$l = r\theta = 10 \times \frac{2\pi}{3} = \frac{20\pi}{3} \text{ cm.}$$

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Concept Application

1. In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc corresponding to the chord.
2. The angles of a triangle are in AP and the number of degrees in the least is to the number of radians in the greatest as 60 to π find the angles in degrees.
3. Assuming that is a person of normal sight can read print at such a distance that the letters subtend an angle of 5' at his eye, find the height of the letters that he can read at a distance of 12 metres.

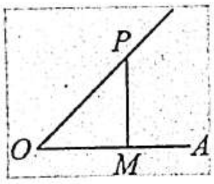
TRIGONOMETRIC RATIOS FOR ACUTE ANGLES

Let a revolving ray OP starts from OA and revolves into the position OP, thus tracing out the angle AOP.

In the revolving ray take any point P and draw PM perpendicular to the initial ray OA.

In the right angle triangle MOP, OP is the hypotenuse, PM is the perpendicular, and OM is the base.

The trigonometrical ratios, or functions, of the angle AOP are defined as follows:



$\frac{MP}{OP}$, i.e., $\frac{\text{Perpendicular}}{\text{Hypotenuse}}$, sin $\angle AOP$

$\frac{OM}{OP}$, i.e. $\frac{\text{Base}}{\text{Hypotenuse}}$, cos $\angle AOP$;

$\frac{MP}{OM}$, i.e. $\frac{\text{Perpendicular}}{\text{Base}}$, tan $\angle AOP$;

$\frac{OM}{MP}$, i.e. $\frac{\text{Base}}{\text{Perpendicular}}$, cot $\angle AOP$;

$\frac{OP}{OM}$, i.e. $\frac{\text{Hypotenuse}}{\text{Base}}$, sec $\angle AOP$;

$\frac{OP}{MP}$, i.e. $\frac{\text{Hypotenuse}}{\text{Perpendicular}}$, cosec $\angle AOP$;

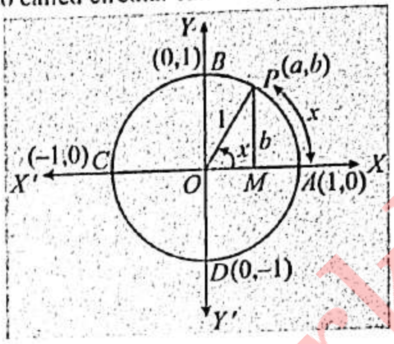
The quantity by which the cosine falls short of unity i.e. $1 - \cos \angle AOP$, is called the Versed Sine of $\angle AOP$; also the quantity $1 - \sin \angle AOP$, by which the sine falls short of unity, is called the Coversed Sine of $\angle AOP$.

It can be noted that the trigonometrical ratios are all real numbers.

The names of these eight ratios are written, for brevity, sin $\angle AOP$, cos $\angle AOP$, tan $\angle AOP$, cot $\angle AOP$, cosec $\angle AOP$, sec $\angle AOP$, vers $\angle AOP$, and covers $\angle AOP$ respectively

TRIGONOMETRIC RATIOS FOR AN ANGLE

We will now extend the definition of trigonometric ratios to any angle in terms of radian measure and study them as trigonometric functions. (also called circular functions)



Consider a unit circle (radius 1 unit) with centre at origin of the coordinate axes. Let $P(a, b)$ be any point on the circle with angle $\angle AOP = x$ radian, i.e., length of arcAP $= x$

We define $\cos x = a$ and $\sin x = b$ Since ΔOMP is a right triangle, we have $OM^2 + MP^2 = OP^2$ or $a^2 + b^2 = 1$ Thus, for every point on the unit circle we have $a^2 + b^2 = 1$ or $\cos^2 x + \sin^2 x = 1$

Since one complete revolution subtends an angle of 2π radian at the centre of the circle, $\angle AOB = \frac{\pi}{2}.$, $\angle AOC = \pi$ and $\angle AOD = \frac{3\pi}{2}$. All angles which are integral multiples of $\frac{\pi}{2}.$ are called quadrantal angles. The coordinates of the points A, B, C and D are, respectively, $(1, 0)$, $(0, 1)$, $(-1, 0)$ and $(0, -1)$. Therefore, for quadrantal angles, we have

$\cos 0 = 1 \qquad \sin 0 = 0,$

$\cos \frac{\pi}{2}. = 0 \qquad \sin \frac{\pi}{2}. = 1$

$\cos \pi = -1 \qquad \sin \pi = 0$

$\cos \frac{3\pi}{2} = 0 \qquad \sin \frac{3\pi}{2} = -1$

$\cos 2\pi = 1 \qquad \sin 2\pi = 0$

Now if we take one complete revolution from the position OP, we again come back to same position OP. Thus, we also observe that if x increases (or decreases) by any integral multiple of 2π, the values of sine and cosine functions do not change. Thus,

$\sin(2n\pi + x) = \sin x, n \in Z$, $\cos(2n\pi + x) = \cos x, n \in Z$

Further, $\sin x = 0$, if $x = 0, \pm\pi, \pm 2\pi, \pm 3\pi$ ....., i.e., when x is an integral multiple of π and $\cos x = 0$, if $x = \pm\frac{\pi}{2}.$, $\pm\frac{3\pi}{2}$, $\pm\frac{5\pi}{2}$, ...i.e., $\cos x$ vanishes when x is an odd multiple of $\frac{\pi}{2}$.

Thus $\sin x = 0$ implies $x = n\pi$, where n is any integer

$\cos x = 0$ implies $x = (2n + 1)\frac{\pi}{2}.$, where n is any integer.

We now define other trigonometric functions in terms of sine and cosine functions:

$\operatorname{cosec} x = \frac{1}{\sin x}$, $x \neq n\pi$, where n is any integer.

$\sec x = \frac{1}{\cos x}$, $x \neq (2n+1)\frac{\pi}{2}$, where n is any integer.

$\tan x = \frac{\sin x}{\cos x}$, $x \neq (2n+1)\frac{\pi}{2}$, where n is any integer.

$\cot x = \frac{\cos x}{\sin x}$, $x \neq n\pi$, where n is any integer.

BASIC TRIGONOMETRIC IDENTITIES

1. $\sin^2\theta + \cos^2\theta = 1$; where $\theta \in R$.
2. $1 + \tan^2\theta = \sec^2\theta$ or $\sec^2\theta - \tan^2\theta = 1$
3. $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ or $\operatorname{cosec}^2\theta - \cot^2\theta = 1$
4. $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cdot\cos^2\theta$
5. $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cdot\cos^2\theta$

Note: (*i*) $(\sec\theta - \tan\theta)$ is reciprocal of $(\sec\theta + \tan\theta)$ and vice-versa.

(*ii*) $(\operatorname{cosec}\theta - \cot\theta)$ is reciprocal of $(\operatorname{cosec}\theta + \cot\theta)$ and vice-versa.

Using above identities hundreds of other identities can be proved. While proving identities you can use rationalization, factorization and many other similar mathematical operations.

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Example 3: If $\operatorname{cosec}\theta - \cot\theta = \frac{1}{5}$ then find the value of $\sin\theta$ (θ lies in Ist quadrant).

Sol. Given $\operatorname{cosec}\theta - \cot\theta = \frac{1}{5}$ ...(*i*)

$\Rightarrow \operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta} = 5$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$2\operatorname{cosec}\theta = 5 + \frac{1}{5} = \frac{26}{5}$

$\Rightarrow \operatorname{cosec}\theta = \frac{13}{5} \Rightarrow \sin\theta = \frac{5}{13}$

Example 4: If $\frac{\cos^4 A}{\cos^2 B} + \frac{\sin^4 A}{\sin^2 B} = 1$, prove that $\cos^2 A\sin^2 B = \sin^2 A\cos^2 B$

Sol. Given, $\frac{\cos^4 A}{\cos^2 B} + \frac{\sin^4 A}{\sin^2 B} = 1 = \cos^2 A + \sin^2 A$

or, $\frac{\cos^4 A}{\cos^2 B} - \cos^2 A = \sin^2 A - \frac{\sin^4 A}{\sin^2 B}$

or, $\frac{\cos^2 A\left(\cos^2 A - \cos^2 B\right)}{\cos^2 B}$

$= \frac{\sin^2 A\left(\sin^2 B - \sin^2 A\right)}{\sin^2 B}$

or, $\frac{\cos^2 A}{\cos^2 B}\left(\cos^2 A - \cos^2 B\right)$

$= \frac{\sin^2 A}{\sin^2 B}\left[\left(1 - \cos^2 B\right) - \left(1 - \cos^2 A\right)\right]$

$\left(\cos^2 A - \cos^2 B\right)\left(\frac{\cos^2 A}{\cos^2 B} - \frac{\sin^2 A}{\sin^2 B}\right) = 0$

When $\cos^2 A - \cos^2 B = 0$, $\cos^2 A = \cos^2 B$

when $\frac{\cos^2 A}{\cos^2 B} - \frac{\sin^2 A}{\sin^2 B} = 0$

$\cos^2 A\sin^2 B = \sin^2 A\cos^2 B$

Example 5: If $\sin\theta + \operatorname{cosec}\theta = 2$, the value of $\sin^{10}\theta + \operatorname{cosec}^{10}\theta$ is

(*a*) 10 (*b*) 2^{10} (*c*) 2^9 (*d*) 2

Sol. (*d*) We have,

$\sin\theta + \operatorname{cosec}\theta = 2 \Rightarrow \sin^2\theta + 1 = 2\sin\theta$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow (\sin\theta - 1)^2 = 0 \Rightarrow \sin\theta = 1$

Required value of $\sin^{10}\theta + \operatorname{cosec}^{10}\theta$

$= (1)^{10} + \frac{1}{(1)^{10}} = 2.$

Example 6: The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as:

(*a*) $\sin A\cos A + 1$

(*b*) $\sec A\operatorname{cosec} A + 1$

(*c*) $\tan A + \cot A$

(*d*) $\sec A + \operatorname{cosec} A$

Sol. (*b*) Given expression can be written as

$\frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} + \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A - \sin A}$

$\left(\because \tan A = \frac{\sin A}{\cos A} \text{ and } \cot A = \frac{\cos A}{\sin A}\right)$

$= \frac{1}{\sin A - \cos A}\left\{\frac{\sin^3 A - \cos^3 A}{\cos A\sin A}\right\}$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

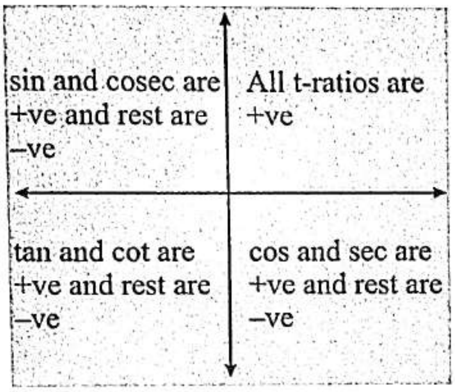
$= \frac{\sin^2 A + \sin A\cos A + \cos^2 A}{\sin A\cos A}$

$= 1 + \sec A\operatorname{cosec} A$

. Concept Application

4. If $\sin x + \sin^2 x = 1$, then find the value of $\cos^8 x + 2\cos^6 x + \cos^4 x$.
5. If $\sec\theta + \tan\theta = 4$ then find $\cot\theta$

IGN CONVENTION OF THE RIGONOMETRIC FUNCTIONS

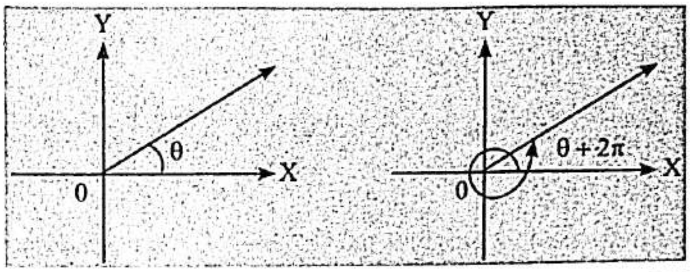

sin and cosec are +ve and rest are –ve
All t-ratios are +ve
tan and cot are +ve and rest are –ve
cos and sec are +ve and rest are –ve

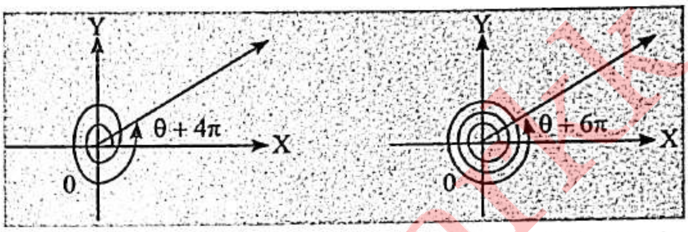
T.Ratio/Angle	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	$\sqrt{\frac{0}{4}}=0$	$\sqrt{\frac{1}{4}}=\frac{1}{2}$	$\sqrt{\frac{2}{4}}=\frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}}=1$
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	*N.D.*

N.D. implies not defined

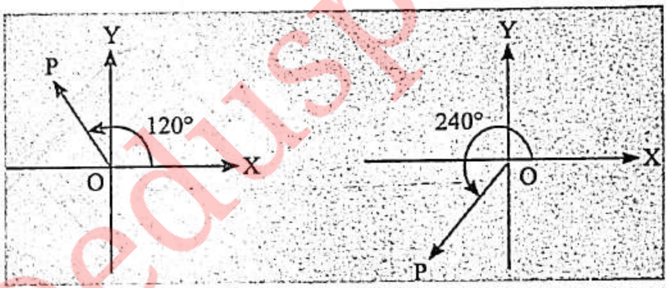
ote: 1. The values of cosec x, sec x and cot x are the reciprocal f the values of sin x, cos x and tan x, respectively.

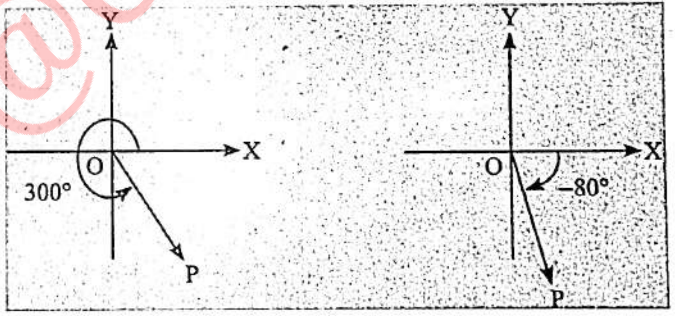
. Any trigonometric ratio of $n \times 360° + A$ ($n \in I$) is equal to the ame trigonometric ratio of A e.g. 420°, –300° are coterminal with 60° and any trigonometric ratio of these angles will equal to the same ratio of θ (as shown in following diagrams also).


Y
X
0
θ
Y
X
0
θ + 2π


Y
X
0
θ + 4π
Y
X
0
θ + 6π

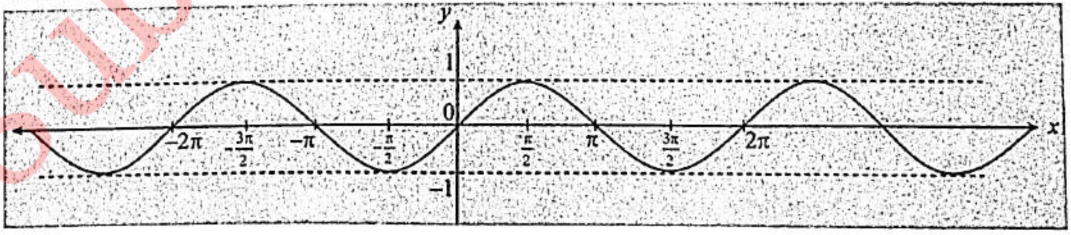
3. An angle can lie in any of four quadrants according to the position of revolving ray for the angle.


Y
P
120°
X
O
Y
240°
X
O
P

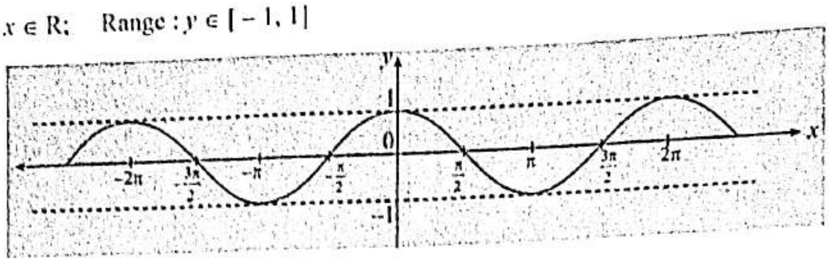

Y
X
O
300°
P
Y
X
O
–80°
P

DOMAIN, RANGE & GRAPHS OF TRIGONOMETRIC FUNCTIONS

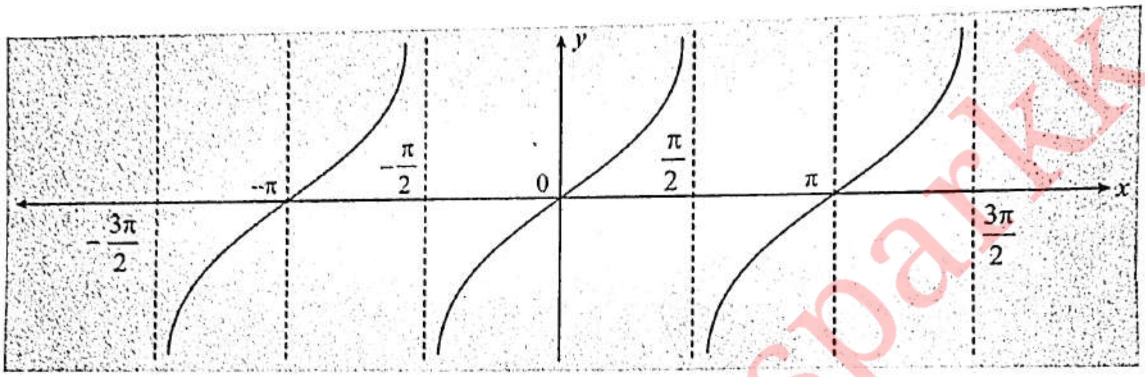
(*i*) $y = \sin x$ Domain : $x \in R$; Range : $y \in [-1, 1]$



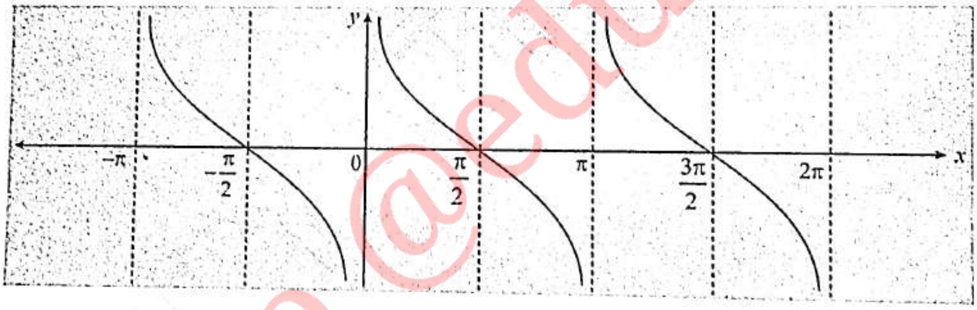
(*ii*) $y = \cos x$ Domain : $x \in R$; Range : $y \in [-1, 1]$



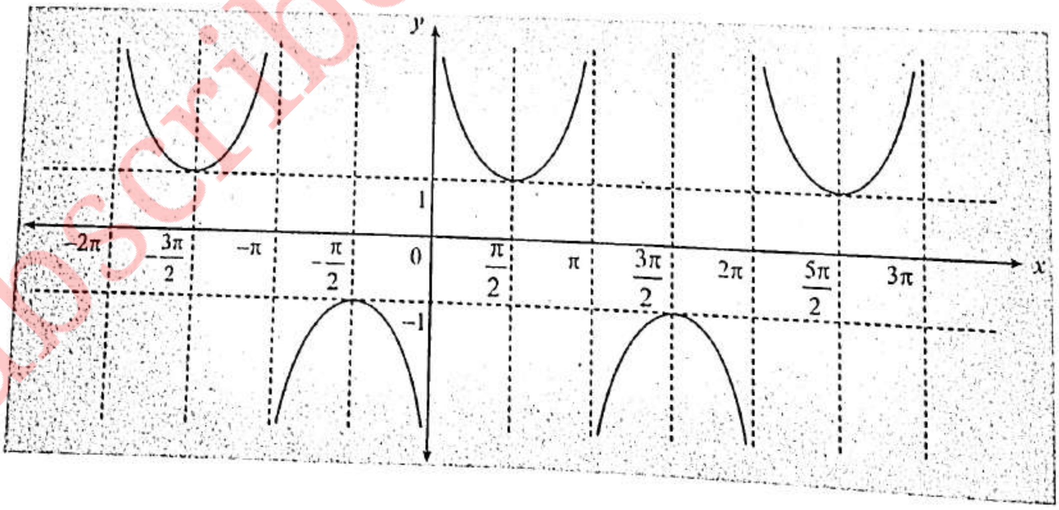
(*iii*) $y = \tan x$ Domain : $x \in R - \left\{(2n+1)\frac{\pi}{2}\right\}, n \in I$; Range : $y \in R$



(*iv*) $y = \cot x$ Domain : $x \in R - \{n\pi\}, n \in I$; Range : $y \in R$



(*v*) $y = \text{cosec}\, x$ Domain : $x \in R - \{n\pi\}, n \in I$; Range : $y \in (-\infty, -1] \cup [1, \infty)$



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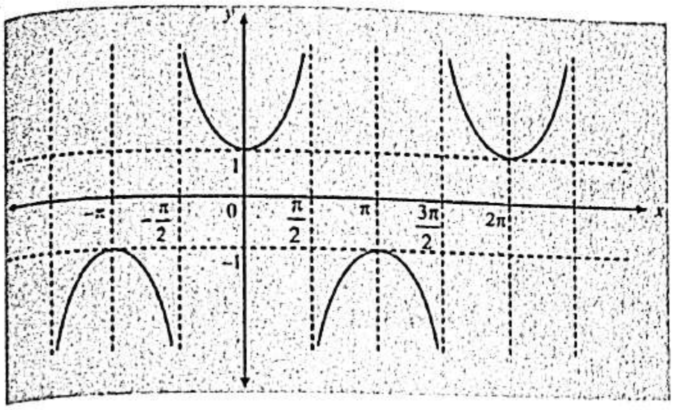
(vi) $y = \sec x$ Domain : $x \in R - \left\{(2n+1)\frac{\pi}{2}\right\}$, $n \in I$; Range: $y \in (-\infty, -1] \cup [1, \infty)$



TRIGONOMETRIC RATIOS OF ALLIED ANGLES

If θ is any angle, then $-\theta$, $\frac{\pi}{2} \pm \theta$, $\pi \pm \theta$, $\frac{3\pi}{2} \pm \theta$, $2\pi \pm \theta$ etc. are called allied angles.

Trigonometric Ratios of (–θ)

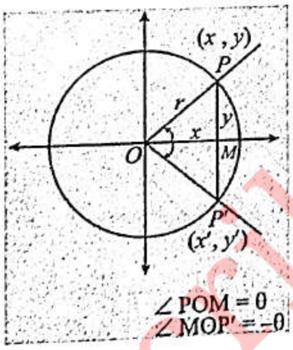
Let θ be an angle in the standard position in the I quadrant. Let its terminal side cuts the circle with centre '*O*' and radius *r* in *P*(*x*, *y*).

Let *P*′(*x*′, *y*′) be the point of intersection of the terminal side of the angle – θ in the standard position with the circle.

Now ∠*MOP* = ∠*MOP*′ (numerically) and *P* and *P*′ have the same projection *M* in the *x*-axis

$\therefore \quad \Delta OMP \cong \Delta OMP' \Rightarrow x = x'$ and $y = -y'$

$\therefore \quad \sin(-\theta) = \frac{y'}{r} = \frac{-y}{r} = -\sin\theta$.;



$\cos(-\theta) = \frac{x'}{r} = \frac{x}{r} = \cos\theta$.

$\tan(-\theta) = \frac{y'}{x'} = -\frac{y}{x} = -\tan\theta$;

$\cot(-\theta) = \frac{x'}{y'} = \frac{x}{-y} = -\cot\theta$.

$\sec(-\theta) = \frac{r}{x'} = \frac{r}{x} = \sec\theta$;

$\text{cosec}(-\theta) = \frac{r}{y'} = \frac{r}{-y} = -\text{cosec}\,\theta$.

Similarly if θ is in the other quadrants then the above results can also be proved.

Trigonometric Ratios of (π – θ)

Let θ be an angle in the standard position in the I quadrant. Let its terminal side cuts the circle with centre '*O*' and radius *r* at *P*(*x*, *y*). Let *P*′(*x*′, *y*′) be the point of intersection of the terminal side of the angle π – θ with the circle. Let *M* and *M*′ be the projections of *P* and *P*′ respectively in the *x*-axis.

Since $\Delta OM'P' \cong \Delta OMP$, $x' = -x$, $y' = y$

$\therefore \sin(\pi - \theta) = \frac{y'}{r} = \frac{y}{r} = \sin\theta$.;

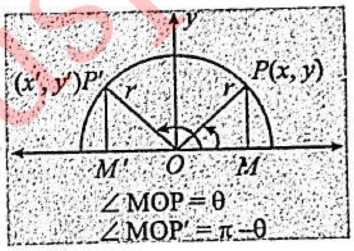
$\cos(\pi - \theta) = \frac{x'}{r} = -\frac{x}{r} = -\cos\theta$.

$\tan(\pi - \theta) = \frac{y'}{x'} = \frac{y}{-x} = -\tan\theta$;

$\cot(\pi - \theta) = \frac{x'}{y'} = -\frac{x}{y} = -\cot\theta$.

$\sec(\pi - \theta) = \frac{r}{x'} = \frac{r}{-x} = -\sec\theta$.;

$\text{cosec}(\pi - \theta) = \frac{r}{y'} = \frac{r}{y} = \text{cosec}\,\theta$.



Trigonometric Ratios of $\left(\frac{\pi}{2} - \theta\right)$

Similarly we can easily prove the following results.

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$, $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$,

$\text{cosec}\left(\frac{\pi}{2} - \theta\right) = \sec\theta$, $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$,

$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$, $\sec\left(\frac{\pi}{2} - \theta\right) = \text{cosec}\,\theta$

Trigonometric Ratios of $\left(\frac{\pi}{2} + \theta\right)$

Similarly we can easily prove the following results.

$\sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$, $\tan\left(\frac{\pi}{2} + \theta\right) = -\cot\theta$,

$\text{cosec}\left(\frac{\pi}{2} + \theta\right) = \sec\theta$, $\cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$,

$\cot\left(\frac{\pi}{2} + \theta\right) = -\tan\theta$, $\sec\left(\frac{\pi}{2} + \theta\right) = -\text{cosec}\,\theta$

Credit - Akshat

Trigonometric Ratios of $(\pi + \theta)$

Similarly we can easily prove the following results.

$$\sin(\pi+\theta) = -\sin\theta,\ \tan(\pi+\theta) = \tan\theta,\ \operatorname{cosec}(\pi+\theta) = -\operatorname{cosec}\theta,$$

$$\cos(\pi+\theta) = -\cos\theta,\ \cot(\pi+\theta) = \cot\theta,$$

$$\sec(\pi+\theta) = -\sec\theta$$

Trigonometric Ratios of $\left(\frac{3\pi}{2}-\theta\right)$

Similarly we can easily prove the following results.

$$\sin\left(\frac{3\pi}{2}-\theta\right) = -\cos\theta,\ \tan\left(\frac{3\pi}{2}-\theta\right) = \cot\theta,$$

$$\operatorname{cosec}\left(\frac{3\pi}{2}-\theta\right) = -\sec\theta,\ \cos\left(\frac{3\pi}{2}-\theta\right) = -\sin\theta,$$

$$\cot\left(\frac{3\pi}{2}-\theta\right) = \tan\theta,\ \sec\left(\frac{3\pi}{2}-\theta\right) = -\operatorname{cosec}\theta,$$

Trigonometric Ratios of $\left(\frac{3\pi}{2}+\theta\right)$

Similarly we can easily prove the following results.

$$\sin\left(\frac{3\pi}{2}+\theta\right) = -\cos\theta,\ \cos\left(\frac{3\pi}{2}+\theta\right) = \sin\theta,$$

$$\tan\left(\frac{3\pi}{2}+\theta\right) = -\cot\theta,\ \cot\left(\frac{3\pi}{2}+\theta\right) = -\tan\theta,$$

$$\sec\left(\frac{3\pi}{2}+\theta\right) = \operatorname{cosec}\theta,\ \operatorname{cosec}\left(\frac{3\pi}{2}+\theta\right) = -\sec\theta$$

There is an easy way to remember these formulae. First of all think of θ as an acute angle. Angles like $180° \pm \theta$, $360° \pm \theta$, $-\theta$ can be considered as angles associated with the horizontal line, angles like $90° - \theta$, $90° + \theta$, $270° \mp \theta$ can be considered as angles associated with vertical line. When associated with the horizontal line, the function does not change, whereas when associated with the vertical line the function changes to the corresponding complementary value (sin ↔ cos, tan ↔ cot, sec ↔ cosec). For example $\sin(180° + \theta)$ will be only $\sin\theta$ (in magnitude) plus or minus and $\cos(90° - \theta)$ will be $\sin\theta$ only in magnitude.

To decide upon the sign, consider the quadrant in which the angle falls and decide the sign by the quadrant rule.

For example, $\sin(180° + \theta)$ is $\sin\theta$ (in magnitude) $(180° + \theta)$ lies in third quadrant and hence $\sin(180° + \theta)$ is negative.

$\therefore \sin(180° + \theta) = -\sin\theta$

Again consider $\tan(90° + \theta)$: This should be $\cot\theta$ and must have a negative sign since $(90° + \theta)$ is in *II* quadrant and hence $\tan(90° + \theta)$ is negative.

$\therefore \tan(90° + \theta) = -\cot\theta$

Train Your Brain

Example 7: Find $\cos(315°)$.

Sol. $\cos(315°) = \cos(360° - 45°) = \cos(-45°)$

$= \cos(45°) = \frac{1}{\sqrt{2}}$

Example 8: Find $\tan(330°)$.

Sol. $\tan(330°) = \tan(360° - 30°) = \tan(-30°)$

$= -\tan 30° = \frac{-1}{\sqrt{3}}$

Example 9: Prove that $\sin 420° \cos 390° + \cos(-300°) \sin(-330°) = 1$.

Sol. $\sin(360° + 60°)\cos(360° + 30°) + \cos(-360° + 60°) \sin(-360° + 30°) = \sin 60° \cos 30° + \cos 60° \sin 30°$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = 1$

Example 10: Prove that $\sin 240° \sin 510° - \sin 330° \cos 390° = 0$

Sol. $\sin(270° - 30°)\sin(540° - 30°) - \sin(360° - 30°) \cos(360° + 30°)$

$= -\cos 30° \sin 30° + \sin 30° \cos 30° = 0$

Example 11: Prove that $\cos A + \sin(270° + A) - \sin(270° - A) + \cos(180° + A) = 0$

Sol. $\cos A + \sin(270° + A) - \sin(270° - A) + \cos(180° + A)$

$= \cos A - \cos A - (-\cos A) + (-\cos A) = 0$

Example 12: Find the value of tan 35°.tan 40°.tan 45°.tan 50°.tan 55°.

Sol. We have,

$\tan 35°.\tan 40°.\tan 45°.\tan 50°.\tan 55°$

$= \{\tan 35° \times \tan 55°\}\{\tan 40° \times \tan 50°\} \times \tan 45°$

$= \{\tan 35° \times \cot 35°\}.\{\tan 40° \times \cot 40°\} \times \tan 45° = 1$

Example 13: Find the value of $\tan(1°).\tan(2°).\tan(3°)\ldots\ldots\ldots\tan(89°)$.

Sol. We have,

$\tan(1°).\tan(2°).\tan(3°)\ldots\ldots\tan(89°)$

$= \tan(1°).\tan(2°).\tan(3°)\ldots\ldots\ldots\tan(44°)$

$\tan(45°)\tan(46°)\ldots\ldots\tan(87°)\tan(88°)\tan(89°)$

$= \{\tan(1°) \times \tan(89°)\}.\{\tan(2°) \times \tan(88°)\}\ldots\ldots$

$\{\tan(44°) \times \tan(46°)\}.\tan(45°) = 1$

Example 14: Find the value of $\cos(1°).\cos(2°).\cos(3°)\ldots\ldots\cos(189°)$.

Sol. We have,

$\cos(1°).\cos(2°).\cos(3°)\ldots\ldots\ldots\cos(189°)$

$= \cos(1°).\cos(2°).\cos(3°)\ldots\ldots\ldots\cos(89°)$

$\cos(90°)\cos(91°)\ldots\ldots\cos(189°)$

$= \cos(1°).\cos(2°).\cos(3°)\ldots\cos(89°) \times 0 \times \cos(91°)\ldots\ldots$

$\cos(189°) = 0$

Concept Application

6. If $5\cos^2\alpha - 2\sin\alpha - 2 = 0$, $(5\pi/4 < \alpha < 7\pi/4)$, then find the value of $\cot \alpha/2$.

7. Let $a = \tan\left(\frac{-2\pi}{3}\right)$, $b = \sin\left(\frac{-17\pi}{4}\right)$, $c = \cot(-300°)$ $d = \cos(-315°)$, then

   (a) $a < b < c < d$

   (b) $a < c < b < d$

   (c) $b < c < d < a$

   (d) $b < c < a < d$

8. Find the value of

$$\cos^2\frac{\pi}{16} + \cos^2\frac{3\pi}{16} + \cos^2\frac{5\pi}{16} + \cos^2\frac{7\pi}{16}$$

9. Match the columns:

Column-I		Column-II	
A.	$\sin(1050°)$	p.	$-1/\sqrt{3}$
B.	$\sin(120°)$	q.	$3/4$
C.	$\cos^2(120°)$	r.	$-\sqrt{3}/2$
D.	$\tan(120°)$	s.	$1/2$
E.	$\sin(135°)$	t.	$1/\sqrt{2}$
F.	$\sin^2 9\pi/4$	u.	$-\sqrt{3}$
G.	$\sin 11\pi/3$	v.	$1/4$
H.	$\cos^2 31\pi/6$	w.	$\sqrt{3}/2$
I.	$\tan 41\pi/6$	x.	$-1/2$

Evaluation Table

Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
Degree	0	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ND	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0
cot	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	ND	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	ND

ND = Not defined

TRIGONOMETRIC RATIOS OF SUM OR DIFFERENCE OF AN ANGLES

(a) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

(b) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

(c) $\sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A = \sin(A + B).\sin(A - B)$

(d) $\cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A = \cos(A + B).\cos(A - B)$

(e) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

(f) $\cot(A \pm B) = \dfrac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$

ADVANCED LEARNING

(g) $\sin(A + B + C) = \sin A \cos B \cos C + \sin B \cos A \cos C + \sin C \cos A \cos B - \sin A \sin B \sin C$

(h) $\cos(A + B + C) = \cos A \cos B \cos C - \cos A \sin B \sin C - \sin A \cos B \sin C - \sin A \sin B \cos C$

(i) $\tan(A + B + C)$

$$= \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$$

(j) $\tan(\theta_1 + \theta_2 + \theta_3 + \ldots\ldots + \theta_n) = \dfrac{S_1 - S_3 + S_5 - \ldots}{1 - S_2 + S_4 - \ldots}$

Train Your Brain

Example 15: Prove that $\cos(9^\circ) + \sin(9^\circ) = \sqrt{2}\sin(54^\circ)$

Sol. We have, $\cos(9^\circ) + \sin(9^\circ)$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos(9^\circ) + \frac{1}{\sqrt{2}}\sin(9^\circ)\right)$$

$$= \sqrt{2}\left(\sin(45^\circ)\cos(9^\circ) + \cos(45^\circ)\sin(9^\circ)\right)$$

$$= \sqrt{2}\left(\sin(45^\circ + 9^\circ)\right) = \sqrt{2}\sin(54^\circ)$$

Example 16: If $A + B = 45^\circ$, then find the value of $(1 + \tan A)(1 + \tan B)$

Sol. We have,

$$A + B = 45^\circ \Rightarrow \tan(A + B) = \tan(45^\circ) \Rightarrow \tan(A + B) = 1$$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\Rightarrow \tan A + \tan B = 1 - \tan A \,.\, \tan B$$

$$\Rightarrow \tan A + \tan B + \tan A \,.\, \tan B = 1$$

$$\Rightarrow 1 + \tan A + \tan B + \tan A \,.\, \tan B = 1 + 1 = 2$$

$$\Rightarrow (1 + \tan A) + \tan B(1 + \tan A) = 2$$

$$\Rightarrow (1 + \tan A)(1 + \tan B) = 2$$

Example 17: Find the value of $(1 + \tan 20^\circ)(1 + \tan 24^\circ)(1 + \tan 25^\circ)(1 + \tan 21^\circ)$

Sol. We have,

$$(1 + \tan 20^\circ)(1 + \tan 24^\circ)(1 + \tan 25^\circ)(1 + \tan 21^\circ)$$

$$= \{(1 + \tan 20^\circ)(1 + \tan 25^\circ)\} \times \{(1 + \tan 24^\circ)(1 + \tan 21^\circ)\}$$

$$= 2 \times 2 = 4$$

Example 18: If $2\tan\alpha = 3\tan\beta$, then show that,

$$\tan(\alpha - \beta) = \frac{2\sin\beta\cos\beta}{4 + 2\sin^2\beta}$$

Sol. We have, $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha.\tan\beta}$

$$= \frac{\frac{3}{2}\tan\beta - \tan\beta}{1 + \frac{3}{2}\tan\beta\tan\beta} = \frac{\tan\beta}{2 + 3\tan^2\beta}$$

$$= \frac{\frac{\sin\beta}{\cos\beta}}{2 + 3\frac{\sin^2\beta}{\cos^2\beta}} = \frac{\sin\beta\cos\beta}{2\cos^2\beta + 3\sin^2\beta}$$

$$= \frac{\sin\beta\cos\beta}{2 + \sin^2\beta} = \frac{2\sin\beta\cos\beta}{4 + 2\sin^2\beta}$$

Concept Application

10. Prove that $\tan(70^\circ) = 2\tan(50^\circ) + \tan(20^\circ)$
11. Show that $\cos^2\theta + \cos^2(\alpha + \theta) - 2\cos\alpha\cos\theta\cos(\alpha + \theta)$ is independent of θ.
12. If $\sin(A - B) = \frac{1}{\sqrt{10}}$, $\cos(A + B) = \frac{2}{\sqrt{29}}$ find the value of $\tan 2A$ where A and B lie between 0 and $\pi/4$.

TRIGONOMETRIC RATIOS OF MULTIPLE AND SUB-MULTIPLE ANGLES

(a) $\sin 2A = 2\sin A\cos A$

Note: $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

(b) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

Note: $2\cos^2\frac{\theta}{2} = 1 + \cos\theta$, $2\sin^2\frac{\theta}{2} = 1 - \cos\theta$.

(c) $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

Note: $\tan\theta = \dfrac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}}$

(d) $\sin 2A = \dfrac{2\tan A}{1 + \tan^2 A}$, $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

(e) $\sin 3A = 3\sin A - 4\sin^3 A$

(f) $\cos 3A = 4\cos^3 A - 3\cos A$

(g) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

(h) $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

(i) $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

(j) $\sin^3 A = \frac{1}{4}(3\sin A - \sin 3A)$

(k) $\sin A\sin(60 - A).\sin(60 + A) = \frac{1}{4}\sin 3A$

(l) $\cos A.\cos(60 - A).\cos(60 + A) = \frac{1}{4}\cos 3A$

(m) $\tan A \,.\, \tan(60 - A) \,.\, \tan(60 + A) = \tan 3A$

(n) $\cot\theta - \tan\theta = 2\cot(2\theta)$

Train Your Brain

Example 19: Show that $\sin 12^\circ \sin 48^\circ \cdot \sin 54^\circ = 1/8$

Sol. L.H.S. $= \frac{1}{2}[\cos 36^\circ - \cos 60^\circ]\sin 54^\circ$

$$\frac{1}{2}\left[\cos 36^\circ \sin 54^\circ - \frac{1}{2}\sin 54^\circ\right]$$

$$= \frac{1}{4}[2\cos 36^\circ \sin 54^\circ - \sin 54^\circ]$$

$$= \frac{1}{4}[\sin 90^\circ + \sin 18^\circ - \sin 54^\circ]$$

$$= \frac{1}{4}[1 - (\sin 54^\circ - \sin 18^\circ)]$$

$$= \frac{1}{4}[1 - 2\sin 18^\circ \cos 36^\circ]$$

$$= \frac{1}{4}\left[1 - \frac{2\sin 18^\circ}{\cos 18^\circ}\cos 18^\circ \cos 36^\circ\right]$$

$$= \frac{1}{4}\left[1 - \frac{\sin 36^\circ \cos 36^\circ}{\cos 18^\circ}\right]$$

$$= \frac{1}{4}\left[1 - \frac{2\sin 36^\circ \cos 36^\circ}{2\cos 18^\circ}\right]$$

$$= \frac{1}{4}\left[1 - \frac{\sin 72^\circ}{2\sin 72^\circ}\right] = \frac{1}{8} = \text{R.H.S.}$$

Example 20: Prove that

(*i*) $\frac{\sin 2A}{1+\cos 2A} = \tan A$

(*ii*) $\tan A + \cot A = 2\operatorname{cosec} 2A$

(*iii*) $\frac{1-\cos A+\cos B-\cos(A+B)}{1+\cos A-\cos B-\cos(A+B)} = \tan\frac{A}{2}\cot\frac{B}{2}$

(*iv*) $\frac{\tan 5\theta+\tan 3\theta}{\tan 5\theta-\tan 3\theta} = 4\cos 2\theta\cos 4\theta$

Sol.

(*i*) L.H.S. $\frac{\sin 2A}{1+\cos 2A} = \frac{2\sin A\cos A}{2\cos^2 A} = \tan A$

(*ii*) L.H.S. $\tan A + \cot A = \frac{1+\tan^2 A}{\tan A}$

$$= 2\left(\frac{1+\tan^2 A}{2\tan A}\right) = \frac{2}{\sin 2A} = 2\operatorname{cosec} 2A$$

(*iii*) L.H.S. $\frac{1-\cos A+\cos B-\cos(A+B)}{1+\cos A-\cos B-\cos(A+B)}$

$$= \frac{2\sin^2\frac{A}{2}+2\sin\frac{A}{2}\sin\left(\frac{A}{2}+B\right)}{2\cos^2\frac{A}{2}-2\cos\frac{A}{2}\cos\left(\frac{A}{2}+B\right)}$$

$$= \tan\frac{A}{2}\left[\frac{\sin\frac{A}{2}+\sin\left(\frac{A}{2}+B\right)}{\cos\frac{A}{2}-\cos\left(\frac{A}{2}+B\right)}\right]$$

$$= \tan\frac{A}{2}\left[\frac{2\sin\frac{A+B}{2}\cos\left(\frac{B}{2}\right)}{2\sin\frac{A+B}{2}\sin\left(\frac{B}{2}\right)}\right] = \tan\frac{A}{2}\cot\frac{B}{2}$$

(*iv*) $\frac{\tan 5\theta+\tan 3\theta}{\tan 5\theta-\tan 3\theta} = \frac{\sin 5\theta\cos 3\theta+\sin 3\theta\cos 5\theta}{\sin 5\theta\cos 3\theta-\sin 3\theta\cos 5\theta}$

$$= \frac{\sin 8\theta}{\sin 2\theta} = 4\cos 2\theta\cos 4\theta$$

Example 21: Prove that: $\frac{\tan\left(\frac{\pi}{4}+A\right)}{\tan\left(\frac{\pi}{4}-A\right)} = \frac{2\cos A+\sin A+\sin 3A}{2\cos A-\sin A-\sin 3A}$

Sol. L.H.S. $= \frac{\frac{1+\tan A}{1-\tan A}}{\frac{1-\tan A}{1+\tan A}} = \frac{(1+\tan A)^2}{(1-\tan A)^2} = \frac{1+\tan^2 A+2\tan A}{1+\tan^2 A-2\tan A}$

$$= \frac{1+\frac{2\tan A}{1+\tan^2 A}}{1-\frac{2\tan A}{1+\tan^2 A}} = \frac{1+\sin 2A}{1-\sin 2A}$$

R.H.S. $= \frac{2\cos A+2\sin 2A.\cos A}{2\cos A-2\sin 2A\cos A}$

$$= \frac{2\cos A(1+\sin 2A)}{2\cos A(1-\sin 2A)} = \frac{1+\sin 2A}{1-\sin 2A}$$

Both sides reduce to the same result.

Example 22: Prove that $\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)\cos\left(\frac{8\pi}{7}\right) = \frac{1}{8}$

Sol. We have $\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)\cos\left(\frac{8\pi}{7}\right)$

$$= \cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)\cos\left(\pi+\frac{\pi}{7}\right)$$

$$= -\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)$$

$$= -\frac{1}{2\sin\left(\frac{\pi}{7}\right)}\left(2\sin\left(\frac{\pi}{7}\right)\cos\left(\frac{\pi}{7}\right)\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)$$

$$= -\frac{1}{2^2\sin\left(\frac{\pi}{7}\right)}\left(2\sin\left(\frac{2\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\right)\cos\left(\frac{4\pi}{7}\right)$$

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$$= -\frac{1}{2^3 \sin\left(\frac{\pi}{7}\right)}\left(2\sin\left(\frac{4\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)\right)$$

$$= -\frac{\sin\left(\frac{8\pi}{7}\right)}{2^3 \sin\left(\frac{\pi}{7}\right)} = -\frac{\sin\left(\pi+\frac{\pi}{7}\right)}{2^3 \sin\left(\frac{\pi}{7}\right)} = \frac{\sin\left(\frac{\pi}{7}\right)}{8\sin\left(\frac{\pi}{7}\right)} = \frac{1}{8}$$

Concept Application

13. If $f(\theta) = \sin^2\theta + \sin^2\left(\theta + \frac{2\pi}{3}\right) + \sin^2\left(\theta + \frac{4\pi}{3}\right)$, then $f\left(\frac{\pi}{15}\right)$ is equal to

(*a*) $\frac{2}{3}$ (*b*) $\frac{3}{2}$ (*c*) $\frac{1}{3}$ (*d*) $\frac{1}{2}$

14. Prove that $\sin(20^\circ)\sin(40^\circ)\sin(80^\circ) = \frac{\sqrt{3}}{8}$

TRIGONOMETRIC RATIOS OF STANDARD ANGLES

(*a*) $\sin n\pi = 0$; $\cos n\pi = (-1)^n$; $\tan n\pi = 0$, where $n \in I$

(*b*) $\sin 15^\circ$ or $\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ$ or $\cos\frac{5\pi}{12}$;

$\cos 15^\circ$ or $\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^\circ$ or $\sin\frac{5\pi}{12}$;

$$\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} = \cot 75^\circ;$$

$$\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \cot 15^\circ$$

To find the trigonometrical functions of an angle of 18°: Let stand for 18°, so that 2θ is 36° and 3θ is 54°.

Hence $2\theta = 90^\circ - 3\theta$ and therefore $\sin 2\theta = \sin(90^\circ - 3\theta) = \cos$

$\therefore \quad 2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

Hence, either $\cos\theta = 0$, which gives $\theta = 90^\circ$, or

$2\sin\theta = 4\cos^2\theta - 3 = 1 - 4\sin^2\theta$

$\therefore \quad 4\sin^2\theta + 2\sin\theta = 1$

By solving this quadratic equation, we have (In our case $\sin\theta$ necessarily a positive quantity. Hence we take the upper sign, a have)

$$\sin\theta = \frac{\sqrt{5}-1}{4} = \sin 18^\circ$$

Hence $\cos 18^\circ = \sqrt{1-\sin^2 18^\circ} = \sqrt{1-\frac{6-2\sqrt{5}}{16}} = \sqrt{\frac{10+2\sqrt{5}}{16}}$

The remaining trigonometrical ratio of 18° may be now found.

Since 72° is the complement of 18°, the value of the ratios for 7 may be obtained.

To find the trigonometrical functions of an angle of 36°

Since $\cos 2\theta = 1 - 2\sin^2\theta$,

$\therefore \quad \cos 36^\circ = 1 - 2\sin^2 18^\circ = \frac{\sqrt{5}+1}{4}$

The remaining trigonometrical functions of 36° may now found. Also, since 54° may be found.

Note: We must be careful while determining the square root trigonometrical function e.g.

$$\sqrt{1+\sin A} = \left|\sin\frac{A}{2} + \cos\frac{A}{2}\right|$$

Values of standard angles:

Angle → ↓ T. Ratio	$\left(\frac{\pi}{12}\right)$ 15°	$\left(\frac{\pi}{10}\right)$ 18°	$\left(\frac{\pi}{8}\right)$ 22.5°	$\left(\frac{\pi}{5}\right)$ 36°	$\left(\frac{3\pi}{8}\right)$ 67.5°	$\left(\frac{5\pi}{12}\right)$ 75°
sin	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$
cos	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$
tan	$2-\sqrt{3}$	$\frac{1}{\sqrt{(5+2\sqrt{5})}}$	$\sqrt{2}-1$	$\sqrt{5-2\sqrt{5}}$	$\sqrt{2}+1$	$2+\sqrt{3}$
cot	$2+\sqrt{3}$	$\sqrt{(5+2\sqrt{5})}$	$\sqrt{2}+1$	$\sqrt{\left(1+\frac{2}{\sqrt{5}}\right)}$	$\sqrt{2}-1$	$2-\sqrt{3}$

Credit - Akshat

Train Your Brain

Example 23: Prove that : $4(\sin(24^\circ)+\cos(6^\circ)) = (1+\sqrt{5})$

Sol. We have, $4(\sin(24^\circ)+\cos(6^\circ))$

$$= 4(\sin(24^\circ)+\sin(84^\circ))$$

$$= 4\left(2\sin\left(\frac{24^\circ+84^\circ}{2}\right)\cos\left(\frac{24^\circ-84^\circ}{2}\right)\right)$$

$$= 8(\sin(54^\circ)\cos(30^\circ)) = 8(\cos(36^\circ)\cos(30^\circ))$$

$$= 8\left(\frac{\sqrt{5}+1}{4}\times\frac{1}{2}\right) = (\sqrt{5}+1)$$

Example 24: Prove that $\sin\frac{\pi}{5}\sin\frac{2\pi}{5}\sin\frac{3\pi}{5}\sin\frac{4\pi}{5} = \frac{5}{16}$

Sol. L.H.S. $= \sin 36^\circ \sin 72^\circ \sin 180^\circ \sin 144^\circ$

$$= \sin 36^\circ \sin 72^\circ \sin(180^\circ - 72^\circ)\sin(180^\circ - 36^\circ)$$

$$= \sin 36^\circ \sin 72^\circ \sin 72^\circ \sin 36^\circ = \sin^2 36^\circ \sin^2 72^\circ$$

$$= \frac{5-\sqrt{5}}{8}\times\frac{5+\sqrt{5}}{8} = \frac{25-5}{64} = \frac{20}{64} = \frac{5}{16}$$

Concept Application

15. If $\alpha = 112^\circ 30'$, find the value of $\sin\alpha$ and $\cos\alpha$.

16. Find the value of $\cos 12^\circ + \cos 84^\circ + \cos 156^\circ + \cos 132^\circ$.

TRANSFORMATION FORMULAE

(*i*) $\sin(A+B)+\sin(A-B) = 2\sin A\cos B$

- $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

(*ii*) $\sin(A+B)-\sin(A-B) = 2\cos A\sin B$

- $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

(*iii*) $\cos(A+B)+\cos(A-B) = 2\cos A\cos B$

- $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

(*iv*) $\cos(A-B)-\cos(A+B) = 2\sin A\sin B$

- $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

Train Your Brain

Example 25: Prove that $\frac{\sin 8\theta\cos\theta - \sin 6\theta\cos 3\theta}{\cos 2\theta\cos\theta - \sin 3\theta\sin 4\theta} = \tan 2\theta$

Sol. $\frac{2\sin 8\theta\cos\theta - 2\sin 6\theta\cos 3\theta}{2\cos 2\theta\cos\theta - 2\sin 3\theta\sin 4\theta}$

$$= \frac{\sin 9\theta + \sin 7\theta - \sin 9\theta - \sin 3\theta}{\cos 3\theta + \cos\theta - \cos\theta + \cos 7\theta}$$

$$= \frac{2\sin 2\theta\cos 5\theta}{2\cos 5\theta\cos 2\theta} = \tan 2\theta$$

Concept Application

17. If $\sin A - \sin B = \frac{1}{2}$ and $\cos A - \cos B = \frac{1}{3}$ then find $\tan\left(\frac{A+B}{2}\right)$.

18. The product cot 123° . cot 133° . cot 137° . cot 147°, when simplified is equal to

(*a*) –1 (*b*) tan 37°

(*c*) cot 33° (*d*) 1

CONDITIONAL IDENTITIES

If $A+B+C=\pi$ then:

(*i*) $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

(*ii*) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(*iii*) $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A\cos B\cos C$

(*iv*) $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

(*v*) $\tan A + \tan B + \tan C = \tan A\tan B\tan C$

(*vi*) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

(*vii*) $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}.\cot\frac{B}{2}.\cot\frac{C}{2}$

(*viii*) $\cot A\cot B + \cot B\cot C + \cot C\cot A = 1$

(*ix*) $A+B+C = \frac{\pi}{2}$ then $\tan A\tan B + \tan B\tan C + \tan C\tan A = 1$

Train Your Brain

Example 26: If $A + B + C = 180°$, prove that, $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A \cos B \cos C$.

Sol. Let $S = \sin^2 A + \sin^2 B + \sin^2 C$

so that

$$2S = 2\sin^2 A + 1 - \cos 2B + 1 - \cos 2C$$

$$= 2\sin^2 A + 2 - 2\cos(B + C)\cos(B - C)$$

$$= 2 - 2\cos^2 A + 2 - 2\cos(B + C)\cos(B - C)$$

$$\therefore S = 2 + \cos A\,[\cos(B - C) + \cos(B + C)]$$

since $\cos A = -\cos(B + C)$

$$\therefore S = 2 + 2\cos A \cos B \cos C$$

Example 27: If $A + B + C = 180°$ prove that $\cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Sol. $(\cos A + \cos B) + \cos C - 1$

$$= 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} + \cos C - 1$$

$$= 2\cos\left(\frac{\pi}{2} - \frac{C}{2}\right)\cos\frac{A-B}{2} + \cos C - 1$$

$$= 2\sin\frac{C}{2}\cos\frac{A-B}{2} + 1 - 2\sin^2\frac{C}{2} - 1$$

$$= 2\sin\frac{C}{2}\cos\frac{A-B}{2} - 2\sin^2\frac{C}{2}$$

$$= 2\sin\frac{C}{2}\left[\cos\frac{A-B}{2} - \sin\left(\frac{\pi}{2} - \frac{A+B}{2}\right)\right]$$

$$= 2\sin\frac{C}{2}\, 2\sin\frac{A}{2}\sin\frac{B}{2} = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

Example 28: If A, B, C are the angles of a triangle, show that $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

Sol. We have

$$\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - (\tan B \tan C + \tan C \tan A + \tan A \tan B)}$$

But $\tan(A + B + C) = \tan 180° = 0$

Hence $0 = \tan A + \tan B + \tan C - \tan A \tan B \tan C$

i.e, $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

This may also be proved independently. For $\tan(A + B) = \tan(180° - C) = -\tan C$.

$$\therefore \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$$

$$\tan A + \tan B = -\tan C + \tan A \tan B \tan C$$

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$

Concept Application

19. In triangle ABC, prove that $\sin(B + C - A) + \sin(C + A - B) + \sin(A + B - C) = 4\sin A \sin B \sin C$.
20. Prove that in triangle ABC, $\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2\sin A \sin B \cos C$.

APPLICATION OF TRIGONOMETRY IN OPTIMISATION (MAXIMISING AND MINIMISING)

Type-I

Maximising and minimising by using the property of boundedness of trigonometric functions.

(*a*) Sine and cosine have bounded values between –1 and 1.

(*b*) Tangent and cotangent are unbounded functions.

(*c*) cosec and sec have values greater than or equal to 1 and less than or equal to –1.

(*d*) $0 \le \sin^2 x \le 1$, $0 \le \cos^2 x \le 1$, $\tan^2 x \ge 0$, $\sec^2 x \ge 1$.

Note: If maximum value of a function is '*b*', and minimum value '*a*' then range is $[a, b]$.

Special Cases

When angle of sine and cosine are same

$$E = a\sin\theta + b\cos\theta$$

$$\Rightarrow E = \sqrt{a^2+b^2}\left\{\frac{a}{\sqrt{a^2+b^2}}\sin\theta + \frac{b}{\sqrt{a^2+b^2}}\cos\theta\right\}$$

Let $\frac{b}{\sqrt{a^2+b^2}} = \sin\alpha$ & $\frac{a}{\sqrt{a^2+b^2}} = \cos\alpha$

$$\Rightarrow E = \sqrt{a^2+b^2}\sin(\theta + \alpha), \text{ where } \tan\alpha = \frac{b}{a}$$

Hence for any real value of θ,

$$-\sqrt{a^2+b^2} \le E \le \sqrt{a^2+b^2}$$

Type-II

If argument of sine and cosine are different or a quadratic in sine/cos is given then we make a perfect square in sine/cosine and interpret.

Type-III

Making use of reciprocal relationship between tan and cot, sin/cosec and cos/sec.

Train Your Brain

Example 29: Find the maximum and minimum values of $f(\theta) = \sin^6\theta + \cos^6\theta$

Sol. We have, $f(\theta) = \sin^6\theta + \cos^6\theta = (\sin^2\theta)^3 + (\cos^2\theta)^3$

$$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$$

$$= 1 - 3\sin^2\theta\cos^2\theta = 1 - \frac{3}{4}(4\sin^2\theta\cos^2\theta)$$

$$= 1 - \frac{3}{4}(\sin^2 2\theta) = 1 + \frac{3}{4}(-\sin^2 2\theta)$$

As we know that, $-1 \le (-\sin^2 2\theta) \le 0$

$$\Rightarrow -\frac{3}{4} \le \frac{3(-\sin^2 2\theta)}{4} \le 0$$

$$\Rightarrow 1 - \frac{3}{4} \le 1 + \frac{3(-\sin^2 2\theta)}{4} \le 1 \Rightarrow \frac{1}{4} \le f(\theta) \le 1$$

Hence, the maximum value = 1

and the minimum value = $\frac{1}{4}$

Example 30: Find minimum and maximum value of $7\cos\theta + 24\sin\theta$.

Sol. $y = 7\cos\theta + 24\sin\theta$.

$$-\sqrt{a^2 + b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2 + b^2}$$

$$\therefore -\sqrt{7^2 + 24^2} \le 7\cos\theta + 24\sin\theta \le \sqrt{7^2 + 24^2}$$

$-25 \le 7\cos\theta + 24\sin\theta \le 25$.

Hence, $y_{min} = -25$

$y_{max} = 25$

Example 31: $y = \cos 2x + 3\sin x$. Find range of y.

Sol. $y = 1 - 2\sin^2 x + 3\sin x$

$$\Rightarrow 1 - 2\left[\sin^2 x - \frac{3}{2}\sin x + \frac{9}{16} - \frac{9}{16}\right]$$

$$\Rightarrow 1 - 2\left[\sin x - \frac{3}{4}\right]^2 + \frac{9}{8};$$

$$y = \frac{17}{8} - 2\left[\sin x - \frac{3}{4}\right]^2$$

$y_{max} = \frac{17}{8}$ at $\sin x = \frac{3}{4}$

$y_{min} = -4$ at $\sin x = -1$

$y \in \left[-4, \frac{17}{8}\right]$; $y_{max} = \frac{17}{8}$; $y_{min} = -4$

Example 32: If $y = a^2\tan^2 x + b^2\cot^2 x$ $(a, b \ge 0)$. Find minimum value of y.

Sol. $y = a^2\tan^2 x + \dfrac{b^2}{\tan^2 x} - 2ab + 2ab$

$$\Rightarrow \left(a\tan x - \frac{b}{\tan x}\right)^2 + 2ab \ge 2ab$$

$y_{min} = 2ab$

Concept Application

21. Find the range of the function $\dfrac{1}{3\sin x + 4\cos x + 2}$

22. Find the maximum and minimum value of $\cos^2\theta - 6\sin\theta\cos\theta + 3\sin^2\theta + 2$.

23. If $A = \sin^2\theta + \cos^4\theta$, then for all real values of θ

(*a*) $1 \le A \le 2$

(*b*) $\frac{3}{4} \le A \le 1$

(*c*) $\frac{13}{16} \le A \le 1$

(*d*) $\frac{3}{4} \le A \le \frac{13}{16}$

SUMMATION OF TRIGONOMETRIC SERIES

Type-I

Sum of the sin and cosine series when the angles are in A.P.

(1) $\sin\alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + ... + \sin(\alpha + (\overline{n-1})\beta)$

$$= \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \times \sin\left[\alpha + (n-1)\frac{\beta}{2}\right]$$

Proof : Let $S = \sin\alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + ... + \sin(\alpha + \overline{n-1}\,\beta)$

Here angle are in A.P. and common difference of angles = β

$\therefore$ multiplying both sides by $2\sin\frac{\beta}{2}$, we get

$2S\sin\frac{\beta}{2} = 2\sin\alpha\sin\frac{\beta}{2} + 2\sin(\alpha + \beta)\sin\frac{\beta}{2} + ... + 2\sin(\alpha + \overline{n-1}\,\beta)\sin\frac{\beta}{2}$ ... (*i*)

Now, $2\sin\alpha\sin\frac{\beta}{2} = \cos\left(\alpha - \frac{\beta}{2}\right) - \cos\left(\alpha + \frac{\beta}{2}\right)$

$2\sin(\alpha + \beta)\sin\frac{\beta}{2} = \cos\left(\alpha + \frac{\beta}{2}\right) - \cos\left(\alpha + \frac{3\beta}{2}\right)$

... ...

... ...

$$2\sin(\alpha+\overline{n-1}\,\beta)\sin\frac{\beta}{2}$$

$$=\cos\left[\alpha+(2n-3)\frac{\beta}{2}\right]-\cos\left[\alpha+(2n-1)\frac{\beta}{2}\right]$$

Adding, we get R.H.S. of equation (*i*)

$$=\cos\left(\alpha-\frac{\beta}{2}\right)-\cos\left[\alpha+(2n-1)\frac{\beta}{2}\right];$$

hence, $S=\dfrac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\left[\alpha+(n-1)\frac{\beta}{2}\right]$

(2) $\cos\alpha+\cos(\alpha+\beta)+\cos(\alpha+2\beta)+\dots+\cos(\alpha+\overline{n-1}\,\beta)$

$$=\frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\times\cos\left[\alpha+(n-1)\frac{\beta}{2}\right]$$

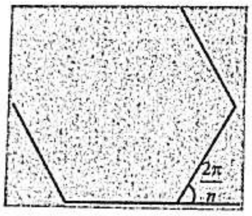
Type-II

For *n* sided regular polygon

Sum of all exterior angles = 2π

(*i*) The value of one exterior angle $=\dfrac{2\pi}{n}$

(*ii*) The value of one interior angle $=\left(\pi-\dfrac{2\pi}{n}\right)=\pi\dfrac{(n-2)}{n}$



(*iii*) Sum of interior angles $=\pi(n-2)$

Type-III

Splitting the sum series as difference of 2 terms.

Here each term of trigonometric series is splitted into difference of two so that most of the terms while adding them gets cancelled.

Type-IV Continued Product of Cosine Series

$$\cos A\cos 2A\cos 4A\cos 8A\ \dots\ \cos 2^{n-1}A=\frac{1}{2^n\sin A}\sin(2^nA)$$

Proof : Multiplying above and below by $2^n\sin A$

$$\therefore \text{LHS}=\frac{2^{n-1}}{2^n\sin A}[2\sin A\cos A\cos 2A\cos 4A\ \dots\ \cos 2^{n-1}A]$$

$$=\frac{2^{n-2}}{2^n\sin A}[2\sin 2A\cos 2A\cos 4A\ \dots\ \cos 2^{n-1}A]$$

$$=\frac{2^{n-3}}{2^n\sin A}[2\sin 4A\cos 4A\ \dots\ \cos 2^{n-1}A]$$

$$=\frac{1}{2^n\sin A}[2\sin 2^{n-1}A\cos 2^{n-1}A]$$

$$=\frac{1}{2^n\sin A}\sin(2\cdot 2^{n-1}A)=\frac{\sin(2^nA)}{2^n\sin A}$$

Train Your Brain

Example 33: Find the summation of the following series

(*i*) $\cos\dfrac{2\pi}{7}+\cos\dfrac{4\pi}{7}+\cos\dfrac{6\pi}{7}$

(*ii*) $\cos\dfrac{\pi}{7}+\cos\dfrac{2\pi}{7}+\cos\dfrac{3\pi}{7}+\cos\dfrac{4\pi}{7}+\cos\dfrac{5\pi}{7}+\cos\dfrac{6\pi}{7}$

Sol.

(*i*) $$\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}=\frac{\cos\dfrac{\left(\frac{2\pi}{7}+\frac{6\pi}{7}\right)}{2}\sin\dfrac{3\pi}{7}}{\sin\dfrac{\pi}{7}}$$

$$=\frac{\cos\frac{4\pi}{7}\sin\frac{3\pi}{7}}{\sin\frac{\pi}{7}}=\frac{-\cos\frac{3\pi}{7}\sin\frac{3\pi}{7}}{\sin\frac{\pi}{7}}=-\frac{\sin\frac{6\pi}{7}}{2\sin\frac{\pi}{7}}=-\frac{1}{2}$$

(*ii*) $$\cos\frac{\pi}{7}+\cos\frac{2\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{5\pi}{7}+\cos\frac{6\pi}{7}$$

$$=\frac{\cos\left(\dfrac{\frac{\pi}{7}+\frac{6\pi}{7}}{2}\right)\sin\frac{6\pi}{14}}{\sin\frac{\pi}{14}}=\frac{\cos\frac{\pi}{2}\sin\frac{6\pi}{14}}{\sin\frac{\pi}{14}}=0$$

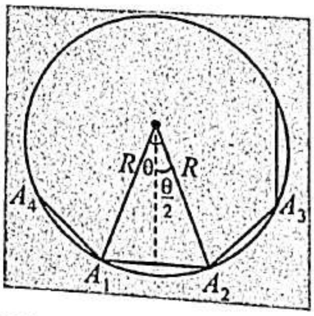
Example 34: In a regular polygon of *n*-sides with $A_1, A_2, \dots A_n$ vertices prove that

$$(A_1A_2)^2+(A_1A_3)^2+(A_1A_4)^2+\dots\dots+(A_1A_n)^2=2nR^2$$

where *R* is the radius of circumcircle circumscribing it.

Sol. $(A_1A_2)^2+(A_1A_3)^2+\dots\dots(A_1A_n)^2=2nR^2$

$$\theta=\frac{2\pi}{n}$$



By trigonometry

$$(A_1A_2)^2=4R^2\cdot\sin^2\frac{\pi}{n}$$

$$(A_1A_3)^2 = 4R^2 \cdot \sin^2 \frac{2\pi}{n}$$

$$(A_1A_4)^2 = 4R^2 \cdot \sin^2 \frac{3\pi}{n} \ldots$$

$$(A_1A_n)^2 = 4R^2 \cdot \sin^2 \frac{(n-1)\pi}{n}$$

$$\therefore (A_1A_2)^2 + (A_1A_3)^2 + \ldots. + (A_1A_n)^2$$

$$4R^2\left[\sin^2\frac{\pi}{n} + \sin^2\frac{2\pi}{n} + \sin^2\frac{3\pi}{n} + \ldots \sin^2\frac{(n-1)\pi}{n}\right]$$

$$\frac{1}{2}4R^2$$

$$\left[1-\cos\frac{2\pi}{n} + 1 - \cos\frac{4\pi}{n} + 1 - \cos\frac{6\pi}{n} + \ldots 1 - \cos\frac{2(n-1)\pi}{n}\right]$$

$$2R^2(n-1) - 2R^2\left[\frac{\sin\frac{(n-1)\pi}{n}}{\sin\left(\frac{\pi}{n}\right)} \cdot \cos\left(\frac{\frac{2\pi}{n} + \frac{2(n-1)\pi}{n}}{2}\right)\right]$$

$$\Rightarrow 2R^2(n-1) - 2R^2(-1) \Rightarrow 2R^2n - 2R^2 + 2R^2 \Rightarrow 2nR^2$$

Example 35: $S = \operatorname{cosec} x + \operatorname{cosec} 2x + \operatorname{cosec} 4x + \ldots. + \operatorname{cosec} 2^n x = \cot \frac{x}{2} - \cot 2^n x$.

Sol. Let $\operatorname{cosec} x = \dfrac{\sin\left(\frac{x}{2}\right)}{\sin\frac{x}{2}\cdot\sin x} = \dfrac{\sin\left(x - \frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)\cdot\sin x}$

$$= \frac{\sin x \cdot \cos\frac{x}{2} - \cos x \cdot \sin\frac{x}{2}}{\sin\frac{x}{2}\cdot\sin x} = \cot\frac{x}{2} - \cot x$$

$$S = \left(\cot\frac{x}{2} - \cot x\right) + (\cot x - \cot 2x) + (\cot 2x - \cot 2^2 x)$$
$$+ \ldots (\cot 2^{n-1}x - \cot 2^n x)$$

$$\Rightarrow S = \left(\cot\frac{x}{2} - \cot 2^n \cdot x\right)$$

Example 36: Prove that $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15} = \frac{1}{128}$

Sol. $\cos\frac{5\pi}{15} = \cos\frac{\pi}{3} = \frac{1}{2}$,

$$\cos\frac{7\pi}{15} = -\cos\frac{8\pi}{15}$$

$$\text{L.H.S.} = \left[\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\left(-\cos\frac{8\pi}{15}\right)\right]$$

$$\left(\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right)\cdot\frac{1}{2} = -\frac{1}{2}\cdot\frac{\sin 2^4\left(\frac{\pi}{15}\right)}{2^4\sin\frac{\pi}{15}}\cdot\frac{\sin\left(2^2\frac{3\pi}{15}\right)}{2^2\sin\frac{3\pi}{15}}$$

$= \frac{1}{128}$, as $\sin(\pi + \theta) = -\sin\theta$

Concept Application

24. $\sin\frac{\pi}{n} + \sin\frac{3\pi}{n} + \sin\frac{5\pi}{n} + \ldots$ up to n terms is equal to

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 0

25. The product

$\left(\cos\frac{x}{2}\right)\cdot\left(\cos\frac{x}{4}\right)\cdot\left(\cos\frac{x}{8}\right)\ldots\ldots\left(\cos\frac{x}{256}\right)$ is equal to:

(*a*) $\dfrac{\sin x}{128\sin\frac{x}{256}}$ (*b*) $\dfrac{1}{256\sin\frac{x}{256}}$

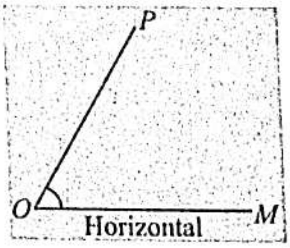
(*c*) $\dfrac{\sin x}{128\sin\frac{x}{128}}$ (*d*) $\dfrac{\sin x}{512\sin\frac{x}{512}}$

26. Find the summation of the following series

$$\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11}$$

HEIGHT AND DISTANCES

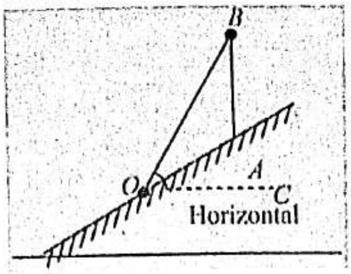
Angle of Elevation



Consider a point P (at higher level) being observed from a point O (usually called observer) at a lower horizontal level. Draw a horizontal line OM through O, OP is called the line of sight and $\angle POM$ is called the angle of elevation of point P as seen from O.

Unless otherwise stated, the height of the observer is neglected because mostly the heights and distances to be measured are very large compared to the height of the observer.

Suppose, there is a flagstaff AB mounted on a sloping level and its top B is observed from O. Then the angle $\angle AOB$ is not the angle of elevation of B because it is not measured w.r.t the horizontal at O. The correct angle of elevation is $\angle COB$.



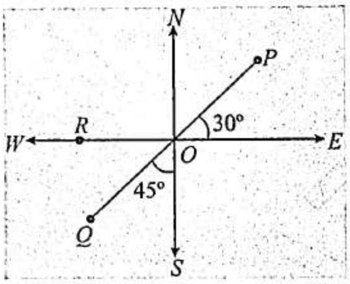
Angle of Depression

Consider a point P being observed from a point O at a higher horizontal level. $\angle POM$ here is called the angle of depression of the point P as seen from O. This is also measured w.r.t. the horizontal at O.

Directions of a Point

When the observer (O) and the object (P) are at the same horizontal level, to specify the direction or location of P w.r.t. O, we take the help of cardinal directions North, East, West and South. Bearing of a point is defined as the angle made by the line of sight with one of the principal directions North, East, West or South.

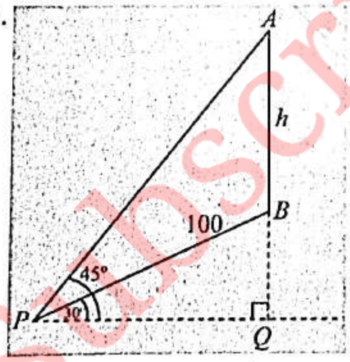
e.g., Bearing of point P can be stated as 30° north of east or 60° east of north, which can be symbolically written as $E30°N$ or $N60°E$. Bearing of Q is $W45°S$ or $S45°W$ or simply WS or SW and that of R is W.



Train Your Brain

Example 37: The angle of elevation of the top of a temple from a point on a hill of slope $\frac{1}{\sqrt{3}}$ is 45°. If the point is at a distance of 100 m from the temple along the hill, find the height of the temple.

Sol.



Given $\tan\angle BPQ = \frac{1}{\sqrt{3}} \Rightarrow \angle BPQ = 30°$

$\angle APQ = 45°$ and $PB = 100$

Now, $BQ = 100 \sin 30° = 50$ m

and $PQ = 100 \cos 30° = 50\sqrt{3}$ m

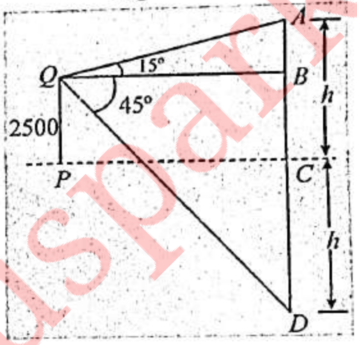
So, $\Delta APQ \Rightarrow \tan 45° = \frac{AQ}{PQ}$

$\Rightarrow h + 50 = 50\sqrt{3} \Rightarrow h = 50(\sqrt{3} - 1) = 36.6$ m.

Example 38: The angle of elevation of a stationary cloud from a point 2500 metres above a lake is 15° and the angle of depression of its reflection in the lake is 45°. What is the height of the cloud above the lake level?

Sol. $\tan 15° = \frac{AB}{BQ}$

$$\Rightarrow \tan(45°-30°) = \frac{h-2500}{BQ} \quad \text{... (i)}$$



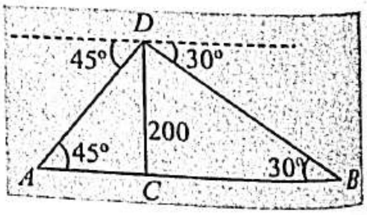
$$\Rightarrow \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}} = \frac{h-2500}{BQ} \text{ and } \tan 45° = \frac{BD}{BQ}$$

$$\Rightarrow 1 = \frac{h+2500}{BQ} \quad \text{... (ii)}$$

$$\text{(i)} \div \text{(ii)} \Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{h-2500}{h+2500} \Rightarrow h = 2500\sqrt{3} \text{ m}$$

Example 39: An observer on the top of a cliff 200 m above the sea level observes the angle of depression of two objects on the opposite sides of the cliff to be 45° and 30°. Find the distance between the objects.

Sol.



$AC = 200 \cot 45°$

$= 200$ m

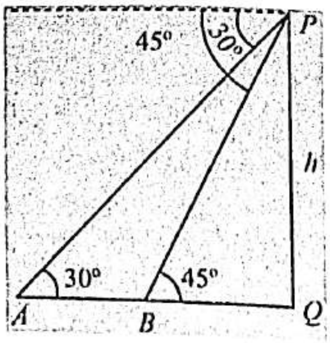
$BC = 200 \cot 30°$

$= 200\sqrt{3}$ m

$\Rightarrow AB = 200(\sqrt{3} + 1) = 546.4$ m.

Example 40: The shadow of a tower standing on a level plane is found to be 60 m longer when the sun's elevation is 30° then when it is 45°. Find the height of tower.

Sol.



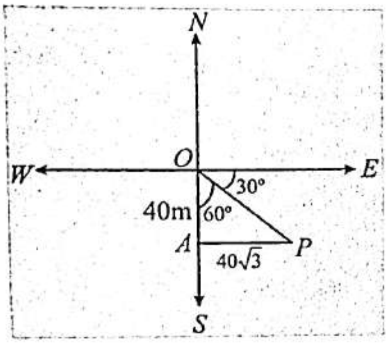
$AB = 60\ m = AQ - BQ$

$= h \cot 30° - h \cot 45°$

$$\Rightarrow \quad h = \frac{60}{\sqrt{3}-1}\text{m} = \frac{60\left(\sqrt{3}+1\right)}{3-1} = 81.96\,\text{m}$$

Example 41: Find the distance and bearing of a point situated 40 m south and $40\sqrt{3}$ m east of the reference point.

Sol.



$$\tan\angle POA = \frac{40\sqrt{3}}{40} = \sqrt{3}$$

$\Rightarrow \quad \angle POA = 60°$

$\therefore$ Bearing of P can be written as $E30°S$ or $S60°E$ or 30° south of east 'or' 60° east of south.

Distance $OP = \sqrt{(40)+\left(40\sqrt{3}\right)^2} = 80$ m

Concept Application

27. Two men are on the opposite side of a tower. They measure the angles of elevation of the top of the tower 45° and 30° respectively. If the height of the tower is 40 m, find the distance between the men.

28. A ladder 5 metre long leans against a vertical wall. The bottom of the ladder is 3 metre far from the wall. If the bottom of the ladder is pulled 1 metre farther from the wall, how much does the top of the ladder slide down the wall.

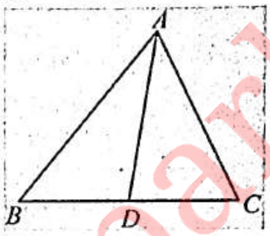
29. The angle of elevation of the top of a pillar at any point A on the ground is 15°. On walking 40 metres towards the pillar, the angle become 30°. The height of the pillar is.

30. The shadow of a tower standing on a level ground is found to be 40 m longer when the sun's altitude is 30° than when it is 60°. The height of the tower is.

Apollonius Theorem

If AD is the median of a ΔABC, then

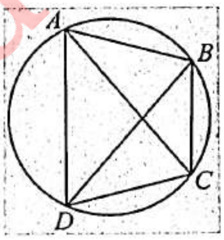
$AB^2 + AC^2 = 2(AD^2 + BD^2)$ or $2(AD^2 + DC^2)$



Ptolemy's Theorem

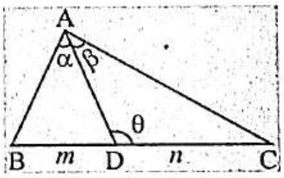
In a cyclic quadrilateral $ABCD$

$AC.BD = AB.CD + AD.BC$



m-n Theorem

If D be the point on the side BC of a ΔABC which divides the side BC in the ratio $m : n$, then according the figure



we have,

(*i*) $(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

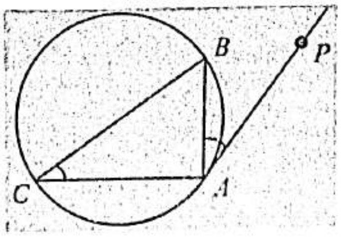
(*ii*) $(m+n)\cot\theta = n\cot B - m\cot C$

Properties of Circles

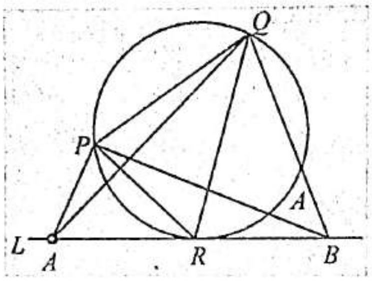
(*i*) If AB subtends equal angles at two points P and Q, the points A, B, P and Q are concyclic. ($\because$ angles on the same segment of a circle are equal)

(*ii*) Angle subtended by a chord at the center is twice the angle subtended at any point on the circumference.

(*iii*) Let AP be the tangent at a point A on the circumference of a circle passing through A, B and C. Then $\angle BAP = \angle ACB$.



(*iv*) A line segment PQ subtends various angles at different points on another line L. Obviously, at some point (say R) PQ will subtend the greatest angle also. Then, the circle passing through P, Q, R touches the line L at R.

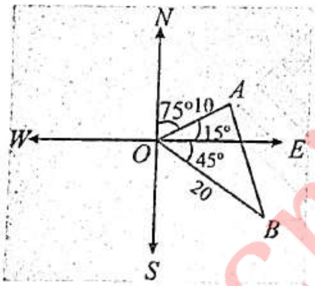


Three dimensional problems in heights and distances are generally solved by decomposing the picture into two 2-dimensional views [top (or plane) view and front view] and then principles of plane trigonometry are used.

Train Your Brain

Example 42: Two cars leave a place at the same time. One travels 10 km in the direction $N75°E$ and the other 20 km in the south-east direction. What is the final distance between the cars?

Sol.



$$\cos\angle AOB=\frac{OA^2+OB^2-AB^2}{2(OA)(OB)}$$

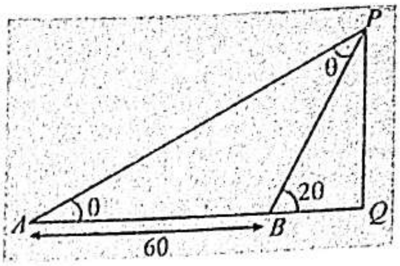
$$\Rightarrow\ AB^2=(10)^2+(20)^2-2(10)(20)\cos 60°$$

$$=100+400-200=300$$

$$\Rightarrow\ AB=10\sqrt{3}\ \text{km}$$

Example 43: A man observes the angle of elevation of the top of a tower to be θ°. He moves a distance 60 m towards the tower and finds that the elevation has doubled. Find the value of sin2θ. If height of the tower is 50 m.

Sol.



In ΔPAB, $\angle ABP=\pi-2\theta$ and $\angle APB=\theta$ (exterior angle is the sum of two opposite interior angles)

Applying sine rule,

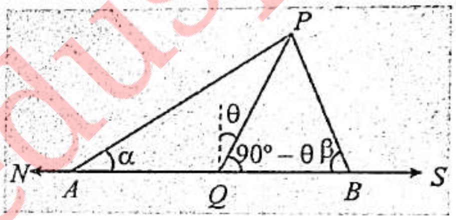
$$\frac{\sin\theta}{AB}=\frac{\sin(\pi-2\theta)}{AP}$$

$$\Rightarrow\ \frac{\sin\theta}{60}=\frac{\sin 2\theta}{\frac{PQ}{\sin\theta}}=\frac{\sin 2\theta\cdot\sin\theta}{50}$$

$$\Rightarrow\ \sin 2\theta=\frac{5}{6}$$

Example 44: A chimney leans towards the south. At equal distances due north and south of it the angles of elevation of the top of chimney are α and β respectively. Find the inclination of the chimney to the vertical.

Sol.



We require the value of θ.

Applying $m-n$ theorem to ΔPAB, We have

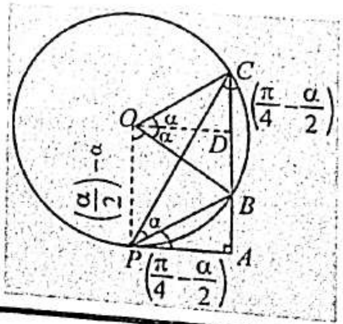
$$(1+1)\cot(90°-\theta)=1.\cot\alpha-1.\cot\beta$$

$$\Rightarrow\ \tan\theta=\frac{\cot\alpha-\cot\beta}{2}$$

$$\Rightarrow\ \theta=\tan^{-1}\left(\frac{\cot\alpha-\cot\beta}{2}\right)$$

Example 45: A man is moving towards a building on which a flagstaff is mounted. The flagstaff subtends its maximum angle α at the man's eye when he is at a distance 'c' from the base of building. Obtain the height of building and flagstaff in terms of c and α.

Sol.


C
$\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$
O
α
α
D
$\left(\frac{\pi}{2}-\alpha\right)$
B
α
P
$\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$
A

Let AB be the building and BC be the flagstaff.

Since angle subtended by BC is maximum at P, a circle drawn through P, B and C touches PA at P.

The perpendicular bisector of chord BC and a line perpendicular to PA at P intersect at the center O of the circle. Consider chord BC.

$\because$ $\angle BPC = \alpha \Rightarrow \angle BOC = 2\alpha \Rightarrow \angle COD = \alpha$

$\therefore$ Height of flagstaff $= BC = 2CD$

$= 2OD \tan \angle COD$

$= 2PA \tan \alpha = 2c \tan \alpha$

Now consider chord PB.

$\because$ $\angle POB = \dfrac{\pi}{2} - \alpha$

$\Rightarrow$ $\angle PCB = \dfrac{\pi}{4} - \dfrac{\alpha}{2}$

$\Rightarrow$ $\angle BPA = \dfrac{\pi}{4} - \dfrac{\alpha}{2}$ (alternate segment theorem)

$\therefore$ Height of building $= AB = AP \tan \angle BPA = C \tan\left(\dfrac{\pi}{4} - \dfrac{\alpha}{2}\right)$

Concept Application

31. A vertical lamp post stands at the mid-point of the edge BC of a park in the shape of a triangle. The angle of elevation of the top of the lamp post from A is 30°. If the sides AB, BC and CA respectively are 40m, $20\sqrt{6}$ m and 20m, obtain the height of lamp post.
32. A rocket of height h metres is fired vertically upwards. Its velocity at time t seconds is $(2t + 3)$ metres/second. If the angle of elevation of the top of the rocket from a point on the ground after 1 second of firing is $\pi/6$ and after 3 seconds it is $\pi/3$ then the distance of the point from the rocket is.
33. A piece of paper in the shape of a sector of a circle of radius 10 cm and of angle 216° just covers the lateral surface of a right circular cone of vertical angle 2θ. Then $\sin\theta$ is.
34. A man standing between two vertical posts finds that the angle subtended at his eyes by the tops of the posts is a right angle. If the heights of the two posts are two times and four times the height of the man, and the distance between them is equal to the length of the longer post, then the ratio of the distances of the man from the shorter and the longer post is.

"To relax is to give yourself the permission to simply be, without the weight of the world on your shoulders."

Aarambh (Solved Examples)

1. If $\frac{\sin^4\alpha}{a}+\frac{\cos^4\alpha}{b}=\frac{1}{a+b}$ then, $\frac{\sin^8\alpha}{a^3}+\frac{\cos^8\alpha}{b^3}=$

(*a*) $\frac{1}{(a+b)^3}$ (*b*) $\frac{1}{(a-b)^3}$

(*c*) $(a+b)^3$ (*d*) $(a-b)^3$

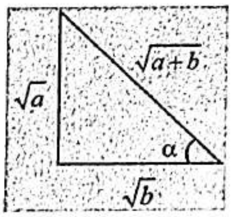
Sol. $\frac{\sin^4\alpha}{a}+\frac{\cos^4\alpha}{b}=\frac{1}{a+b}$

$$\frac{(a+b)}{a}\sin^4\alpha+\frac{(a+b)}{b}\cos^4\alpha=1$$

$$\Rightarrow \sin^4\alpha+\frac{b}{a}\sin^4\alpha+\frac{a}{b}\cos^4\alpha+\cos^4\alpha=1$$

$$\Rightarrow 1-2\sin^2\alpha\cos^2\alpha+\frac{b}{a}\sin^4\alpha+\frac{a}{b}\cos^4\alpha=1$$

$$\Rightarrow\left(\sqrt{\frac{b}{a}}\sin^2\alpha-\sqrt{\frac{a}{b}}\cos^2\alpha\right)^2=0$$



$$\Rightarrow \tan^2\alpha=\frac{a}{b}$$

$$\Rightarrow \text{Now, } \frac{\sin^8\alpha}{a^3}+\frac{\cos^8\alpha}{b^3}$$

$$=\frac{a^4}{(a+b)^4}\frac{1}{a^3}+\frac{b^4}{(a+b)^4}\frac{1}{b^3}=\frac{1}{(a+b)^3}$$

Therefore, option (*a*) is the correct answer.

2. If $X=\sin\left(\theta+\frac{7\pi}{12}\right)+\sin\left(\theta-\frac{\pi}{12}\right)+\sin\left(\theta+\frac{3\pi}{12}\right)$,

$Y=\cos\left(\theta+\frac{7\pi}{12}\right)+\cos\left(\theta-\frac{\pi}{12}\right)+\cos\left(\theta+\frac{3\pi}{12}\right)$, then

$\frac{X}{Y}-\frac{Y}{X}=$

(*a*) $2\tan2\theta$ (*b*) $2\tan^2\theta$

(*c*) $2\cot^2\theta$ (*d*) None of these

Sol. $X=\sin\left(\theta+\frac{7\pi}{12}\right)+\sin\left(\theta-\frac{\pi}{12}\right)+\sin\left(\theta+\frac{3\pi}{12}\right)$

$$=2\sin\left(\frac{\theta+\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)+\sin\left(\theta+\frac{\pi}{4}\right) \quad =2\sin\left(\theta+\frac{\pi}{4}\right)$$

$$Y=\cos\left(\theta+\frac{7\pi}{12}\right)+\cos\left(\theta-\frac{\pi}{12}\right)+\cos\left(\theta+\frac{3\pi}{12}\right)$$

$$=2\cos\left(\theta+\frac{\pi}{4}\right)\cdot\frac{1}{2}+\cos\left(\theta+\frac{\pi}{4}\right)$$

$$=2\cos\left(\theta+\frac{\pi}{4}\right)$$

Now $\frac{X}{Y}-\frac{Y}{X}=\frac{2\sin(\theta+\pi/4)}{2\cos(\theta+\pi/4)}-\frac{2\cos(\theta+\pi/4)}{2\sin(\theta+\pi/4)}$

$$=\left[\frac{-2\cos(2\theta+\pi/2)}{\sin(2\theta+\pi/2)}\right]=2\tan2\theta$$

Therefore, option (*a*) is the correct answer.

3. Evaluate: $\cos^6\frac{\pi}{16}+\cos^6\frac{3\pi}{16}+\cos^6\frac{5\pi}{16}+\cos^6\frac{7\pi}{16}$

(*a*) 5/8 (*b*) 5/4 (*c*) 7/16 (*d*) 5/32

Sol. $\left(\cos^6\frac{\pi}{16}+\sin^6\frac{\pi}{16}\right)+\left(\cos^6\frac{3\pi}{16}+\sin^6\frac{3\pi}{16}\right)$

$$=2-3\cos^2\frac{\pi}{16}\sin^2\frac{\pi}{16}-3\cos^2\frac{3\pi}{16}\sin^2\frac{3\pi}{16}$$

$$=2-\frac{3}{4}\left[\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}\right]$$

$$=2-\frac{3}{4}\left[\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}\right]=\frac{5}{4}$$

Therefore, option (*b*) is the correct answer.

4. If $\frac{x}{\cos\theta}=\frac{y}{\cos\left(\theta+\frac{2\pi}{3}\right)}=\frac{z}{\cos\left(\theta-\frac{2\pi}{3}\right)}$, then $x+y+z$ i

equal to:

(*a*) -1 (*b*) 1 (*c*) 0 (*d*) 2

Sol. Given $\frac{x}{\cos\theta}=\frac{y}{\cos\left(\theta+\frac{2\pi}{3}\right)}=\frac{z}{\cos\left(\theta-\frac{2\pi}{3}\right)}=\lambda$ (say)

$$\Rightarrow x+y+z=\lambda\left\{\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)+\cos\left(\theta-\frac{2\pi}{3}\right)\right\}$$

$$=\lambda\left\{\cos\theta+2\cos\theta\cos\frac{2\pi}{3}\right\}=0$$

Therefore, option (*c*) is the correct answer.

5. If $\alpha+\beta+\gamma+\delta=2\pi$, then the value of $\cos\alpha+\cos\beta+$ co

$\gamma+\cos\delta+4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\gamma}{2}\right)\cos\left(\frac{\alpha+\delta}{2}\right)$ is

(*a*) 3/2 (*b*) 5/4 (*c*) 3 (*d*) 0

Sol. Given $\alpha+\beta+\gamma+\delta=2\pi$

Tanking L.H.S. we have

$\cos\alpha+\cos\beta+\cos\gamma+\cos\delta+$

$$4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\gamma}{2}\right)\cos\left(\frac{\alpha+\delta}{2}\right.$$

$$\Rightarrow\ 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}+2\cos\frac{\gamma+\delta}{2}\cos\frac{\gamma-\delta}{2}+$$

$$4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\gamma}{2}\right)\cos\left(\frac{\alpha+\delta}{2}\right.$$

$\gamma+\delta=2\pi-(\alpha+\beta)$

so $\cos\dfrac{\gamma+\delta}{2}=\cos\left(\pi-\dfrac{\alpha+\beta}{2}\right)=-\cos\left(\dfrac{\alpha+\beta}{2}\right)$

$$=2\cos\left(\frac{\alpha+\beta}{2}\right)\left[\cos\left(\frac{\alpha-\beta}{2}\right)-\cos\left(\frac{\gamma-\delta}{2}\right)\right]$$

$$+4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\gamma}{2}\right)\cos\left(\frac{\alpha+\delta}{2}\right)-2\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$\left[2\cos\left(\frac{\alpha+\gamma-(\beta+\delta)}{4}\right)\sin\frac{(\gamma+\beta)-(\alpha+\delta)}{4}\right]$$

$$+4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\gamma}{2}\right)\cos\left(\frac{\alpha+\delta}{2}\right)$$

$\because \beta+\delta=2\pi-(\alpha+\gamma)$ and $\gamma+\beta=2\pi-(\alpha+\delta)$

so, $\sin\dfrac{\alpha+\gamma-(\beta+\delta)}{4}$

$=\sin\dfrac{2(\alpha+\gamma)-2\pi}{4}=-\sin\left(\dfrac{\pi}{2}-\dfrac{\alpha+\gamma}{2}\right)$

$=-\cos\dfrac{\alpha+\gamma}{2}$ and $\sin\dfrac{\gamma+\beta-(\alpha+\delta)}{4}$

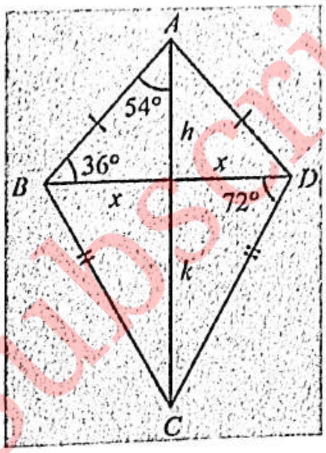
$=\sin\left(\dfrac{2\pi-2(\alpha+\delta)}{4}\right)=\cos\left(\dfrac{\alpha+\delta}{2}\right)\quad\Rightarrow 2\cos\left(\dfrac{\alpha+\beta}{2}\right)$

Therefore, option (*d*) is the correct answer.

6. In a kite $ABCD$, $AB=AD$ and $CB=CD$. If $\angle A=108^\circ$ and $\angle C=36^\circ$ then the ratio of the area of ΔABD to the area of ΔCBD can be written in the form $\dfrac{a-b\tan^2 36^\circ}{c}$ where a, b and c are relatively prime positive integers. Determine the ordered triple (a, b, c).

(*a*) (1, 2, 1) (*b*) (2, 1, 1) (*c*) (4, 3, 2) (*d*) (1, 1, 2)

Sol. Since the triangles ABD and CBD have a common base, hence the ratio of their areas equals the ratio of their heights.



Since $\tan 36^\circ=\dfrac{h}{x}$, then $h=x\tan 36^\circ$

$\Rightarrow \tan 72^\circ=\dfrac{k}{x}$, then $k=x\tan 72^\circ$

Hence, $\dfrac{h}{k}=\dfrac{x\tan 36^\circ}{x\tan 72^\circ}=\dfrac{\tan 36^\circ}{\dfrac{2\tan 36^\circ}{1-\tan^2 36^\circ}}=\dfrac{1-\tan^2 36^\circ}{2}$

Then ordered triple (a, b, c) is $(1, 1, 2)$.

Therefore, option (*d*) is the correct answer.

7. Which of the following is/are correct?

(*a*) $(\tan x)^{\ln(\sin x)}>(\cot x)^{\ln(\sin x)},\ \forall x\in(0,\pi/4)$

(*b*) $(1/2)^{\ln(\cos x)}<(1/3)^{\ln(\cos x)},\ \forall x\in(0,\pi/2)$

(*c*) $4^{\ln(\operatorname{cosec} x)}<5^{\ln(\operatorname{cosec} x)},\ \forall x\in(0,\pi/2)$

(*d*) $2^{\ln(\tan x)}>2^{\ln(\sin x)},\ \forall x\in(0,\pi/2)$

Sol. (*a*) For $x\in\left(0,\dfrac{\pi}{4}\right)$, $\tan x<\cot x$

Also $\ln(\sin x)<0\Rightarrow(\tan x)^{\ln(\sin x)}>(\cot x)^{\ln(\sin x)}$

(*b*) $x\in\left(0,\dfrac{\pi}{2}\right)\Rightarrow\cos x\in(0,1)$

$\Rightarrow\ln(\cos x)<0$

Also $\dfrac{1}{2}>\dfrac{1}{3}\Rightarrow\left(\dfrac{1}{2}\right)^{\ln(\cos x)}<\left(\dfrac{1}{3}\right)^{\ln(\cos x)}$

(*c*) $x\in\left(0,\dfrac{\pi}{2}\right)$, $\operatorname{cosec} x\geq 1\Rightarrow\ln(\operatorname{cosec} x)\geq 0$

$\Rightarrow 4^{\ln(\operatorname{cosec} x)}<5^{\ln(\operatorname{cosec} x)}$

(*d*) $x\in\left(0,\dfrac{\pi}{2}\right)$

Since $\sin x<\tan x$, we get $\ln(\sin x)<\ln(\tan x)$ |

$\Rightarrow 2^{\ln\sin x}<2^{\ln\tan x}$

Therefore, option (*a*,*b*,*c*,*d*) is the correct answers.

8. If the equation $\cot^4 x-2\operatorname{cosec}^2 x+a^2=0$ has at least one solution then possible integral values of a can be:

(*a*) −1 (*b*) 0 (*c*) 1 (*d*) 2

Sol. $\Rightarrow\cot^4 x-2(1+\cot^2 x)+a^2=0$

$\Rightarrow\cot^4 x-2\cot^2 x+a^2-2=0$

$\Rightarrow(\cot^2 x-1)^2=3-a^2$

to have atleast one solution

$\Rightarrow 3-a^2\geq 0$

$\Rightarrow a^2-3\leq 0$

$\therefore\ a\in\left[-\sqrt{3},\sqrt{3}\right]$

Therefore, option (*a*,*b*,*c*) is the correct answers.

9. $(a+2)\sin\alpha+(2a-1)\cos\alpha=(2a+1)$ if $\tan\alpha=$

(*a*) $\dfrac{3}{4}$ (*b*) $\dfrac{4}{3}$ (*c*) $\dfrac{2a}{a^2+1}$ (*d*) $\dfrac{2a}{a^2-1}$

Sol.

Let $\tan\dfrac{\alpha}{2}=t$

$(a+2)\,2t+(2a-1)(1-t^2)=(2a+1)(t^2+1)$

$\Rightarrow 2a\,t+4t+2a-2a\,t^2-1+t^2=2a+1+2a\,t^2+t^2$

$\Rightarrow 4a\,t^2-2t(2+a)+2=0\Rightarrow 2a\,t^2-2t-a\,t+1=0$

$\Rightarrow 2t(at-1) - 1\,(at-1) = 0 \Rightarrow t = 1/2,\ t = 1/a$

$\Rightarrow \tan\alpha = \dfrac{2\tan\alpha/2}{1-\tan^2\alpha/2}$

$\Rightarrow \tan\alpha = \dfrac{2\times 1/2}{1-1/4} = 4/3$

or $\tan\alpha = \dfrac{2/a}{1-1/a^2} = \dfrac{2a}{a^2-1}$

Therefore, option (*b,d*) is the correct answers.

10. If 3 sin β=sin (2α+β), then tan (α+β) – 2 tan α is

(*a*) Independent of α

(*b*) Independent of β

(*c*) Dependent of both α and β

(*d*) Independent of α but dependent of β

Sol. $3\sin\beta = \sin(2\alpha+\beta)$

$\Rightarrow \dfrac{\sin(2\alpha+\beta)}{\sin\beta} = \dfrac{3}{1}$

Applying componendo & dividendo

$\Rightarrow \dfrac{\sin(2\alpha+\beta)+\sin\beta}{\sin(2\alpha+\beta)-\sin\beta} = \dfrac{3+1}{3-1} = \dfrac{2}{1}$

$\Rightarrow \dfrac{2\sin(\alpha+\beta).\cos\alpha}{2\cos(\alpha+\beta)\sin\alpha} = 2$

$\Rightarrow \tan(\alpha+\beta) = 2\tan\alpha$

$\Rightarrow \tan(\alpha+\beta) - 2\tan\alpha = 0$

Hence, independent of α & β both.

Therefore, option (*a,b*) is the correct answers.

11. If α + β = *c* where α, β > 0 each lying between 0 and π/2 and *c* is a constant, then

(*a*) The maximum value of $\sin\alpha + \sin\beta = 2\sin\dfrac{c}{2}$

(*b*) The minimum value of $\tan\alpha + \tan\beta = 2\tan\dfrac{c}{2}$

(*c*) The maximum value of $\sin\alpha + \sin\beta = 4\sin\dfrac{c}{2}$

(*d*) The minimum value of $\tan\alpha + \tan\beta = 4\tan\dfrac{c}{2}$

Sol. $\sin\alpha + \sin\beta = 2\sin\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$

$= 2\sin\dfrac{c}{2}\cos\left(\dfrac{\alpha-\beta}{2}\right)$

Its max. value is $2\sin\dfrac{c}{2}$

$\tan\alpha + \tan\beta = \dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$

$= \dfrac{2\sin c}{\cos(\alpha+\beta)+\cos(\alpha-\beta)}$

$= \dfrac{\sin c}{\cos^2\dfrac{c}{2} - \sin^2\left(\dfrac{\alpha-\beta}{2}\right)}$

This is minimum when denominator is maximum i.e. whe

$\sin^2\left(\dfrac{\alpha-\beta}{2}\right)$ is zero.

$\dfrac{2\sin\dfrac{c}{2}\cos\dfrac{c}{2}}{\cos^2\dfrac{c}{2}} = 2\tan\dfrac{c}{2}$

Therefore, option (*a,b*) is the correct answers.

12. Match the Column:

Column-I		Column-II	
A.	If for some real x, the equation $x + \dfrac{1}{x} = 2\cos\theta$ holds, then cos θ is equal to	p.	2
B.	If sin θ + cosec θ = 2, then $\sin^{2023}\theta + \text{cosec}^{2023}\theta$ is equal to	q.	1
C.	Maximum value of $\sin^4\theta + \cos^4\theta$ is	r.	0
D.	Least value of $2\sin^2\theta + 3\cos^2\theta$ is	s.	–1

(*a*) A → q, s; B → p; C → q; D → p

(*b*) A → p; B → q, s; C → q; D → p

(*c*) A → s; B → p; C → q, s; D → p

(*d*) A → s; B → p; C → q; D → q, s

Sol. (*a*) $x + \dfrac{1}{x} = 2\cos\theta \geq 2$ or ≤ -2

$\Rightarrow \cos\theta = 1$ or -1

(*b*) $\sin\theta + \text{cosec}\,\theta = 2$

$\because \sin\theta + \dfrac{1}{\sin\theta} \geq 2$ or ≤ -2

but given that $\sin\theta + \text{cosec}\,\theta = 2 \Rightarrow \sin\theta + \dfrac{1}{\sin\theta} =$

which is possible only when $\sin\theta = 1$

$\therefore \sin^{2023}\theta + \text{cosec}^{2023}\theta = \sin^{2023} + \dfrac{1}{\sin^{2023}}$

$= 1 + 1 = 2$

(*c*) $\sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$= 1 - \dfrac{1}{2}\sin^2 2\theta$

$\because 0 \leq \sin^2 2\theta \leq 1$

$\therefore \dfrac{1}{2} \leq 1 - \dfrac{1}{2}\sin^2 2\theta \leq 1$

$\therefore$ maximum value = 1

(*d*) $2\sin^2\theta + 3\cos^2\theta = 2\sin^2\theta + 3 - 3\sin^2\theta = 3 - \sin^2\theta$

$\because 0 \leq \sin^2\theta \leq 1$

$\therefore 2 \leq 3 - \sin^2\theta \leq 3$

$\therefore$ least value = 2

Therefore, option (*a*) is the correct answer.

Comprehension Question (13 - 14)

Let α is a root of the equation $(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$, β is a root of the equation $3\cos^2 x - 10\cos x + 3 = 0$ and γ is a root of the equation $1 - \sin 2x = \cos x - \sin x$. $0 \le \alpha, \beta, \gamma \le \frac{\pi}{2}$, then

13. $\cos\alpha + \cos\beta + \cos\gamma$ can be equal to

(*a*) $\frac{3\sqrt{6}+2\sqrt{2}+6}{6\sqrt{2}}$ (*b*) $\frac{3\sqrt{3}+8}{6}$

(*c*) $\frac{3\sqrt{3}+2}{6}$ (*d*) None of these

Sol. Now, $(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$

$\Rightarrow (1 + \cos x)[2\sin x - \cos x - 1 + \cos x] = 0$

$\Rightarrow (1 + \cos x)(2\sin x - 1) = 0$

$\Rightarrow \cos x = -1$ or $\sin x = \frac{1}{2}$

So, $\sin\alpha = \frac{1}{2}$ $\left[\text{as } 0 \le \alpha \le \frac{\pi}{2}\right]$

$\therefore \cos\alpha = \frac{\sqrt{3}}{2}$

$3\cos^2 x - 10\cos x + 3 = 0$

$\cos x = \frac{1}{3}, \cos x \ne 3$

$\cos\beta = \frac{1}{3}, \sin\beta = \frac{2\sqrt{2}}{3}$

and $1 - \sin 2x = \cos x - \sin x$

$\Rightarrow \sin^2 x + \cos^2 x - 2\sin x\cos x = \cos x - \sin x$

$\Rightarrow (\cos x - \sin x)(\cos x - \sin x - 1) = 0$

$\Rightarrow \sin x = \cos x = \frac{1}{\sqrt{2}}$

or $\cos x - \sin x = 1$

$\Rightarrow \cos x = 1, \sin x = 0$

$\Rightarrow \cos\gamma = 1, \sin\gamma = 0$

$\cos\alpha + \cos\beta + \cos\gamma = \frac{\sqrt{3}}{2} + \frac{1}{3} + 1 = \frac{3\sqrt{3}+8}{6}$

Therefore, option (*b*) is the correct answer.

14. $\sin(\alpha - \beta)$ is equal to

(*a*) 1 (*b*) 0

(*c*) $\frac{1-2\sqrt{6}}{6}$ (*d*) $\frac{\sqrt{3}-2\sqrt{2}}{6}$

Sol. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \frac{1}{2} \times \frac{1}{3} - \frac{\sqrt{3}}{2} \times \frac{2\sqrt{2}}{3} = \frac{1-2\sqrt{6}}{6}$

Therefore, option (*c*) is the correct answer.

15. The value of $f(x) = x^4 + 4x^3 + 2x^2 - 4x + 7$, when $x = \cot\frac{11\pi}{8}$ is

Sol. $x = \cot\frac{11\pi}{8} = \cot\left(\pi + \frac{3\pi}{8}\right) = \cot\frac{3\pi}{8} = \sqrt{2} - 1$

$(x + 1)^2 = 2$

$x^2 + 2x - 1 = 0$

$f(x) = x^4 + 4x^3 + 2x^2 - 4x + 7$

$= x^2(x^2 + 2x - 1) + 2x^3 + 3x^2 - 4x + 7$

$= 2x(x^2 + 2x - 1) - x^2 - 2x + 7 = -x^2 - 2x + 7$

$= -(x^2 + 2x - 1) + 6 = 0 + 6 = 6$

Therefore, 6 is the correct answer.

16. If $\cos(\theta - \phi), \cos\theta, \cos(\theta + \phi)$, are in H. P, then the value of $\sqrt{2}\cos\theta . \sec\frac{\phi}{2}$ is $\left(0 < \theta, \phi < \frac{\pi}{2}\right)$

Sol. $\cos\theta = \frac{2\cos(\theta-\phi).\cos(\theta+\phi)}{\cos(\theta-\phi)+\cos(\theta+\phi)} = \frac{2(\cos^2\theta - \sin^2\phi)}{2\cos\theta.\cos\phi}$

$\Rightarrow \cos^2\theta . \cos\phi = \cos^2\theta - \sin^2\phi$

$\Rightarrow \sin^2\phi . \cos^2\theta (1 - \cos\phi)$

$\Rightarrow 4\sin^2\frac{\phi}{2} . \cos^2\frac{\phi}{2} = 2\cos^2\theta . \sin^2\frac{\phi}{2}$

$\Rightarrow \cos\theta \sec\frac{\phi}{2} = \sqrt{2}$

Therefore, 2 is the correct answer.

17. The maximum value of $5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$ is

Sol. $5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$

$= 5\cos\theta + 3\left(\cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3}\right) + 3$

$= 5\cos\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$= \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$\therefore$ maximum value $= 3 + \sqrt{\left(\frac{13}{2}\right)^2 + \left(-\frac{3\sqrt{3}}{2}\right)^2}$

$= 3 + \sqrt{\frac{196}{4}} = 3 + 7 = 10$

Therefore, 10 is the correct answer.

18. If $x^2 + y^2 = 4$ and $a^2 + b^2 = 8$. Find sum of minimum and maximum value of $(ax + by)$.

Sol. Let $x = r_1\cos\theta, y = r_1\sin\theta$ and $a = r_2\cos\phi; b = r_2\sin\phi$

$\therefore r_1 = 2, r_2 = 2\sqrt{2}$

Then $(ax + by) = r_1 r_2\cos(\theta - \phi)$, $-r_1 r_2 \le (ax + by) \le r_1 r_2$

$-4\sqrt{2} \le (ax + by) \le 4\sqrt{2}$

$y_{\max} = 4\sqrt{2}$ and $y_{\min} = -4\sqrt{2}$

Therefore, 0 is the correct answer.

School Level Problems

SINGLE CORRECT TYPE QUESTIONS

1. If $\tan\theta = \frac{-4}{3}$, then $\sin\theta$ is

(a) $\frac{-4}{5}$ but not $\frac{4}{5}$ (b) $\frac{-4}{5}$ or $\frac{4}{5}$

(c) $\frac{4}{5}$ but not $-\frac{4}{5}$ (d) None of these

2. The value of $\sin 20° \sin 40° \sin 60° \sin 80°$ is

(a) $\frac{-3}{16}$ (b) $\frac{5}{16}$

(c) $\frac{3}{16}$ (d) $\frac{1}{16}$

3. If $\sin\theta + \cos\theta = 1$, then the value of $\sin 2\theta$ is equal to

(a) 1 (b) $\frac{1}{2}$

(c) 0 (d) -1

4. The value of $\sin(45° + \theta) - \cos(45° - \theta)$ is

(a) $2\cos\theta$ (b) $2\sin\theta$

(c) 1 (d) 0

5. The value of $\cot\left(\frac{\pi}{4} + \theta\right)\cot\left(\frac{\pi}{4} - \theta\right)$ is

(a) -1 (b) 0 (c) 1 (d) Not defined

6. If $\tan A = \frac{1}{2}, \tan B = \frac{1}{3}$, then $\tan(2A + B)$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

7. The value of $\frac{1 - \tan^2 15°}{1 + \tan^2 15°}$ is

(a) 1 (b) $\sqrt{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) 2

8. Which of the following is correct?

(a) $\sin 1° > \sin 1$ (b) $\sin 1° < \sin 1$

(c) $\sin 1° = \sin 1$ (d) $\sin 1° = \frac{\pi}{18°}\sin 1$

9. The value of $\tan 75° - \cot 75°$ is equal to

(a) $2\sqrt{3}$ (b) $2 + \sqrt{3}$

(c) $2 - \sqrt{3}$ (d) 1

VERY SHORT ANSWER TYPE QUESTIONS

10. Find the value of $\sqrt{3}\operatorname{cosec} 20° - \sec 20°$

11. If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$, then show that $\frac{\tan x}{\tan y} = \frac{a}{b}$

12. If θ lies in the second quadrant, then show that

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = -2\sec\theta$$

13. If $\cot\theta + \tan\theta = 2\operatorname{cosec}\theta$, then find the general value of θ.

14. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then prove th
$m^2 - n^2 = 4\sin\theta\tan\theta$

15. If $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$, where α lie betwee
0 and $\frac{\pi}{4}$, find the value of $\tan 2\alpha$.

16. Prove that $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$

LONG ANSWER TYPE QUESTIONS

17. Find the value of

$$\left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 + \cos\frac{5\pi}{8}\right)\left(1 + \cos\frac{7\pi}{8}\right)$$

18. Show that

$$2\sin^2\beta + 4\cos(\alpha + \beta)\sin\alpha\sin\beta + \cos 2(\alpha + \beta) = \cos 2\alpha$$

19. Find the value of the expression

$$\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$$

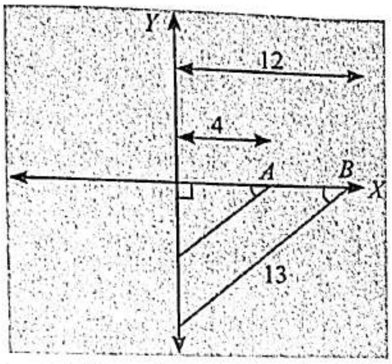
20. If $\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$, then prove th
$\cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta) = 1 - 2a^2 - 2b^2$

21. If $a\cos 2\theta + b\sin 2\theta = c$ has α and β as its roots, then prov
that $\tan\alpha + \tan\beta = \frac{2b}{a+c}$.

CASE STUDY BASED QUESTIONS

CASE STUDY-I

Rajiv constructs two right angled triangles in the fourth quadrant in such a way that the measure of triangle gives $\cos A = \frac{4}{5}$ and $\cos B = \frac{12}{13}$, where $\frac{3\pi}{2} < A$ and $B < 2\pi$.



Based on the above information, answer the following questions.

22. Find the value of $\cos(A + B)$

(a) $\frac{13}{16}$ (b) $\frac{13}{65}$ (c) $\frac{33}{65}$ (d) $\frac{1}{16}$

23. Find the value of $\tan(A + B)$

(a) $\frac{36}{33}$ (b) $\frac{56}{65}$

(c) $-\frac{56}{33}$ (d) $\frac{56}{33}$

CASE STUDY-II

In a class test of class XI, a teacher asked to students to consider $A + B = \frac{\pi}{4}$, where A and B are acute angles.

Based on the above information, answer the following questions.

24. Find the value of $(1 + \tan A)(1 + \tan B)$?

(a) -1 (b) 0

(c) 1 (d) 2

25. Find the value of $\sin(A + B) - \cos(A + B) + \tan(A + B)$.

(a) -1 (b) 0

(c) 1 (d) 2

Prarambh (Topicwise)

ANGLE AND ITS MEASUREMENT

1. An equilateral triangle has side length 8. The area of the region containing all points outside the triangle but not more than 3 units from a point on the triangle is

(a) $9(8 + \pi)$ (b) $8(9 + \pi)$

(c) $9\left(8+\frac{\pi}{2}\right)$ (d) $8\left(9+\frac{\pi}{2}\right)$

2. The perimeter of a certain sector of a circle is equal to half that of the circle of which it is a sector. The circular measure of one angle of the sector is

(a) $(\pi - 2)$ radian (b) $(\pi + 2)$ radian

(c) π radian (d) $(\pi - 3)$ radian

BASIC TRIGONOMETRIC IDENTITIES

3. Number of values of θ for which $\cos\theta = 0.707$ $(\theta \in (0, 2\pi))$

(a) 1 (b) 3

(c) 4 (d) 2

4. If $\frac{2\sin\alpha}{1+\sin\alpha+\cos\alpha} = \lambda$ then $\frac{1+\sin\alpha-\cos\alpha}{1+\sin\alpha}$ is equal to

(a) $\frac{1}{\lambda}$ (b) λ (c) $1-\lambda$ (d) $1+\lambda$

5. If $\sin A \tan A = \cos^2 A$ then $\cos^3 A + \cos^2 A$ is equal to

(a) 1 (b) 2

(c) 4 (d) None of these

6. If $\operatorname{cosec} A + \cot A = \frac{11}{2}$, then $\tan A$ is

(a) $\frac{21}{22}$ (b) $\frac{15}{16}$ (c) $\frac{44}{117}$ (d) $\frac{117}{43}$

7. Which of the following statement is incorrect?

(a) Tangent of odd integral multiple of $\frac{\pi}{2}$ is not defined.

(b) Cotangent of integral multiple of π is not defined.

(c) Tangent of odd integral multiple of π is 0.

(d) Cotangent of integral multiple of $\frac{\pi}{2}$ is not defined.

8. $\frac{\sin^3\theta-\cos^3\theta}{\sin\theta-\cos\theta}-\frac{\cos\theta}{\sqrt{1+\cot^2\theta}}-2\tan\theta\cot\theta=-1$ if

(a) $\theta \in \left(0, \frac{\pi}{2}\right)$ (b) $\theta \in \left(\frac{\pi}{2}, \pi\right)$

(c) $\theta \in \left(\pi, \frac{3\pi}{2}\right)$ (d) $\theta \in \left(\frac{3\pi}{2}, 2\pi\right)$

9. If $\frac{\tan^3 A}{1+\tan^2 A}+\frac{\cot^3 A}{1+\cot^2 A}=p\sec A\,\mathrm{cosec}A+q\sin A\cos A$, then

(a) $p=2, q=1$ (b) $p=1, q=2$

(c) $p=1, q=-2$ (d) $p=2, q=-1$

SIGN OF TRIGONOMETRIC RATIO AND ALLIED ANGLE AND REDUCTON FORMULAE

10. The expression $3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi+\alpha)\right]$ is equal to

(a) 0 (b) 1

(c) 3 (d) $\sin 4\alpha + \sin 6\alpha$

11. $\cos(540° - \theta) - \sin(630° - \theta)$ is equal to

(a) 0 (b) $2\cos\theta$

(c) $2\sin\theta$ (d) $\sin\theta - \cos\theta$

12. The value of $\sin(\pi + \theta)\sin(\pi - \theta)\,\mathrm{cosec}^2\theta$ is equal to

(a) −1 (b) 0

(c) $\sin\theta$ (d) None of these

13. $\cos 0+\cos\frac{\pi}{7}+\cos\frac{2\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{5\pi}{7}+\cos\frac{6\pi}{7}=$

(a) 1/2 (b) −1/2

(c) 0 (d) 1

14. The value of $\cos 10° - \sin 10°$ is

(a) Positive (b) Negative

(c) 0 (d) 1

15. Angle in 3rd quadrant whose sine and cosine are equal

(a) $3\pi/2$ (b) $5\pi/4$ (c) 2π (d) $\pi/2$

16. If $\sin\theta = \frac{24}{25}$ and θ lies in the second quadrant, then $\sec\theta + \tan\theta =$

(a) −3 (b) −5

(c) −7 (d) −9

17. If A lies in the second quadrant and $3\tan A + 4 = 0$, the value of $2\cot A - 5\cos A + \sin A$ is equal to

(a) $\frac{-53}{10}$ (b) $\frac{-7}{10}$

(c) $\frac{7}{10}$ (d) $\frac{23}{10}$

18. The value of $e^{\log_{10}\tan 1° + \log_{10}\tan 2° + \log_{10}\tan 3° + \dots\dots + \log_{10}\tan 89°}$ is

(a) 0 (b) e

(c) $1/e$ (d) 1

19. $\cos A + \sin(270° + A) - \sin(270° - A) + \cos(180° + A) =$

(a) −1 (b) 0

(c) 1 (d) 2

20. $\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}+\sin^2\frac{5\pi}{8}+\sin^2\frac{7\pi}{8}=$

(a) 1 (b) −1

(c) 0 (d) 2

21. If ABCD is a cyclic quadrilateral, then the value of $\cos A - \cos B + \cos C - \cos D =$

(a) 0 (b) 1

(c) $2(\cos B - \cos D)$ (d) $2(\cos A - \cos C)$

22. The value of $\sin 1\cdot\cos 2\cdot\tan 3\cdot\cot 4\cdot\sec 5.\mathrm{cosec}\, 6$ is

(a) Positive

(b) Negative

(c) Zero

(d) May be positive and Negative

23. The value of expression $\sum_{\theta=0}^{8}\frac{1}{1+\tan^3(10\theta)}$, equal is to

(a) 5 (b) $\frac{21}{4}$ (c) $\frac{14}{3}$ (d) $\frac{9}{2}$

24. If $a = \cos(2012\pi)$, $b = \sec(2013\pi)$ and $c = \tan(2014$ then

(a) $a < b < c$ (b) $b < c < a$

(c) $c < b < a$ (d) $a = b < c$

TRIGONOMETRIC IDENTITIES (SUM TO PRODUCT AND PRODUCT TO SUM

25. The value of is $\frac{\sin 24°\;\cos 6° - \sin 6°\sin 66°}{\sin 21°\;\cos 39° - \cos 51°\;\sin 69°}$ is

(a) −1 (b) 1

(c) 2 (d) None of these

26. If $3\sin\alpha = 5\sin\beta$, then $\frac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$ is equal to

(a) 1 (b) 2

(c) 3 (d) 4

27. If $\tan A$ and $\tan B$ are the roots of the quadratic equatio $x^2 - ax + b = 0$, then the value of $\sin^2(A + B)$.

(a) $\frac{a^2}{a^2+(1-b)^2}$ (b) $\frac{a^2}{a^2+b^2}$

(c) $\frac{a^2}{(b+c)^2}$ (d) $\frac{a^2}{b^2(1-a)^2}$

28. In a triangle ABC if $\tan A < 0$ then:

(a) $\tan B.\tan C > 1$ (b) $\tan B.\tan C < 1$

(c) $\tan B.\tan C = 1$ (d) None of these

Credit - Akshat

29. If $\tan A - \tan B = x$ and $\cot B - \cot A = y$, then $\cot(A - B)$ is equal to

(*a*) $\frac{1}{y} - \frac{1}{x}$ (*b*) $\frac{1}{x} - \frac{1}{y}$

(*c*) $\frac{1}{x} + \frac{1}{y}$ (*d*) None of these

30. If $\tan 25° = x$, then $\frac{\tan 155° - \tan 115°}{1 + \tan 155° \tan 115°}$ is equal to

(*a*) $\frac{1-x^2}{2x}$ (*b*) $\frac{1+x^2}{2x}$

(*c*) $\frac{1+x^2}{1-x^2}$ (*d*) $\frac{1-x^2}{1+x^2}$

31. If $\sin\alpha \sin\beta - \cos\alpha \cos\beta + 1 = 0$, then the value of $1 + \cot\alpha \tan\beta$ is

(*a*) 1 (*b*) −1

(*c*) 2 (*d*) None of these

32. If $\tan^2\theta = 2\tan^2\phi + 1$, then the value of $\cos 2\theta + \sin^2\phi$ is

(*a*) 1 (*b*) 2

(*c*) − 1 (*d*) Independent of ϕ

33. The value of the expression $\left(1 + \cos\frac{\pi}{10}\right)\left(1 + \cos\frac{3\pi}{10}\right)$ $\left(1 + \cos\frac{7\pi}{10}\right)\left(1 + \cos\frac{9\pi}{10}\right)$ is

(*a*) $\frac{1}{8}$ (*b*) $\frac{1}{16}$ (*c*) $\frac{1}{4}$ (*d*) 0

34. The expression $\frac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{\cos 5x + 5\cos 3x + 10\cos x}$ is equal to

(*a*) $\cos 2x$ (*b*) $2\cos x$

(*c*) $\cos^2 x$ (*d*) $1 + \cos x$

35. If $\cos(A - B) = 3/5$ and $\tan A \tan B = 2$, then

(*a*) $\cos A \cos B = -\frac{1}{5}$ (*b*) $\sin A \sin B = -\frac{2}{5}$

(*c*) $\cos(A+B) = -\frac{1}{5}$ (*d*) $\sin A \cos B = \frac{4}{5}$

36. $\frac{(\sin^2 A - \sin^2 B)}{(\sin A \cos A - \sin B \cos B)} =$

(*a*) $\tan(A - B)$ (*b*) $\tan(A + B)$

(*c*) $\cot(A - B)$ (*d*) $\cot(A + B)$

37. $\tan 3A - \tan 2A - \tan A =$

(*a*) $\tan 3A \tan 2A \tan A$

(*b*) $-\tan 3A \tan 2A \tan A$

(*c*) $\tan A \tan 2A - \tan 2A \tan 3A - \tan 3A \tan A$

(*d*) 0

38. If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$, then $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 =$

(*a*) 3 (*b*) 2 (*c*) 1 (*d*) 0

39. The value of $\sin\frac{\pi}{10}\sin\frac{13\pi}{10}$

(*a*) $\frac{1}{2}$ (*b*) $\frac{-1}{2}$

(*c*) $\frac{-1}{4}$ (*d*) 1

40. If $\tan\frac{\alpha}{2}$ and $\tan\frac{\beta}{2}$ are the roots of the equation $8x^2 - 26x + 15 = 0$ then $\cos(\alpha + \beta)$ is equal to

(*a*) $-\frac{627}{725}$ (*b*) $\frac{627}{725}$

(*c*) $-\frac{725}{627}$ (*d*) −1

41. The value of the expression

$\frac{2(\sin 1° + \sin 2° + \sin 3° + \cdots + \sin 89°)}{2(\cos 1° + \cos 2° + \cdots + \cos 44°) + 1}$ equals

(*a*) $\sqrt{2}$ (*b*) $1/\sqrt{2}$ (*c*) 1/2 (*d*) 1

42. Given that $\left(1 + \sqrt{1+x}\right)\tan y = 1 + \sqrt{1-x}$. Then $\sin 4y$ is equal to

(*a*) $4x$ (*b*) $2x$

(*c*) x (*d*) None of these

TRIGONOMETRIC RATIO OF MULTIPLE ANGLES

43. If $\cos A = 3/4$, then the value of $16\cos^2(A/2) - 32\sin(A/2)\sin(5A/2)$ is

(*a*) − 4 (*b*) − 3 (*c*) 3 (*d*) 4

44. If $A = \tan 6° \tan 42°$ and $B = \cot 66° \cot 78°$, then

(*a*) $A = 2B$ (*b*) $A = \frac{1}{3}B$

(*c*) $A = B$ (*d*) $3A = 2B$

45. The value of $\cos\frac{\pi}{10}\cos\frac{2\pi}{10}\cos\frac{4\pi}{10}\cos\frac{8\pi}{10}\cos\frac{16\pi}{10}$ is:

(*a*) $\frac{\sqrt{10 + 2\sqrt{5}}}{64}$ (*b*) $\frac{\cos(\pi/10)}{8}$

(*c*) $\frac{\cos(\pi/10)}{16}$ (*d*) $-\frac{\sqrt{10 + 2\sqrt{5}}}{64}$

46. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha - \beta}{2}$ is

(*a*) $-\frac{3}{\sqrt{130}}$ (*b*) $\frac{3}{\sqrt{130}}$

(*c*) $\frac{6}{65}$ (*d*) $-\frac{6}{65}$

Credit - Akshat

47. If $\tan x+\tan\left(\frac{\pi}{3}+x\right)+\tan\left(\frac{2\pi}{3}+x\right)=3$, then

(*a*) $\tan x=1$ (*b*) $\tan 2x=1$
(*c*) $\tan 3x=1$ (*d*) $\tan x=2$

48. $\sqrt{2+\sqrt{2+2\cos 4\theta}}$ = [where $\theta\in(0,\pi/4)$]

(*a*) $\cos\theta$ (*b*) $\sin\theta$
(*c*) $2\cos\theta$ (*d*) $2\sin\theta$

49. If $x+\frac{1}{x}=2\cos\theta$, then $x^3+\frac{1}{x^3}=$

(*a*) $\cos 3\theta$ (*b*) $2\cos 3\theta$
(*c*) $\cos 3\theta$ (*d*) $\cos 3\theta$

50. If $\tan\theta=\sqrt{n}$ where $n\in N\geq 2$, then $\sec 2\theta$ is always

(*a*) a rational number (*b*) an irrational number
(*c*) a positive integer (*d*) a negative integer

51. If $\sin 2\theta=\cos 3\theta$ and θ is an acute angle, then $\sin\theta$ equals

(*a*) $\frac{\sqrt{5}-1}{4}$ (*b*) $-\left(\frac{\sqrt{5}-1}{4}\right)$
(*c*) $\frac{\sqrt{5}+1}{4}$ (*d*) $\frac{-\sqrt{5}-1}{4}$

52. In a ΔABC, $\angle B<\angle C$ and the values of B and C satisfy the equation $2\tan x-k(1+\tan^2 x)=0$, where $(0<k<1)$. Then, the measure of $\angle A$ is

(*a*) $\frac{\pi}{3}$ (*b*) $\frac{2\pi}{3}$ (*c*) $\frac{\pi}{2}$ (*d*) $\frac{3\pi}{4}$

TRIGONOMETRIC IDENTITIES IN A △

53. If $\alpha+\beta+\gamma=2\pi$, then

(*a*) $\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}+\tan\frac{\gamma}{2}=\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(*b*) $\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}+\tan\frac{\gamma}{2}=\tan\frac{\gamma}{2}+\tan\frac{\beta}{2}\tan\frac{\alpha}{2}$

(*c*) $\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}+\tan\frac{\gamma}{2}=-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(*d*) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}+\tan\frac{\beta}{2}\tan\frac{\gamma}{2}+\tan\frac{\gamma}{2}\tan\frac{\alpha}{2}=0$

54. If $A+B+C=\frac{3\pi}{2}$, then $\cos 2A+\cos 2B+\cos 2C$ is equal to

(*a*) $1-4\cos A\cos B\cos C$
(*b*) $4\sin A\sin B\sin C$
(*c*) $1+2\cos A\cos B\cos C$
(*d*) $1-4\sin A\sin B\sin C$

55. If $A+B+C=\pi$ and $\cos A=\cos B\cdot\cos C$, then $\tan B\cdot\tan C$ is equal to

(*a*) 1 (*b*) $\frac{1}{2}$ (*c*) 2 (*d*) 3

GREATEST AND LEAST VALUE

56. If $f(\theta)=\sin^4\theta+\cos^2\theta$, then range of $f(\theta)$ is

(*a*) $\left[\frac{1}{2},1\right]$ (*b*) $\left[\frac{1}{2},\frac{3}{4}\right]$
(*c*) $\left[\frac{3}{4},1\right]$ (*d*) None of these

57. Maximum value of $f(x)=\sin x+\cos x$ is

(*a*) 1 (*b*) 2 (*c*) $\frac{1}{\sqrt{2}}$ (*d*) $\sqrt{2}$

58. The number of integral value of k for which the equatio
$7\cos x+5\sin x=2k+1$ has a solution is

(*a*) 4 (*b*) 8 (*c*) 10 (*d*) 12

59. If $a\leq 3\cos x+5\sin(x-\pi/6)\leq b$ for all x, then (a,b) is

(*a*) $\left(-\sqrt{19},\sqrt{19}\right)$ (*b*) $(-17,17)$
(*c*) $\left(-\sqrt{21},\sqrt{21}\right)$ (*d*) None of these

MISCELLANEOUS

60. If $\alpha+\beta+\gamma=\pi$ then : $\tan(\beta+\gamma-\alpha)+\tan(\gamma+\alpha-\beta)$ + $\tan(\alpha+\beta-\gamma)$ is :

(*a*) $\tan(\beta-\gamma-\alpha)\cdot\tan(\gamma+\alpha-\beta)\cdot\tan(\alpha+\beta-\gamma)$
(*b*) $\cot(\beta+\gamma-\alpha)\cdot\cot(\alpha+\gamma-\beta)\cdot\cot(\alpha+\beta-\gamma)$
(*c*) $\cot(\beta+\gamma-\alpha)\cdot\cot(\alpha+\gamma-\beta)\cdot\tan(\alpha+\beta-\gamma)$
(*d*) $\tan(\beta+\gamma-\alpha)\cdot\tan(\gamma-\alpha-\beta)\cdot\tan(\alpha-\beta-\gamma)$

61. If an angle α is divided into two parts A and B such tha
$A-B=x$ and $\tan A:\tan B=k:1$, then the value of $\sin x$ i

(*a*) $\left(\frac{k+1}{k-1}\right)\sin\alpha$ (*b*) $\left(\frac{k}{k+1}\right)\sin\alpha$
(*c*) $\left(\frac{k-1}{k+1}\right)\sin\alpha$ (*d*) $\left(\frac{k+1}{k}\right)\sin\alpha$

62. Suppose x and y are real numbers such that $\tan x$ + ta
$y=42$ and $\cot x+\cot y=49$. Find the value of $\tan(x+y)$.

(*a*) 300 (*b*) 294 (*c*) 8 (*d*) 5

63. If the expression $\cos\left(x-\frac{3\pi}{2}\right)+\sin\left(\frac{3\pi}{2}+x\right)+\sin(32\pi+$
$-18\cos(19\pi-x)+\cos(56\pi+x)-9\sin(x+17\pi)$ is expresse
in the form of $a\sin x+b\cos x$ find the value of $a+b$.

(*a*) 5 (*b*) 27 (*c*) 10 (*d*) 50

64. $\cos A\sin(B-C)+\cos B\sin(C-A)+\cos C\sin(A-B)$

(*a*) 1 (*b*) 0 (*c*) 8 (*d*) 5

65. If $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}=\frac{p}{q}$ then $\tan\alpha\cdot\cot\beta$ has the value equal t

(*a*) $\frac{p+q}{p-q}$ (*b*) $\frac{p-q}{p+q}$ (*c*) $\frac{p+q}{q}$ (*d*) $\frac{p-q}{q}$

Prabal (JEE Main Level)

1. The value of $\tan 7\frac{1}{2}°$ is equal to

(a) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}+1}$ (b) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}-1}$

(c) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{2}+1}$ (d) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{2}-1}$

2. If $a \sin x + b \cos(c + x) + b \cos(c - x) = \alpha$, $a < \alpha$, then the minimum value of $|\cos c|$ is:

(a) $\sqrt{\frac{\alpha^2-a^2}{b^2}}$ (b) $\sqrt{\frac{\alpha^2-a^2}{2b^2}}$

(c) $\sqrt{\frac{\alpha^2-a^2}{3b^2}}$ (d) $\sqrt{\frac{\alpha^2-a^2}{4b^2}}$

3. If $\frac{\sin A}{\sin B}=\frac{\sqrt{3}}{2}$ and, $\frac{\cos A}{\cos B}=\frac{\sqrt{5}}{2}$ $0 < A,\ B < \frac{\pi}{2}$ then $\tan A + \tan B$ is equal to:

(a) $\sqrt{\frac{3}{5}}$ (b) $\sqrt{\frac{5}{3}}$

(c) $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{5}}$ (d) $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}}$

4. If $\alpha + \gamma = 2\beta$ then the expression $\frac{\sin\alpha - \sin\gamma}{\cos\gamma - \cos\alpha}$ simplifies to:

(a) $\tan\beta$ (b) $-\tan\beta$

(c) $\cot\beta$ (d) $-\cot\beta$

5. If $\log_3 \sin x - \log_3 \cos x - \log_3(1 - \tan x) - \log_3(1+\tan x) = -1$, then $\tan 2x$ is equal to (wherever defined)

(a) –2 (b) 3/2 (c) 2/3 (d) 6

6. $\sum_{r=1}^{n-1}\cos^2\frac{r\pi}{n}$ is equal to

(a) $\frac{n}{2}$ (b) $\frac{n-1}{2}$ (c) $\frac{n}{2}-1$ (d) $\frac{n+1}{2}$

7. The expression 2 sin 2° + 4 sin 4° + 6 sin 6° + ... + 180 sin 180° equals

(a) cot 1° (b) 90 cot 1°

(c) sin 1° (d) 90 cos 1°

8. The value of $\tan\frac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)\ldots(1+\sec 2^n\theta)$ is

(a) $\tan 2^n\theta$ (b) $\tan 2^{n-1}\theta$

(c) $\tan 2^{n+1}\theta$ (d) $\tan 2^{n-2}\theta$

9. The minimum value of $27^{\cos 3x}\cdot 81^{\sin 3x}$ is

(a) 1 (b) $\frac{1}{81}$ (c) $\frac{1}{243}$ (d) $\frac{1}{27}$

10. The value of 'a' for which the equation $\sin x(\sin x + \cos x) = a$ has a real solution are

(a) $1-\sqrt{2} \le a \le 1+\sqrt{2}$ (b) $2-\sqrt{3} \le a \le 2+\sqrt{3}$

(c) $0 \le a \le 2+\sqrt{3}$ (d) $\frac{1-\sqrt{2}}{2} \le a \le \frac{1+\sqrt{2}}{2}$

11. For $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$, $\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}$ lies in the interval

(a) $(-\infty, \infty)$ (b) $(-2, 2)$ (c) $(0, \infty)$ (d) $(-1, 1)$

12. If $f(\alpha,\beta)=\cos^2\alpha+\sin^2\alpha\cdot\cos 2\beta$, then which of the following is incorrect?

(a) $f\left(\frac{\pi}{5},\frac{2\pi}{5}\right) \ne f\left(\frac{2\pi}{5},\frac{\pi}{5}\right)$

(b) $f\left(\frac{\pi}{12},\frac{\pi}{3}\right) = f\left(\frac{\pi}{3},\frac{\pi}{12}\right)$

(c) $3f\left(\frac{\pi}{5},\frac{\pi}{3}\right) \ne f\left(\frac{\pi}{3},\frac{\pi}{5}\right)$

(d) $f\left(\frac{\pi}{4},\frac{\pi}{18}\right) \ne 3f\left(\frac{\pi}{18},\frac{\pi}{4}\right)$

13. Let $f(x) = \cos 10x + \cos 8x + 3\cos 4x + 3\cos 2x$ and $g(x) = 8\cos x\cdot\cos^3 3x$, then for all x we have

(a) $f(x) = g(x)$ (b) $2f(x) = 3g(x)$

(c) $f(x) = 2g(x)$ (d) $2f(x) = g(x)$

14. The value of cot 70°+ 4 cos 70° is

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) $2\sqrt{3}$ (d) $\frac{1}{2}$

15. If $\cos^2 A + \cos^2 B + \cos^2 C = 1$, then ΔABC is

(a) Equilateral (b) Isosceles

(c) Right angled (d) None of these

16. Let $f_k(\theta) = \sin^k(\theta) + \cos^k(\theta)$, then find the value of

$\frac{1}{6}f_6(\theta)-\frac{1}{4}f_4(\theta)$

(a) –1/12 (b) 0 (c) 1/3 (d) 1

17. If $270° < \theta < 360°$, then $\sqrt{2+\sqrt{2(1+\cos\theta)}}$ find

(a) $-2\sin\left(\frac{\theta}{4}\right)$ (b) $2\sin\left(\frac{\theta}{4}\right)$

(c) $\pm 2\sin\frac{\theta}{4}$ (d) $2\cos\frac{\theta}{4}$

Credit - Akshat

18. If $\tan\alpha$, $\tan\beta$ are the roots of the equation $x^2 + px + q = 0$, then the value of $\sin^2(\alpha+\beta) + p\sin(\alpha+\beta)\cos(\alpha+\beta) + q\cos^2(\alpha+\beta)$ is

(*a*) Independent of p but dependent on q

(*b*) Independent of q but dependent on p

(*c*) Independent of both p and q

(*d*) Dependent on both p and q

19. The value of $(\cos^4 1° + \cos^4 2° + \cos^4 3° + ... + \cos^4 179°) - (\sin^4 1° + \sin^4 2° + \sin^4 3° + ... + \sin^4 179°)$ equals to

(*a*) $2\cos 1°$ (*b*) -1 (*c*) $2\sin 1°$ (*d*) 0

20. If in a triangle ABC, $\cos 3A + \cos 3B + \cos 3C = 1$, then one angle must be exactly equal to

(*a*) $\frac{\pi}{3}$ (*b*) $\frac{2\pi}{3}$ (*c*) π (*d*) $\frac{4\pi}{3}$

21. The value of $\cos 2(\theta+\phi) + 4\cos(\theta+\phi)\sin\theta\sin\phi + 2\sin^2\phi$ is

(*a*) Independent of θ only

(*b*) Independent of ϕ only

(*c*) Independent of both θ and ϕ

(*d*) Dependent on θ and ϕ

22. Which of the following statement is incorrect

(*a*) The orthocentre of ΔABC with sides 12, 35, 37 is one of the vertex

(*b*) $\sin 314° > 0$

(*c*) $\cos 1 - \sin 1 < 0$

(*d*) $\sin 4 + \sin 6 < 0$

23. The exact value of cosec 10°+ cosec50°– cosec70° is :

(*a*) 4 (*b*) 5 (*c*) 6 (*d*) 8

24. Let $t_1 = (\sin\alpha)^{\cos\alpha}, t_2 = (\sin\alpha)^{\sin\alpha}, t_3 = (\cos\alpha)^{\cos\alpha}, t_4 = (\cos\alpha)^{\sin\alpha}$, where $\alpha \in \left(0, \frac{\pi}{4}\right)$ then which of the following is correct

(*a*) $t_3 > t_1 > t_2$ (*b*) $t_4 > t_2 > t_1$

(*c*) $t_4 > t_1 > t_2$ (*d*) $t_1 > t_3 > t_2$

25. If x_1 and x_2 are two distinct roots of the equation $a\cos x + b\sin x = c$, then $\tan\frac{x_1 + x_2}{2}$ is equal to

(*a*) $\frac{a}{b}$ (*b*) $\frac{b}{a}$ (*c*) $\frac{c}{a}$ (*d*) $\frac{a}{c}$

26. If $\tan\alpha$ is equal to the integral solution of the inequality $4x^2 - 16x + 15 < 0$ and $\cos\beta$ is equal to the slope of the bisector of the first quadrant, then $\sin(\alpha+\beta)\sin(\alpha-\beta)$ is equal to

(*a*) $\frac{3}{5}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{2}{\sqrt{5}}$ (*d*) $\frac{4}{5}$

27. $\sin\theta \cdot \sec 3\theta + \sin 3\theta \cdot \sec 3^2\theta + \sin 3^2\theta \cdot \sec 3^3\theta + ...$ upto n terms =

(*a*) $\frac{1}{2}\left[\tan 3^n\theta - \tan\theta\right]$ (*b*) $\frac{1}{2}\left[\tan 3^n\theta + \tan\theta\right]$

(*c*) $\left[\tan 3^n\theta + \tan\theta\right]$ (*d*) $\left[\tan 3^n\theta - \tan\theta\right]$

28. $\frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x} =$

(*a*) $\frac{1}{2}[\tan 27x - \tan x]$ (*b*) $\frac{1}{2}[\tan 27x + \tan x]$

(*c*) $[\tan 27x + \tan x]$ (*d*) $[\tan 27x - \tan x]$

29. If $\tan\frac{\theta}{2} = \sqrt{\frac{a-b}{a+b}}\tan\frac{\varphi}{2}$, then $\frac{a\cos\varphi + b}{a + b\cos\varphi} =$

(*a*) $\cos\theta$ (*b*) 1

(*c*) 0 (*d*) $\sin\theta$

30. If $\theta \in R$, the expression $a\sin^2\theta + b\sin\theta\cos\theta + c\cos^2\theta$ lies between

(*a*) $\frac{-1}{2}\sqrt{b^2 + (a-c)^2}$ and $\frac{1}{2}\sqrt{b^2 + (a-c)^2}$

(*b*) $\frac{a+c}{2}$ and $\frac{a-c}{2}$

(*c*) $\frac{a+c}{2} - \frac{1}{2}\sqrt{b^2 + (a-c)^2}$ and $\frac{a+c}{2} + \frac{1}{2}\sqrt{b^2 + (a-c)^2}$

(*d*) None of these

31. $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} =$

(*a*) 1 (*b*) $-1/2$ (*c*) 0 (*d*) 2

32. Find set of all possible values of α in $[-\pi, \pi]$ such th $\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha}}$ is equal to $(\sec\alpha - \tan\alpha)$

(*a*) $0 < \alpha < \frac{\pi}{3}$ (*b*) $-\pi < \alpha < \frac{\pi}{4}$

(*c*) $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ (*d*) $-\pi < \alpha < \pi$

33. Find the maximum and minimum value of $\sin^6 x + \cos^6 x$

(*a*) 1 and $\frac{1}{4}$ (*b*) 1 and $\frac{3}{4}$

(*c*) 0 and $\frac{1}{4}$ (*d*) 0 and $\frac{3}{4}$

34. $\sin\alpha + \sin 2\alpha + \sin 4\alpha + \sin 5\alpha =$

(*a*) $4\cos\frac{\alpha}{2}\cos\frac{3\alpha}{2}\sin 3\alpha$ (*b*) 0

(*c*) 1 (*d*) $2\cos\frac{\alpha}{2}\cos\frac{3\alpha}{2}\sin 3\alpha$

35. $\cos(36° - A)\cos(36° + A) + \cos(54° + A)\cos(54° - A)$

(*a*) $\cos 2A$ (*b*) 1

(*c*) 0 (*d*) $\sin 2A$

36. $\mathrm{cosec}\,\theta + \mathrm{cosec}\,2\theta + \mathrm{cosec}\,2^2\theta + ... + \mathrm{cosec}\,2^{n-1}\theta =$

(*a*) $\cot(\theta/2) - \cot 2^{n-1}\theta$ (*b*) $\cot(\theta/2) + \cot 2^{n-1}\theta$

(*c*) 0 (*d*) $\frac{\tan 2^n\theta}{\tan\theta}$

37. If $A + B + C = \pi$, then $\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2}$

(*a*) ≤ 1 (*b*) always 0
(*c*) ≤ 2 (*d*) ≥ 1

38. Let $P = \cos(A + B + C) + \cos(-A + B + C) + \cos(A - B + C) + \cos(A + B - C)$ and
$Q = \sin(A + B + C) + \sin(-A + B + C) - \sin(A - B + C) + \sin(A + B - C)$ then $\frac{P}{Q} =$

(*a*) $\cos 2B$ (*b*) $\cot B$ (*c*) 0 (*d*) $\sin B$

39. Distance between orthocentre and circumcentre in a triangle with sides 17, 15, 8.

(*a*) 9 (*b*) $\frac{17}{2}$ (*c*) 15/2 (*d*) 4

40. Value of the expression $\log_{1/2}(\sin 6° \cdot \sin 42° \cdot \sin 45° \cdot \sin 66° \cdot \sin 78°)$

(*a*) Lies between 3 and 5
(*b*) is an integer
(*c*) Is irrational which is a simple surd
(*d*) Is irrational which is a mixed surd

INTEGER TYPE QUESTIONS

41. If $\sin x + \sin^2 x = 1$, then the value of $\cos^{12}x + 3\cos^{10}x + 3\cos^8 x + \cos^6 x - 1$ is equal to

42. If $\tan\alpha = \frac{\sqrt{3}+1}{\sqrt{3}-1}$, then the expression

$\cos 2\alpha + (2 + \sqrt{3})\sin 2\alpha$ is

43. The maximum value of $1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2\cos\left(\frac{\pi}{4} - \theta\right)$ for real values of θ is

44. If $\cos(\alpha + \beta) + \sin(\alpha - \beta) = 0$ and $\tan\beta = \frac{1}{2025}$; then $\tan 3\alpha$ is:

45. The value of $\tan 9° - \tan 27° - \tan 63° + \tan 81°$ is

46. If $\frac{\sin\theta}{\cos(3\theta)} + \frac{\sin(3\theta)}{\cos(9\theta)} + \frac{\sin(9\theta)}{\cos(27\theta)} + \frac{\sin(27\theta)}{\cos(81\theta)} = \frac{\sin(k\theta)}{2\cos\theta\cos 81\theta}$, then k equals to

47. $\cot 16° \cdot \cot 44° + \cot 44° \cdot \cot 76° - \cot 76° \cdot \cot 16° =$

48. The number of all possible 5-tuples $(a_1, a_2, a_3, a_4, a_5)$ such that $a_1 + a_2\sin x + a_3\cos x + a_4\sin 2x + a_5\cos 2x = 0$ holds for all x is

49. If m and n are positive integers satisfying $1 + \cos 2\theta + \cos 4\theta + \cos 6\theta + \cos 8\theta + \cos 10\theta = \frac{\cos m\theta \cdot \sin n\theta}{\sin\theta}$ then $m + n$ is equal to

50. $\log[\sec(2022\pi) + \sec(2023\pi) + \cos(12345678\pi)] =$

51. If cosine of odd integral multiple of $\frac{\pi}{2}$ is a, tangent of integral multiple of π is b, sine of integral multiple of π is c, cosine of odd integral multiple of π is d and even integral multiple of e and let $\sin\left(2n\pi + \frac{\pi}{2}\right) = f, \sin\left(2n\pi - \frac{\pi}{2}\right) = g, (n \in I)$ and then $a + b + c + d + e + f + g =$

Parikshit (JEE Advanced Level)

SINGLE CORRECT TYPE QUESTIONS

1. If $\tan\frac{\pi}{9}, x$ and $\tan\frac{5\pi}{18}$ are in A.P. and $\tan\frac{\pi}{9}, y$ and $\tan\frac{7\pi}{18}$ are also in A.P., then

(*a*) $2x = y$ (*b*) $x > 2$
(*c*) $x = y$ (*d*) None of these

2. It is known that $\sin\beta = \frac{4}{5}$ and $0 < \beta < \pi$ then the value of

$$\frac{\sqrt{3}\sin(\alpha+\beta) - \frac{2}{\cos(\pi/6)}\cos(\alpha+\beta)}{\sin\alpha}$$ is

(*a*) Independent of α for all β in $(0, \pi)$
(*b*) $\frac{5}{13}$ for $\tan\beta < 0$
(*c*) $\frac{3(7+24\cot\alpha)}{15}$ for $\tan\beta > 0$
(*d*) None of these

3. The sum $\sum_{x=2}^{44}(2\sin x.\sin 1)\,(1 + \sec(x - 1)\sec(x + 1))$ can be written in the form as $\sum_{n=1}^{4}(-1)^n\frac{\phi^2(\theta_n)}{\psi(\theta_n)}$ where ϕ and ψ are trigonometric functions and $\theta_1, \theta_2, \theta_3, \theta_4$ are in degree $\in [0, 45]$, then the value of $|\theta_1 + \theta_3 - \theta_2 - \theta_4|$ is (where | | represents modules function)

(*a*) 82° (*b*) 90° (*c*) 86° (*d*) 45°

4. The value of $P = (\sqrt{3}+\tan 1^0)(\sqrt{3}+\tan 2^0)(\sqrt{3}+\tan 3^0)$ ......$(\sqrt{3}+\tan 28^0)(\sqrt{3}+\tan 29^0)$ is

(*a*) 2^{28} (*b*) $2^{15}+2-\sqrt{3}$
(*c*) 2^{29} (*d*) $2^{14}+2-\sqrt{3}$

5. If $\tan\theta_i$, where $i = 1, 2, 3, 4$ are the roots of equation $x^4 - x^3 \sin 2\beta + x^2 \cos 2\beta = x\cos\beta + \sin\beta$ then $\tan\left(\sum_{i=1}^{4}\theta_i\right)$

(*a*) $-\cot\beta$ (*b*) $\cot\left(\frac{\beta}{2}\right)$ (*c*) $\tan\beta$ (*d*) $\cot\beta$

6. Let $\dfrac{\tan\left(\frac{\pi}{4}+\alpha\right)}{5} = \dfrac{\tan\left(\frac{\pi}{4}+\beta\right)}{3} = \dfrac{\tan\left(\frac{\pi}{4}+\gamma\right)}{2}$. Then $12\sin^2(\alpha-\beta) + 15\sin^2(\beta-\gamma) - 7\sin^2(\gamma-\alpha)$ is equal to

(*a*) $-\frac{1}{2}$ (*b*) $\frac{1}{2}$ (*c*) 1 (*d*) 0

7. If $a = \sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}$ and x is the solution the of equation $y = 2[x] + 2$ and $y = 3[x-2]$, where $[x]$ denotes the integral part of x, then a is equal to

(*a*) $[x]$ (*b*) $\frac{1}{[x]}$ (*c*) $2[x]$ (*d*) $[x]^2$

8. If $\tan\frac{\pi}{10}\sin\frac{\pi}{5}\cdot\cos\frac{3\pi}{10}\tan\frac{2\pi}{5}\tan\frac{3\pi}{5}\cos\frac{7\pi}{10}\sin\frac{4\pi}{4}$

$\tan\frac{9\pi}{10}\cos\pi$

$$= \frac{k\sin^2 18^0}{\cos 5^0}\left[\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+2\cos 1360^0}}}}\right]$$

Then k is

(*a*) Greater than 1 (*b*) Less than 0
(*c*) Irrational number (*d*) Rational number

9. $3\tan^6\frac{\pi}{18} - 27\tan^4\frac{\pi}{18} + 33\tan^2\frac{\pi}{18}$ equals

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

MULTIPLE CORRECT TYPE QUESTIONS

10. Let $f:(-1,1)\to R$ be such that $f(\cos 4\theta) = \dfrac{2}{2-\sec^2\theta}$. $\theta\in\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{4},\frac{\pi}{2}\right)$. Then the values of $f\left(\frac{1}{3}\right)$ is /are

(*a*) $1-\sqrt{\frac{3}{2}}$ (*b*) $1+\sqrt{\frac{3}{2}}$ (*c*) $1-\sqrt{\frac{2}{3}}$ (*d*) $1+\sqrt{\frac{2}{3}}$

11. Which of the following is/are +ve?

(*a*) $\log_{\sin 1}\tan 1$ (*b*) $\log_{\cos 1}(1+\tan 3)$
(*c*) $\log_{\log_{10}5}(\cos\theta+\sec\theta)$ (*d*) $\log_{\tan 15^\circ}(2\sin 18^\circ)$

12. Let $P(k) = \left(1+\cos\frac{\pi}{4k}\right)\left(1+\cos\frac{(2k-1)\pi}{4k}\right)\left(1+\cos\frac{(2k+1)\pi}{4k}\right)\left(1+\cos\frac{(4k-1)\pi}{4k}\right)$. Then

(*a*) $P(3) = \frac{1}{16}$ (*b*) $P(4) = \frac{2-\sqrt{2}}{16}$
(*c*) $P(5) = \frac{3-\sqrt{5}}{32}$ (*d*) $P(6) = \frac{2-\sqrt{3}}{16}$

13. If $\tan x = \frac{2b}{a-c}$, $(a \neq c)$ $y = a\cos^2 x + 2b\sin x\cos x + c\sin^2 x$ and $z = a\sin^2 x - 2b\sin x\cos x + c\cos^2 x$, then

(*a*) $y = z$ (*b*) $y + z = a + c$
(*c*) $y - z = a - c$ (*d*) $y - z = (a-c)^2 + 4b^2$

14. The value of $\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n + \left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n$ is

(*a*) $2\tan^n\frac{A-B}{2}$ (*b*) $2\cot^n\frac{A-B}{2}$; n is even
(*c*) 0 ; n is odd (*d*) None of these

15. In a triangle $\tan A + \tan B + \tan C = 6$ and $\tan A\tan B = 2$, then the values of $\tan A$, $\tan B$ and $\tan C$ are

(*a*) 1, 2, 3 (*b*) 2, 1, 3
(*c*) 1, 2, 0 (*d*) None of these

16. If $3\sin\beta = \sin(2\alpha+\beta)$, then :

(*a*) $(\cot\alpha + \cot(\alpha+\beta))(\cot\beta - 3\cot(2\alpha+\beta)) = 6$
(*b*) $\sin\beta = \cos(\alpha+\beta)\sin\alpha$
(*c*) $\tan(\alpha+\beta) = 2\tan\alpha$
(*d*) $2\sin\beta = \sin(\alpha+\beta)\cos\alpha$

17. For $0 < \theta < \pi/2$, $\tan\theta + \tan 2\theta + \tan 3\theta = 0$ if

(*a*) $\tan\theta = 0$ (*b*) $\tan 2\theta = 0$
(*c*) $\tan 3\theta = 0$ (*d*) $\tan\theta\tan 2\theta = 2$

18. If $\tan A + \tan B + \tan C = \tan A.\tan B.\tan C$, then

(*a*) A, B, C may be angles of a triangle
(*b*) $A + B + C$ is an integral multiple of π
(*c*) Sum of any two of A, B, C is equal to third
(*d*) None of these

19. If the sides of a right angled triangle are $\{\cos 2\alpha + \cos 2\beta + 2\cos(\alpha+\beta)\}$ and $\{\sin 2\alpha + \sin 2\beta + 2\sin(\alpha+\beta)\}$, then the length of the hypotenuse is:

(*a*) $2[1+\cos(\alpha-\beta)]$ (*b*) $2[1-\cos(\alpha+\beta)]$
(*c*) $4\cos^2\frac{\alpha-\beta}{2}$ (*d*) $4\sin^2\frac{\alpha+\beta}{2}$

20. If $x + y = z$, then $\cos^2 x + \cos^2 y + \cos^2 z - 2\cos x\cos y\cos z$ is equal to

(*a*) $\cos^2 z$ (*b*) $\sin^2 z$
(*c*) $\cos(x+y-z)$ (*d*) 1

21. The equation $\sin^6 x + \cos^6 x = a^2$ has real solution if

(a) $a \in (-1, 1)$ (b) $a \in \left[-1, \frac{-1}{2}\right]$

(c) $a \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $a \in \left[\frac{1}{2}, 1\right]$

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 22 to 24): If $\cos\alpha + \cos\beta = a$ and $\sin\alpha + \sin\beta = b$ and $2\theta = \alpha + \beta$, then $\sin 2\theta + \cos 2\theta = 1 + \frac{nb(a-b)}{a^2+b^2}$ where n is some integer. Also if a, b, c are 3 non-zero terms in G.P then $b^2 = ac$. So answer the following questions:

22. The value of n is

(a) 0 (b) 1 (c) 2 (d) −2

23. If for n obtained in above question, $\sin^n A = x$, then $\sin A \sin 2A \sin 3A \sin 4A$ is a polynomial in x, of degree

(a) 5 (b) 6 (c) 7 (d) 8

24. If degree of polynomial obtained in previous questions is p and $(p-5) + \sin x$, $\cos x$, $\tan x$ are in G.P., then value of $(\cos^9 x + \cos^6 x + 3\cos^5 x - 1)$ is

(a) −1 (b) 0

(c) 1 (d) None of these

Comprehension (Q. 25 to 27): Let p be the product of the sines of the angles of a triangle ABC and q is the product of the cosines of the angles.

25. In this triangle $\tan A + \tan B + \tan C$ is equal to

(a) $p+q$ (b) $p-q$

(c) p/q (d) None of these

26. $\tan A \tan B + \tan B \tan C + \tan C \tan A$ is equal to

(a) $1+q$ (b) $\frac{1+q}{q}$ (c) $1+p$ (d) $\frac{1+p}{p}$

27. The value of $\tan^3 A + \tan^3 B + \tan^3 C$ is

(a) $\frac{p^3 - 3pq^2}{q^3}$ (b) $\frac{q^3}{p^3}$

(c) $\frac{p^3}{q^3}$ (d) $\frac{p^3 - 3pq}{q^3}$

MATCH THE COLUMN TYPE QUESTIONS

28. Match the column:

	Column-I		Column-II
A.	The value of $\frac{\sin 22^\circ}{\sin 56^\circ \sin 34^\circ \cot 68^\circ}$ equals to	p.	5
B.	The value of $\left(\cos 65^\circ + \sqrt{3}\sin 5^\circ + \sin 85^\circ\right)^2 = \mu \cos^2 25^\circ$, then value of μ is	q.	2
C.	If $f(\theta) = 2\sin 2\theta - \sin^2\theta + 3\cos^2\theta$, then number of integers in the range of $f(\theta)$ equals to	r.	3
D.	If $f(\theta) = \cos(\sin\theta) + \sin(\sin\theta)$, then number of integers in the Range of $f(\theta)$ equals to	s.	1

(a) A → r; B → s; C → p; D → q

(b) A → q; B → r; C → p; D → q

(c) A → q; B → r; C → q; D → p

(d) A → q; B → r; C → s; D → p

29. If α and β are the roots of the equation, $a\cos\theta + b\sin\theta = c$ then match the entries of Column-I with the entries of Column-II.

	Column-I		Column-II
A.	$\sin\alpha + \sin\beta$	p.	$\frac{2b}{a+c}$
B.	$\sin\alpha \cdot \sin\beta$	q.	$\frac{c-a}{c+a}$
C.	$\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}$	r.	$\frac{2bc}{a^2+b^2}$
D.	$\tan\frac{\alpha}{2} \cdot \tan\frac{\beta}{2} =$	s.	$\frac{c^2-a^2}{a^2+b^2}$

(a) A → r; B → s; C → p; D → q

(b) A → q; B → r; C → p; D → q

(c) A → q; B → r; C → q; D → p

(d) A → q; B → r; C → s; D → p

30. Match the columns:

	Column-I		Column-II
A.	$\cot 35^\circ + \cot 145^\circ + 3$ is equal to	p.	1
B.	$\frac{1}{\sec^2 120^\circ} + \frac{1}{\operatorname{cosec}^2 120^\circ}$ is equal to	q.	3
C.	If $(\sin 2)^x + (\cos 2)^x = 1$ then x is	r.	0
D.	$\tan\frac{\pi}{11} + \tan\frac{2\pi}{11} + \tan\frac{4\pi}{11} + \tan\frac{7\pi}{11}$ $\tan\frac{9\pi}{11} + \tan\frac{10\pi}{11}$ is equal to	s.	2
E.	$\cos 10^\circ + \cos 20^\circ + \cos 30^\circ + \ldots\ldots + \cos 80^\circ + \cos 100^\circ + \cos 150^\circ + \cos 160^\circ + \cos 170^\circ + 4$ is equal to	t.	4

(a) A → q; B → p; C → s; D → r; E → t

(b) A → r; B → s; C → p; D → q; E → q

(c) A → r; B → s; C → p; D → t; E → q

(d) A → r; B → s; C → p; D → q; E → t

31. Match the column:

Column-I		Column-II	
A.	$\cos^2 52\frac{1^\circ}{2} - \sin^2 22\frac{1^\circ}{2}$	p.	1
B.	$\cos^2\frac{3\pi}{5} + \cos^2\frac{4\pi}{5}$	q.	$\frac{3-\sqrt{3}}{4\sqrt{2}}$
C.	$\sin 24^\circ + \cos 6^\circ$	r.	$\frac{3}{4}$
D.	$\sin^2 50^\circ + \cos^2 130^\circ$	s.	$\frac{\sqrt{15}+\sqrt{3}}{4}$

(*a*) A → q; B → r; C → s; D → p

(*b*) A → r; B → s; C → q; D → p

(*c*) A → s; B → p; C → r; D → q

(*d*) A → q; B → r; C → p; D → s

NTEGER TYPE QUESTIONS

32. If $\operatorname{cosec}\theta - \sin\theta = a^3$ and $\sec\theta - \cos\theta = b^3$, then $a^2 b^2 (a^2 + b^2) =$

33. If the expression $\tan(55^\circ)\tan(65^\circ)\tan(75^\circ)$ simplifies to $\cot(x^\circ)$ and m is the numerical value of the expression $\tan(27^\circ) + \tan(18^\circ) + \tan(27^\circ)\tan(18^\circ)$, then find the value of $(m + x + 1)$.

34. Given that for $a, b, c, d \in R$: if $a \sec(190^\circ) - c \tan(190^\circ) = d$ and $b \sec(190^\circ) + d \tan(190^\circ) = c$, then find the value of $\left(\frac{a^2+b^2+c^2+d^2}{bd-ac}\right)\sin 20^\circ$.

35. If $P = \sec^6\theta - \tan^6\theta - 3\sec^2\theta\tan^2\theta$, $Q = \operatorname{cosec}^6\theta - \cot^6\theta - 3\operatorname{cosec}^2\theta\cot^2\theta$ and $R = \sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$, then find the value of $(P+Q+R)^{(P+Q+R)}$

36. If the angles α & β satisfy the relation $\frac{\sin\beta}{\sin(2\alpha+\beta)} = \frac{n}{m}(|m|>|n|) \Rightarrow \frac{a+\frac{\tan\beta}{\tan\alpha}}{m+n} = \frac{a-\tan\alpha\tan\beta}{m-n}$, then $a =$

37. Let $k = 1^\circ$, and $\sum_{n=0}^{88}\sec(nk)\sec(n+1)k = \frac{\alpha\cos k}{\sin^2 k}$, where $\alpha \in Z$, then $\alpha =$

38. If $\alpha + \beta + \gamma = \pi$ and $\tan\left(\frac{\beta+\gamma-\alpha}{4}\right)\cdot\tan\left(\frac{\gamma+\alpha-\beta}{4}\right)\cdot\tan\left(\frac{\alpha+\beta-\gamma}{4}\right) = 1 \Rightarrow a + \cos\alpha + \cos\beta + \cos\gamma = 0$, then $a =$

39. If $A + B + C = 2S$ and
$\sin(S-A) + \sin(S-B) + \sin(S-C) - \sin S$
$= \alpha\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$, (where $\alpha \in Z$) then $\alpha =$

40. If $A + B + C = 2S$, and
$\cos^2 A + \cos^2 B + \cos^2 C + 2\cos A\cos B\cos C$
$= 1 + \alpha\cos S\cos(S-A)\cos(S-B)\cos(S-C)$, then $\alpha =$

41. If $\cos(-A+B+C) + \cos(A-B+C) + \cos(A+B-C) +$
$\cos(A+B+C) = \lambda\cos A\cos B\cos C$. Then find λ.

42. If $x + y + z = xyz$, then
$\frac{3x-x^3}{1-3x^2} + \frac{3y-y^3}{1-3y^2} + \frac{3z-z^3}{1-3z^2} = \frac{\lambda x-x^3}{1-3x^2}\cdot\frac{\lambda y-y^3}{1-3y^2}\cdot\frac{\lambda z-z^3}{1-3z^2}$.
Find λ.

43. For ΔABC
$\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} - 1 = \lambda\sin\frac{\pi-A}{4}\sin\frac{\pi-B}{4}\sin\frac{\pi-C}{4}$.
Then λ =

44. If $\frac{\cos 5A}{\cos A} + \frac{\sin 5A}{\sin A} = a + b\cos 4A$, then find the value c
$(a+b)$.

45. If $4\sin 54^\circ + \cot\left(7\frac{1^\circ}{2}\right)$
$= \sqrt{a_1} + \sqrt{a_2} + \sqrt{a_3} + \sqrt{a_4} + \sqrt{a_5} + \sqrt{a}$
then, find the value of $\sum_{i=1}^{6} a_i^3$

46. The absolute value of the expression
$\tan\frac{\pi}{16} + \tan\frac{5\pi}{16} + \tan\frac{9\pi}{16} + \tan\frac{13\pi}{16}$ is

47. The value of $\left(1+\tan\frac{3\pi}{8}\tan\frac{\pi}{8}\right) + \left(1+\tan\frac{5\pi}{8}\tan\frac{3\pi}{8}\right) + \left(1+\tan\frac{7\pi}{8}\tan\frac{5\pi}{8}\right) + \left(1+\tan\frac{9\pi}{8}\tan\frac{7\pi}{8}\right)$

48. If $\tan\left(\frac{\pi}{4}+\frac{y}{2}\right) = \tan^3\left(\frac{\pi}{4}+\frac{x}{2}\right) \Rightarrow \frac{\sin y}{\sin x} = \frac{k+\sin^2 x}{1+k\sin^2 x}$. Th
find k.

49. If $A + B + C = \pi$ and $\sum\left(\frac{\tan A}{\tan B.\tan C}\right) = \sum(\tan$
$- k\sum(\cot A)$. Then find k.

50. Let $A_1, A_2, A_3, ..., A_n$ are the vertices of a regular n sid
polygon inscribed in a circle of radius R. If $(A_1A_2)^2 + (A_1A$
$+ ... + (A_1A_n)^2 = 14R^2$, find the number of sides in the polyg

51. Let $A_1, A_2, ..., A_n$ be the vertices of an n-sided regular polyg
such that; $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$. Find the value of n.

52. If $\frac{x}{\tan(\theta+\alpha)} = \frac{y}{\tan(\theta+\beta)} = \frac{z}{\tan(\theta+\gamma)} \Rightarrow \sum\frac{x+y}{x-y}$ si
$(\alpha-\beta) = k$, then $k =$

Credit - Akshat

PYQ's (Past Year Questions)

BASED ON TRIGONOMETRIC FORMULAE

1. If $\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$, then the value of $\left(a + \frac{1}{a}\right)$ is: **[30 Jan, 2023 (Shift-I)]**

(a) 4 (b) $4 - 2\sqrt{3}$

(c) 2 (d) $5 - \frac{3}{2}\sqrt{3}$

2. $96 \cos\frac{\pi}{33} \cos\frac{2\pi}{33} \cos\frac{4\pi}{33} \cos\frac{8\pi}{33} \cos\frac{16\pi}{33}$ is equal to **[10 April, 2023 (Shift-I)]**

(a) 3 (b) 2

(c) 4 (d) 1

3. $16\sin(20^\circ)\sin(40^\circ)\sin(80^\circ)$ is equal to: **[26 June, 2022 (Shift-II)]**

(a) $\sqrt{3}$ (b) $2\sqrt{3}$

(c) 3 (d) $4\sqrt{3}$

4. The value of $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$ is equal to: **[27 June, 2022 (Shift-I)]**

(a) –1 (b) $-\frac{1}{2}$ (c) $-\frac{1}{3}$ (d) $-\frac{1}{4}$

5. $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to **[25 July, 2022 (Shift-II)]**

(a) $\frac{3}{16}$ (b) $\frac{1}{16}$ (c) $\frac{1}{32}$ (d) $\frac{9}{32}$

6. If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha + \beta)$ and the quadrant in which $\alpha + \beta$ lies, respectively are **[28 June, 2022 (Shift-II)]**

(a) $-\frac{1}{7}$ and IVth quadrant (b) 7 and Ist quadrant

(c) –7 and IVth quadrant (d) $\frac{1}{7}$ and Ist quadrant

7. The value of $2\sin(12^\circ) - \sin(72^\circ)$ is: **[25 June, 2022 (Shift-II)]**

(a) $\frac{\sqrt{5}\left(1-\sqrt{3}\right)}{4}$ (b) $\frac{1-\sqrt{5}}{8}$

(c) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{2}$ (d) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{4}$

8. If $\sin^2(10^\circ)\sin(20^\circ)\sin(40^\circ)\sin(50^\circ)\sin(70^\circ) = \alpha - \frac{1}{16}\sin(10^\circ)$ then $16 + \alpha^{-1}$ is equal to..... **[26 June, 2022 (Shift-I)]**

9. Let α and β be real numbers such that $-\frac{\pi}{4} < \beta < 0 < \alpha < \frac{\pi}{4}$. If $\sin(\alpha+\beta) = \frac{1}{3}$ and $\cos(\alpha - \beta) = \frac{2}{3}$ then the greatest integer less than or equal to $\left(\frac{\sin\alpha}{\cos\beta} + \frac{\cos\beta}{\sin\alpha} + \frac{\cos\alpha}{\sin\beta} + \frac{\sin\beta}{\cos\alpha}\right)^2$ is ____. **[JEE Adv, 2022]**

10. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x+y) = 3/2$, Then, $\sin x + \cos y$ is equal to: **[25 Feb, 2021 (Shift-II)]**

(a) $\frac{1+\sqrt{3}}{2}$ (b) $\frac{1-\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

11. The value of $\cot\left(\frac{\pi}{24}\right)$ is: **[25 July, 2021 (Shift-II)]**

(a) $\sqrt{2} + \sqrt{3} + 2 - \sqrt{6}$ (b) $\sqrt{2} + \sqrt{3} + 2 + \sqrt{6}$

(c) $\sqrt{2} - \sqrt{3} - 2 + \sqrt{6}$ (d) $3\sqrt{2} - \sqrt{3} - \sqrt{6}$

12. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in: **[25 Feb, 2021 (Shift-I)]**

(a) $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

(b) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

(c) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(d) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

13. The value of $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right).\sin\left(\frac{7\pi}{8}\right)$ is: **[26 Aug, 2021(Shift-II)]**

(a) $\frac{1}{8\sqrt{2}}$ (b) $\frac{1}{4\sqrt{2}}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

14. If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and $M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then **[5 Sep, 2020 (Shift-II)]**

(a) $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$ (b) $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

(c) $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\frac{\pi}{8}$ (d) $L = \frac{1}{4\sqrt{2}} - \frac{1}{4}\cos\frac{\pi}{8}$

15. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$ $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, then tan (α + 2 β) is equal to **[8 Jan, 2020 (Shift-II)]**

16. The value of $\cos^3\left(\frac{\pi}{8}\right)\cdot\cos\left(\frac{3\pi}{8}\right)+\sin^3\left(\frac{\pi}{8}\right)\cdot\sin\left(\frac{3\pi}{8}\right)$ is: **[9 Jan, 2020 (Shift-I)]**

(*a*) $\frac{1}{2\sqrt{2}}$ (*b*) $\frac{1}{\sqrt{2}}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{1}{2}$

17. If $\cos(\alpha+\beta)=\frac{3}{5}, \sin(\alpha-\beta)=\frac{5}{13}$ and $0<\alpha, \beta<\frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to: **[8 April, 2019 (Shift-I)]**

(*a*) $\frac{21}{16}$ (*b*) $\frac{63}{52}$ (*c*) $\frac{33}{52}$ (*d*) $\frac{63}{16}$

18. The value of sin 10° sin 30° sin 50° sin 70° is **[9 April, 2019 (Shift-II)]**

(*a*) $\frac{1}{36}$ (*b*) $\frac{1}{32}$ (*c*) $\frac{1}{18}$ (*d*) $\frac{1}{16}$

19. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is **[9 April, 2019 (Shift-I)]**

(*a*) $\frac{3}{2}(1+\cos 20°)$ (*b*) $\frac{3}{4}$

(*c*) $\frac{3}{4}+\cos 20°$ (*d*) $\frac{3}{2}$

20. The equation $y = \sin x \sin(x + 2) - \sin^2(x + 1)$ represents a straight line lying in: **[12 April, 2019 (Shift-I)]**

(*a*) Second and third quadrants only
(*b*) Third and fourth quadrants only
(*c*) First, third and fourth quadrants
(*d*) First, second and fourth quadrants

21. For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$, the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals: **[9 Jan, 2019 (Shift-I)]**

(*a*) $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$
(*b*) $13 - 4\cos^6\theta$
(*c*) $13 - 4\cos^2\theta + 6\cos^4\theta$
(*d*) $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

22. The value of $\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\cdot\ldots\cdot\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$ is **[10 Jan, 2019 (Shift-I**

(*a*) $\frac{1}{512}$ (*b*) $\frac{1}{1024}$ (*c*) $\frac{1}{256}$ (*d*) $\frac{1}{2}$

23. For non-negative integers n, let

$$f(n)=\frac{\sum_{k=0}^{n}\sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n}\sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1} x$ takes values in [0, π], which of following options is/are correct? **[JEE Adv, 201**

(*a*) $\sin(7\cos^{-1} f(5)) = 0$
(*b*) $f(4) = \frac{\sqrt{3}}{2}$
(*c*) $\lim_{x\to\infty} f(n) = \frac{1}{2}$
(*d*) If $\alpha = \tan(\cos^{-1} f(6))$, then $\alpha^2 + 2\alpha - 1 = 0$

MAXIMUM AND MINIMUM VALUES OF TRIGONOMETRIC FUNCTION

24. The set of all values of λ for which the equation $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$ **[29 Jan, 2023 (Shift-I**

(*a*) [–2, –1] (*b*) $\left[-2, -\frac{3}{2}\right]$

(*c*) $\left[-1, -\frac{1}{2}\right]$ (*d*) $\left[-\frac{3}{2}, -1\right]$

25. The number of elements in the set

$$S=\left\{x\in\mathbb{R}: 2\cos\left(\frac{x^2+x}{6}\right)=4^x+4^{-x}\right\}$$

[29 July, 2022 (Shift-I

(*a*) 1 (*b*) 3 (*c*) 0 (*d*) Infinite

26. The maximum value of $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any r value of θ is: **[12 Jan, 2019 (Shift-**

(*a*) $\sqrt{19}$ (*b*) $\frac{\sqrt{79}}{2}$ (*c*) $\sqrt{34}$ (*d*) $\sqrt{31}$

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PW Challengers

1. $\sqrt[3]{\cos\frac{2\pi}{7}}+\sqrt[3]{\cos\frac{4\pi}{7}}+\sqrt[3]{\cos\frac{8\pi}{7}}=\sqrt[3]{\frac{1}{2}(a-3\sqrt[3]{b})}$, then find $a+b$.

2. For every positive integer n,

$$\tan\frac{\pi}{2n+1}.\tan\frac{2\pi}{2n+1}\ldots\tan\frac{n\pi}{2n+1}=\sqrt{\lambda n+\mu}$$. Find $\lambda+\mu$.

3. $\cos\theta=\frac{a}{b+c},\cos\phi=\frac{b}{a+c},\cos\psi=\frac{c}{a+c}$. where θ, ϕ, $\psi \in (0, \pi)$ and a, b, c are sides of triangle ABC then $\tan^2\frac{\theta}{2}+\tan^2\frac{\phi}{2}+\tan^2\frac{\psi}{2}$ is equal to ________.

4. If $a=\frac{2\pi}{2023}$ and $\cos a\cos 2a\cos 3a\ldots\cos 1011a=\frac{1}{k}$, then $\frac{\log_2 k}{1011}$ is equal to ______

5. For $0\le\theta<2\pi$, let

$$P=\frac{1}{2}\cos\theta-\frac{1}{4}\sin 2\theta-\frac{1}{8}\cos 3\theta+\frac{1}{16}\sin 4\theta+\frac{1}{32}\cos 5\theta-\frac{1}{64}\sin 6\theta-\frac{1}{128}\cos 7\theta+\ldots$$ and

$$Q=1-\frac{1}{2}\sin\theta-\frac{1}{4}\cos 2\theta+\frac{1}{8}\sin 3\theta+\frac{1}{16}\cos 4\theta-\frac{1}{32}\sin 5\theta-\frac{1}{64}\cos 6\theta+\frac{1}{128}\sin 7\theta+\ldots$$

so that $\frac{P}{Q}=\frac{2\sqrt{2}}{7}$. Then $\sin\theta=-\frac{m}{n}$, where m and n are relatively prime positive integer, then $\left(\frac{m+n}{9}\right)$ is equal to

6. The value the expression

$$\left(\tan^2\frac{\pi}{7}+\tan^2\frac{2\pi}{7}+\tan^2\frac{3\pi}{7}\right)\left(\cot^2\frac{\pi}{7}+\cot^2\frac{2\pi}{7}+\cot^2\frac{3\pi}{7}\right)$$

is equal to.

7. The value of $\left[\tan\frac{3\pi}{11}+4\sin\frac{2\pi}{11}\right]$, (where $[\cdot]$ represents G.I.F) is equal to.

8. Sum of integral value(s) in the range of $f(x)=\cos x\left\{\sin x+\sqrt{\sin^2 x+\sin^2\alpha}\right\}$ $\{x\in R\}$ is equal to

9. If $x\sin a+y\sin 2a+z\sin 3a=\sin 4a$

$x\sin b+y\sin 2b+z\sin 3b=\sin 4b$

$x\sin c+y\sin 2c+z\sin 3c=\sin 4c$

then roots of equation $t^3-\left(\frac{z}{2}\right)t^2-\left(\frac{y+2}{4}\right)t+\left(\frac{z-x}{8}\right)=0$, $a,b,c\ne n\pi$, are

(*a*) $\sin a$, $\sin b$, $\sin c$ (*b*) $\cos a$, $\cos b$, $\cos c$

(*c*) $\sin 2a$, $\sin 2b$, $\sin 2c$ (*d*) $\cos 2a$, $\cos 2b$, $\cos 2c$

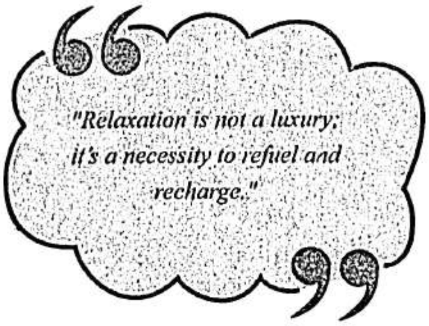
10. Let x, y, z be real numbers with $x\ge y\ge z\ge\frac{\pi}{12}$ such that $x+y+z=\frac{\pi}{2}$

(*a*) Maximum $(\cos x\cdot\sin y\cdot\cos z)=\frac{2+\sqrt{3}}{4}$

(*b*) Minimum $(\cos x\cdot\sin y\cdot\cos z)=\frac{1}{8}$

(*c*) Minimum value occurs when $x=y=\frac{5\pi}{24}$ and $z=\frac{\pi}{12}$

(*d*) Maximum $(\cos x\cdot\sin y\cdot\cos z)=\frac{2+\sqrt{3}}{8}$



"Relaxation is not a luxury; it's a necessity to refuel and recharge."

ANSWER KEY

CONCEPT APPLICATION

1. [20π/3] 2. [90,60,30] 3. [1.7] 4. [1] 5. [8/15] 6. [−1] 7. *(c)* 8. [2]

9. A → x; B → w; C → v; D → u; E → t; F → s; G → r; H → q; I → p 12. [17] 13. *(b)*

15. $\left[\sin\alpha=\frac{\sqrt{2+\sqrt{2}}}{2},\cos\alpha=-\frac{\sqrt{2-\sqrt{2}}}{2}\right]$ 16. [−1/2] 17. [−2/3] 18. *(d)* 21. $\left(-\infty,-\frac{1}{3}\right]\cup\left[\frac{1}{7},\infty\right)$

22. $(4-\sqrt{10},4+\sqrt{10})$ 23. *(b)* 24. *(d)* 25. *(b)* 26. [1/2] 27. 109.28 m 28. 1 m

29. 20 m 30. $20\sqrt{3}$ m 31. $h=\frac{20}{\sqrt{3}}$ m. 32. $AP=7\sqrt{3}$ m 33. $\sin\theta=\frac{3}{5}$

34. 3 : 1 and 1 : 3

SCHOOL LEVEL PROBLEMS

1. *(b)*	2. *(c)*	3. *(c)*	4. *(d)*	5. *(c)*	6. *(c)*	7. *(c)*	8. *(b)*	9. *(a)*	22. (c)
23. *(c)*	24. *(d)*	25. *(c)*							

PRARAMBH (TOPICWISE)

1. *(a)*	2. *(a)*	3. *(d)*	4. *(b)*	5. *(a)*	6. *(c)*	7. *(d)*	8. *(b)*	9. *(c)*	10. *(b)*
11. *(a)*	12. *(a)*	13. *(d)*	14. *(a)*	15. *(b)*	16. *(c)*	17. *(d)*	18. *(d)*	19. *(b)*	20. *(d)*
21. *(a)*	22. *(b)*	23. *(a)*	24. *(b)*	25. *(a)*	26. *(d)*	27. *(a)*	28. *(b)*	29. *(c)*	30. *(a)*
31. *(d)*	32. *(d)*	33. *(b)*	34. *(b)*	35. *(c)*	36. *(b)*	37. *(a)*	38. *(d)*	39. *(c)*	40. *(a)*
41. *(a)*	42. *(c)*	43. *(c)*	44. *(c)*	45. *(d)*	46. *(a)*	47. *(c)*	48. *(c)*	49. *(b)*	50. *(a)*
51. *(a)*	52. *(c)*	53. *(a)*	54. *(d)*	55. *(c)*	56. *(c)*	57. *(d)*	58. *(b)*	59. *(a)*	60. *(c)*
61. *(c)*	62. *(b)*	63. *(b)*	64. *(b)*	65. *(a)*					

PRABAL (JEE MAIN LEVEL)

1. *(a)*	2. *(d)*	3. *(c)*	4. *(c)*	5. *(c)*	6. *(c)*	7. *(b)*	8. *(a)*	9. *(c)*	10. *(d)*
11. *(a)*	12. *(a)*	13. *(a)*	14. *(b)*	15. *(c)*	16. *(a)*	17. *(b)*	18. *(a)*	19. *(b)*	20. *(b)*
21. *(b)*	22. *(b)*	23. *(c)*	24. *(b)*	25. *(b)*	26. *(d)*	27. *(a)*	28. *(a)*	29. *(a)*	30. *(c)*
31. *(b)*	32. *(c)*	33. *(a)*	34. *(a)*	35. *(a)*	36. *(a)*	37. *(d)*	38. *(b)*	39. *(b)*	40. *(a)*
41. [0]	42. [1]	43. [4]	44. [1]	45. [4]	46. [80]	47. [3]	48. [1]	49. [11]	50. [0]
51. [0]									

PARIKSHIT (JEE ADVANCED LEVEL)

1. *(a)*	2. *(d)*	3. *(c)*	4. *(c)*	5. *(d)*	6. *(d)*	7. *(b)*	8. *(d)*	9. *(b)*	10. *(a,b)*
11. *(b,d)*	12. *(a,b,c,d)*	13. *(b,c)*	14. *(b,c)*	15. *(a,b)*	16. *(a,b,c,d)*	17. *(c,d)*	18. *(a,b)*	19. *(a,c)*	20. *(c,d)*
21. *(b,d)*	22. *(c)*	23. *(a)*	24. *(b)*	25. *(c)*	26. *(b)*	27. *(d)*	28. *(b)*	29. *(a)*	30. *(a)*
31. *(a)*	32. [1]	33. [7]	34. [2]	35. [27]	36. [1]	37. [1]	38. [1]	39. [4]	40. [4]
41. [4]	42. [3]	43. [4]	44. [6]	45. [441]	46. [4]	47. [0]	48. [3]	49. [2]	50. [7]
51. [7]	52. [0]								

PYQ's (PAST YEAR QUESTIONS)

1. *(a)*	2. *(a)*	3. *(b)*	4. *(b)*	5. *(b)*	6. *(a)*	7. *(d)*	8. [80]	9. [1]	10. *(a)*
11. *(b)*	12. *(b)*	13. *(d)*	14. *(b)*	15. [1]	16. *(a)*	17. *(d)*	18. *(d)*	19. *(b)*	20. *(b)*
21. *(b)*	22. *(a)*	23. *(a,b,d)*	24. *(d)*	25. *(a)*	26. *(a)*				

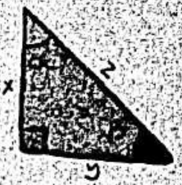
PW CHALLENGERS

1. [12]	2. [3]	3. [1]	4. [1]	5. [4]	6. [105]	7. [3]	8. [0]	9. *(b)*	10. *(b,d)*

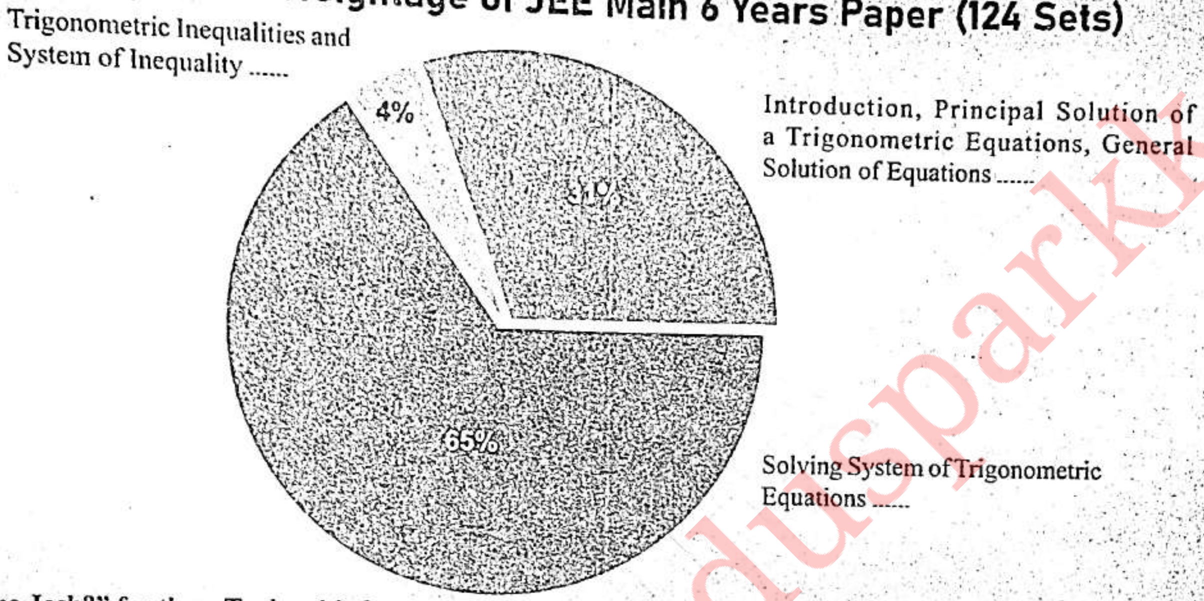
CHAPTER

4

Trigonometric Equation



Topicwise Weightage of JEE Main 6 Years Paper (124 Sets)



"How's the Josh?" for these Topics: *Mark your confidence level in the blank space around the topic (Low-L, Medium-M, High-H)*

INTRODUCTION

A solution of trigonometric equation is the value of the unknown angle that satisfies the equation.

e.g. if $\sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \ldots\ldots$

SOLUTION OF TRIGONOMETRIC EQUATIONS

The trigonometric equation may have infinite number of solutions (because of their periodic nature) and can be classified as:

1. Principal solution
2. General solution.

PRINCIPAL SOLUTIONS

The solutions of trigonometric equation which lie in the interval $[0, 2\pi)$ are called **principal solutions.**

Example: The Principal solutions of the equation $\sin x = \frac{1}{2}$ is $\frac{\pi}{6}$ and $\frac{5\pi}{6}$.

GENERAL SOLUTION

Since trigonometric functions are periodic function, therefore solution of trigonometric equations can be generalised with the help of periodicity of a trigonometrical function. The solution consisting of all possible solution of a trigonometrical equation is called the general solution.

The expression involving an integer 'n' which gives all solutions of a trigonometric equation is called **General solution.**

Some Important General Solutions of Trigonometric Equations

1. $\sin\theta = 0 \Rightarrow \theta = n\pi, n \in I$
2. $\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}, n \in I$
3. $\tan\theta = 0 \Rightarrow \theta = n\pi, n \in I$
4. $\sin\theta = 1 \Rightarrow \theta = (4n+1)\frac{\pi}{2}, n \in I$
5. $\cos\theta = 1 \Rightarrow \theta = 2n\pi, n \in I$
6. $\tan\theta = 1 \Rightarrow \theta = (4n+1)\frac{\pi}{4}, n \in I$
7. $\sin\theta = -1 \Rightarrow \theta = (4n-1)\frac{\pi}{2}, n \in I$

Credit - Akshat

8. $\cos\theta = -1 \Rightarrow \theta = (2n-1)\pi, n \in I$

9. $\tan\theta = -1 \Rightarrow \theta = (4n-1)\frac{\pi}{4}, n \in I$

General Solution of Some Standard Trigonometric Equations

(*a*) If $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$, where, $n \in I$.

(*b*) If $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$, where, $n \in I$.

(*c*) If $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$, where, $n \in I$.

(*d*) If $\sin^2\theta = \sin^2\alpha \Rightarrow \theta = n\pi \pm \alpha$, where, $n \in I$.

(*e*) If $\cos^2\theta = \cos^2\alpha \Rightarrow \theta = n\pi \pm \alpha$, where, $n \in I$.

(*f*) If $\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha$, where, $n \in I$.

[**Note:** α is called the principal angle]

Proof:

(*i*) $\sin\theta = \sin\alpha \Rightarrow \sin\theta - \sin\alpha = 0$

$$\Rightarrow 2\cos\left(\frac{\theta+\alpha}{2}\right)\sin\left(\frac{\theta-\alpha}{2}\right) = 0$$

$$\Rightarrow \cos\left(\frac{\theta+\alpha}{2}\right) = 0 \text{ or } \sin\left(\frac{\theta-\alpha}{2}\right) = 0$$

$$\Rightarrow \frac{\theta+\alpha}{2} = (2m+1)\frac{\pi}{2}$$

or $\frac{\theta-\alpha}{2} = m\pi$, where $m \in I$.

$\Rightarrow \theta = (2m+1)\pi - \alpha$

or $\theta = 2m\pi + \alpha$, where $m \in I$.

$\Rightarrow \theta = (2m+1)\pi + (-1)^{2m+1}\alpha$

or $\theta = 2m\pi + (-1)^{2m}\alpha$

$\Rightarrow \theta = n\pi + (-1)^n\alpha, n \in I$.

(*ii*) $\cos\theta = \cos\alpha$

$\Rightarrow \cos\alpha - \cos\theta = 0$

$$\Rightarrow 2\sin\left(\frac{\alpha+\theta}{2}\right)\sin\left(\frac{\theta-\alpha}{2}\right) = 0$$

$$\Rightarrow \sin\frac{\theta+\alpha}{2} = 0$$

or $\sin\frac{\theta-\alpha}{2} = 0, \Rightarrow \frac{\theta+\alpha}{2} = n\pi$ or $\frac{\theta-\alpha}{2} = n\pi$

$\Rightarrow \theta = 2n\pi - \alpha$, or $\theta = 2n\pi + \alpha\ n \in I$

$\Rightarrow \theta = 2n\pi \pm \alpha$.

(*iii*) $\tan\theta = \tan\alpha \Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{\sin\alpha}{\cos\alpha}$

$\Rightarrow \sin\theta\cos\alpha - \cos\theta.\sin\alpha = 0$

$\Rightarrow \sin(\theta - \alpha) = 0$

$\Rightarrow \theta - \alpha = n\pi$

$\Rightarrow \theta = n\pi + \alpha$, where $n \in I$

(*iv*) $\sin^2\theta = \sin^2\alpha$

$\sin^2\theta - \sin^2\alpha = \sin(\theta+\alpha)\sin(\theta-\alpha) = 0$

$\sin(\theta+\alpha) = 0$ or $\sin(\theta-\alpha) = 0$

$\theta + \alpha = n\pi$ or $\theta - \alpha = n\pi, n \in I$

$\theta = n\pi \pm \alpha, n \in I$

(*v*) $\cos^2\theta = \cos^2\alpha$

$\Rightarrow 1 - \sin^2\theta = 1 - \sin^2\alpha$

$\Rightarrow \sin^2\theta = \sin^2\alpha$

$\theta = n\pi \pm \alpha, n \in I$

(*vi*) $\tan^2\theta = \tan^2\alpha$

$\Rightarrow \tan\theta = \pm\tan\alpha = \tan(\pm\alpha)$

$\Rightarrow \theta = n\pi \pm \alpha$, where $n \in I$

Train Your Brain

Example 1: Solve $4\sec^2\theta = 5 + \tan^2\theta$

Sol. $4\sec^2\theta = 5 + \tan^2\theta$ ...(*i*)

For equation (*i*) to be defined $\theta \neq (2n+1)\frac{\pi}{2}, n \in I$

Equation (*i*) can be written as:

$4(1 + \tan^2\theta) = 5 + \tan^2\theta$

$\tan^2\theta = \tan^2\frac{\pi}{6}$

$\theta = n\pi \pm \frac{\pi}{6}, n \in I$

Example 2: Solve $\sin 2\theta = \sin\theta$

Sol. We have, $\sin 2\theta = \sin\theta$

$\Rightarrow 2\sin\theta\cos\theta = \sin\theta$

$\Rightarrow \sin\theta(2\cos\theta - 1) = 0$

$\Rightarrow \sin\theta = 0$ and $\cos\theta = \frac{1}{2}$

$\Rightarrow \theta = n\pi$ and $\theta = 2n\pi \pm \frac{\pi}{3}$, where $n \in I$.

Example 3: The general solution of the equation $\sin^2 x + \cos^2 3x = 1$ is equal to: (where $n \in I$)

(*a*) $x = \frac{n\pi}{2}$

(*b*) $x = n\pi + \frac{\pi}{4}$

(*c*) $x = \frac{n\pi}{4}$

(*d*) $x = n\pi + \frac{\pi}{2}$

Sol. (*c*) $\sin^2 x = \sin^2 3x \Rightarrow 3x = n\pi \pm x$

$x = \frac{n\pi}{4}, x = \frac{n\pi}{2}$, hence general solution is $\frac{n\pi}{4}$

Example 4: Solve : $\sqrt{3}\sec 2\theta = 2$.

Sol. $\cos 2\theta = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6} \Rightarrow 2\theta = 2n\pi \pm \frac{\pi}{6}$

$\theta = n\pi \pm \frac{\pi}{12}, n \in I$

Concept Application

1. Find the set of values of x for which $\frac{\tan 3x - \tan 2x}{1 + \tan 3x . \tan 2x} = 1$
2. The number of value of x lying between 0 and 2π satisfying the equation: $\sin x + \sin 3x = 0$ is/are:
   (*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

SOLUTIONS OF DIFFERENT TYPES OF TRIGONOMETRIC EQUATION

Type-I: Solutions of Equations by Factorising

Solution of trigonometric equation by factorization or equation which are expressed in quadratic form or which can be expressed in quadratic form.

Train Your Brain

Example 5: Solve $(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$ in $[0, 2\pi]$.

Sol. $(1 + \cos x)(2 \sin x - \cos x) - (1 - \cos^2 x) = 0$

$(1 + \cos x)[(2 \sin x - \cos x) - (1 - \cos x)] = 0$

$(1 + \cos x)(2 \sin x - 1) = 0$

$\sin x = \frac{1}{2}, \cos x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \pi.$

Example 6: Find the general solutions of equation $1 - \cos x \cot x = \cot x - \cos x$.

Sol. $(1 + \cos x)(1 - \cot x) = 0$

$\Rightarrow \cos x = -1$ or $\cot x = 1$

$\Rightarrow (2n + 1)\pi, n\pi + \pi/4, n \in I$

Example 7: Find the solutions of the equation, $\log_{\sqrt{2}\sin x}(1 + \cos x) = 2$ in the interval $x \in [0, 2\pi]$.

Sol. $2 \sin^2 x = 1 + \cos x$; $2 \cos^2 x + \cos x - 1 = 0$

$\Rightarrow \cos x = \frac{1}{2}$ or -1

$\Rightarrow x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ but $x = \pi$ and $\frac{5\pi}{3}$ are rejected $\Rightarrow x = \frac{\pi}{3}$

Example 8: If $\frac{1}{6}\sin\theta$, $\cos\theta$ and $\tan\theta$ are in GP, then find the general solution for θ

Sol. Since, $\frac{1}{6}\sin\theta, \cos\theta, \tan\theta$ are in GP.

$\therefore \cos^2\theta = \frac{1}{6}\sin\theta . \tan\theta$

$\Rightarrow 6\cos^3\theta + \cos^2\theta - 1 = 0$

Note that $\cos\theta = \frac{1}{2}$ satisfies the equation (by trial).

$\therefore (2\cos\theta - 1)(3\cos^2\theta + 2\cos\theta + 1) = 0$

$\Rightarrow \cos\theta = \frac{1}{2}$ (other values of $\cos\theta$ are imaginary)

$\Rightarrow \cos\theta = \cos\frac{\pi}{3} \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}, n \in Z$

Concept Application

3. The number of values of x in the interval $[0, 5\pi]$ satisfying the equation $3\sin^2 x - 7\sin x + 2 = 0$ is:
   (*a*) 0 (*b*) 2
   (*c*) 6 (*d*) 8
4. Solve $\cos^3 x + \cos^2 x - 4\cos^2\frac{x}{2} = 0$
5. Solve for x $\frac{\sin^3\frac{x}{2} - \cos^3\frac{x}{2}}{2 + \sin x} = \frac{\cos x}{3}$
6. Solve for x : $\sin^3 x - \cos^3 x = 1 + \sin x \cos x$.

Type-II: Solutions of Equations Reducible to Quadratic Equations

Rules:

1. Transform the terms to be in only one trigonometric ratio involving angles in same form.
2. Factorize the equation and express it in $f(x) \times g(x) = 0$
   $\Rightarrow f(x) = 0$ or $g(x) = 0$
3. Solve both the equations one by one to get the general value of the variables.

Train Your Brain

Example 9: Number of values of 'x' in $(-2\pi, 2\pi)$ satisfying the equation $2^{\sin^2 x} + 4.2^{\cos^2 x} = 6$ is:
(*a*) 8 (*b*) 6 (*c*) 4 (*d*) 2

Sol. $2^{\sin^2 x} + 4.2^{\cos^2 x} = 6$

Let, $2^{\sin^2 x} = t$; $\therefore t + \frac{8}{t} = 6 \Rightarrow (t - 4)(t - 2) = 0$

$2^{\sin^2 x} = 4$ and $2^{\sin^2 x} = 2$

$\Rightarrow \sin x = \pm\sqrt{2}$ (reject) and $\sin x = \pm 1$

$\therefore$ in $(-2\pi, 2\pi)$ there are 4 solutions.

Example 10: Solve the equation $\sin^4 x + \cos^4 x = \frac{7}{2}\sin x \cos x$.

Sol. We have, $\sin^4 x + \cos^4 x = \frac{7}{2}\sin x . \cos x$.

$$\Rightarrow (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x = \frac{7}{2}\sin x . \cos x$$

$$\Rightarrow 1 - \frac{1}{2}(\sin 2x)^2 = \frac{7}{4}(\sin 2x) \Rightarrow 2\sin^2 2x + 7\sin 2x - 4 = 0$$

$\Rightarrow (2\sin 2x - 1)(\sin 2x + 4) = 0 \Rightarrow \sin 2x = \frac{1}{2}$ or $\sin 2x = -4$ (which is not possible)

$$\Rightarrow 2x = n\pi + (-1)^n \frac{\pi}{6}, n \in I \text{ ; i.e., } x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12}, n \in I.$$

Concept Application

7. The complete solution of the equation $7\cos^2 x + \sin x \cos x - 3 = 0$ is given by-

   (a) $n\pi + \frac{\pi}{2}; (n \in I)$ (b) $n\pi - \frac{\pi}{4}; (n \in I)$

   (c) $n\pi + \tan^{-1}\frac{4}{3}; (n \in I)$

   (d) $n\pi + \frac{3\pi}{4}, k\pi + \tan^{-1}\frac{4}{3}; (n, k \in I)$

8. Find the general solution set of the equation $\log_{\tan x}(2 + 4\cos^2 x) = 2$.

Type-III: Solution to simultaneous equation

When there is a system of two trigonometric equations satisfied simultaneously, the system is called simultaneous equation.

Illustration 1: Find the most general values of θ which satisfies the equations $\sin\theta = -1/2$ and $\tan\theta = 1/\sqrt{3}$.

Solution: As discussed above, first solve both the equations separately.

$\sin\theta = -1/2 \Rightarrow \theta = 7\pi/6$ or $11\pi/6$

and $\tan\theta = 1/\sqrt{3} \Rightarrow \theta = \pi/6$ or $7\pi/6$ [values between 0 and 2π].

Common value of $\theta = 7\pi/6$

Hence, the required solution is $\theta = 2n\pi + 7\pi/6$.

Type-IV: Solving equations by introducing an auxiliary argument

A Trigonometric Equation is of the form

$a\cos\theta \pm b\sin\theta = c$

Rule:

1. Divide by $\sqrt{a^2 + b^2}$ on both the sides
2. Reduce the given equation into either $\sin(\theta \pm \alpha)$ or $\cos(\theta \pm \alpha)$
3. Simplify the given equation.

Train Your Brain

Example 11: Solve $\sin x + \cos x = \sqrt{2}$

Sol. ∵ $\sin x + \cos x = \sqrt{2}$ ...(i)

∴ Divide both sides of equation (i) by $\sqrt{2}$.

We get $\sin x . \frac{1}{\sqrt{2}} + \cos x . \frac{1}{\sqrt{2}} = 1$

$$\Rightarrow \sin x . \sin\frac{\pi}{4} + \cos x . \cos\frac{\pi}{4} = 1$$

$$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = 1$$

$$\Rightarrow x - \frac{\pi}{4} = 2n\pi, n \in I$$

$$\Rightarrow x = 2n\pi + \frac{\pi}{4}, n \in I$$

∴ Solution of given equation is $2n\pi + \frac{\pi}{4}, n \in I$

Example 12: Solve the equation

$\cos 7x - \sin 5x = \sqrt{3}(\cos 5x - \sin 7x)$.

Sol. Rewrite the equation in the form

$$\frac{1}{2}\cos 7x + \frac{\sqrt{3}}{2}\sin 7x = \frac{\sqrt{3}}{2}\cos 5x + \frac{1}{2}\sin 5x$$

$$\text{or } \sin\frac{\pi}{6}\cos 7x + \cos\frac{\pi}{6}\sin 7x = \sin\frac{\pi}{3}\cos 5x + \cos\frac{\pi}{3}\sin 5x,$$

$$\text{i.e. } \sin\left(\frac{\pi}{6} + 7x\right) = \sin\left(\frac{\pi}{3} + 5x\right).$$

But $\sin\alpha = \sin\beta$ if and only if either $\alpha - \beta = 2k\pi$ or $\alpha + \beta = (2m + 1)\pi$ ($k, m = 0, \pm 1, \pm 2, ...$).

$$\text{Hence } \frac{\pi}{6} + 7x - \frac{\pi}{3} - 5x = 2k\pi$$

$$\text{or } \frac{\pi}{6} + 7x + \frac{\pi}{3} + 5x = (2m + 1)\pi.$$

Thus, the roots for the equation are

$$\left.\begin{aligned} x &= \frac{\pi}{12}(12k + 1), \\ x &= \frac{\pi}{24}(4m + 1) \end{aligned}\right\} (k, m = 0, \pm 1, \pm 2, ...).$$

Concept Application

9. Prove that the equation $k \cos x - 3 \sin x = k + 1$ possesses a solution if $k \in (-\infty, 4]$

10. The number of solution of the equation $\sqrt{3}\sin x + \cos x = 1$ in the interval $0 \le x \le 2\pi$
    (*a*) 3 (*b*) 2
    (*c*) 4 (*d*) None of these

11. Solve $\sqrt{3}\cos\theta - 3\sin\theta = 4\sin 2\theta \cos 3\theta$

Type-V:

Solving equations by transforming a sum of trigonometric functions into a product

Train Your Brain

Example 13: General solution of $\sin x + \sin 5x = \sin 2x + \sin 4x$ is

(*a*) $\frac{n\pi}{3}$ (*b*) $\frac{2n\pi}{3}$
(*c*) $2n\pi$ (*d*) $n\pi$

Sol. (*a*) $2 \sin 3x \cos 2x = 2\sin 3x \cos x$

$\Rightarrow \sin 3x = 0$ or $\cos 2x = \cos x$

$\Rightarrow 3x = n\pi$ or $2x = 2n\pi \pm x$

$\therefore x = \frac{n\pi}{3}, 2n\pi, \frac{2n\pi}{3} \Rightarrow x = \frac{n\pi}{3}$

Example 14: Solve $\cos 3x + \sin 2x - \sin 4x = 0$

Sol. $\cos 3x + \sin 2x - \sin 4x = 0$

$\Rightarrow \cos 3x + 2 \cos 3x \,.\, \sin(-x) = 0$

$\Rightarrow \cos 3x - 2 \cos 3x \,.\, \sin x = 0$

$\Rightarrow \cos 3x\,(1 - 2 \sin x) = 0$

$\Rightarrow \cos 3x = 0$ or $1 - 2 \sin x = 0$

$\Rightarrow 3x = (2n + 1)\frac{\pi}{2}, n \in I$ or $\sin x = \frac{1}{2}$

$\Rightarrow x = (2n + 1)\frac{\pi}{6}, n \in I$ or $x = n\pi + (-1)^n \frac{\pi}{6}, n \in I$

$\therefore$ Solution of given equation is $(2n + 1)\frac{\pi}{6}, n \in I$

or $n\pi + (-1)^n \frac{\pi}{6}, n \in I$

Example 15: Find the number of solutions of the equations, $\sin x + 2 \sin 2x = 3 + \sin 3x$ in $[0, \pi]$

(*a*) No solution
(*b*) Infinite solution
(*c*) Exactly one solution
(*d*) Two solutions

Sol. (*a*) We have,

$\Rightarrow \sin 3x - \sin x - 2 \sin 2x + 3 = 0$

$\Rightarrow 2 \cos 2x \cdot \sin x - 4 \sin x \cdot \cos x + 3 = 0$

$\Rightarrow \sin x\,(2 \cos 2x - 4 \cos x) + 3 = 0$

$\Rightarrow \sin x\,(4 \cos^2 x - 4 \cos x - 2) + 3 = 0$

$\Rightarrow \sin x\,(4 \cos^2 x - 4 \cos x + 1) + 3 - 3 \sin x = 0$

$\Rightarrow \sin x\,(2 \cos x - 1)^2 + 3\,(1 - \sin x) = 0$ ...(*i*)

since $x \in [0, \pi]$,

$\therefore \sin x \ge 0$ and $1 - \sin x \ge 0$

$\therefore$ each part in equation (*i*) must be zero.

i.e. $\sin x\,(2 \cos x - 1)^2 = 0$ and $3\,(1 - \sin x) = 0$

from the second equation of system we have, $\sin x = 1$

$\Rightarrow \cos x = 0$ hence $\sin x\,(2 \cos^2 x - 1)^2 \ne 0$

$\therefore$ not a single solution of the second equation is a solution of the first.

Hence the original equation has no real solution.

Concept Application

12. The number of solution of $\cos x + \cos 2x + \cos 4x = 0$, where $0 \le x \le \pi$.
    (*a*) 2 (*b*) 3
    (*c*) 4 (*d*) None of these

13. Solve $\cos\theta + \cos 3\theta - 2 \cos 2\theta = 0$

Type-VI:

Solving equations by transforming a product of trigonometric functions into a sum

Train Your Brain

Example 16: Solve $\cos x \cos 2x \cos 3x = 1/4$.

Sol. $\cos x \cos 2x \cos 3x = 1/4$

$\Rightarrow 2\,(2\cos x \cos 3x) \cos 2x = 1$

$\Rightarrow 2\,(\cos 4x + \cos 2x) \cos 2x = 1$

$\Rightarrow 2\,(2\cos^2 2x - 1 + \cos 2x) \cos 2x = 1$

$\Rightarrow 4\cos^3 2x + 2\cos^2 2x - 2\cos 2x - 1 = 0$

$\Rightarrow (2\cos^2 2x - 1)(2\cos 2x + 1) = 0$

$\Rightarrow \cos 4x\,(2 \cos 2x + 1) = 0$

$\Rightarrow \cos 4x = 0$ or $\cos 2x = -1/2$

$\Rightarrow 4x = (2n+1)\frac{\pi}{2}$ or $2x = 2m\pi \pm \frac{2\pi}{3} \cdot m, n \in Z$

$\Rightarrow x = (2n+1)\frac{\pi}{8}$ or $x = m\pi \pm \frac{\pi}{3}$

Example 17: Number of solutions of the trigonometric equation in $[0, \pi]$, $\sin 3\theta = 4\sin\theta\cdot\sin 2\theta\cdot\sin 4\theta$.

(*a*) 4 (*b*) 6

(*c*) 8 (*d*) 10

Sol. (*c*) $\sin 3\theta = 4\sin\theta\sin(3\theta - \theta)\sin(3\theta + \theta)$

$= 4\sin\theta(\sin^2 3\theta - \sin^2\theta)$

$\Rightarrow \sin 3\theta + 4\sin^3\theta = 4\sin\theta\sin^2 3\theta$

$\Rightarrow 3\sin\theta = 4\sin\theta\,\sin^2 3\theta$

$\Rightarrow \sin\theta = 0$ or $\sin^2 3\theta = \frac{3}{4}$

$\Rightarrow \theta = n\pi$ or $3\theta = n\pi \pm \frac{\pi}{3}$

$\Rightarrow \theta = n\pi$ or $\theta = \frac{n\pi}{3} \pm \frac{\pi}{9}, n \in I$

$\Rightarrow \theta = 0, \pi, \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}$

Concept Application

14. Find the number of solutions for,

$\sin 5\theta.\cos 3\theta = \sin 9\theta.\cos 7\theta$ in $\left[0, \frac{\pi}{2}\right]$

15. Solve $\sin 3\alpha = 4\sin\alpha\sin(x + \alpha)\sin(x - \alpha)$.

Type-VII:

Solving equations by a change of variable

(*i*) Equations of the form

$P(\sin x \pm \cos x, \sin x.\cos x) = 0,$

where $P(y, z)$ is a polynomial, can be solved by the change

$\cos x \pm \sin x = t$

$\Rightarrow 1 \pm 2\sin x\cdot\cos x = t^2$

(*ii*) Equations of the form of $a.\sin x + b.\cos x + d = 0$, where a, b & d are real numbers & $a, b \neq 0$ can be solved by changing $\sin x$ & $\cos x$ into their corresponding tangent of half the angle.

(*iii*) Many equations can be solved by introducing a new variable. eg. the equation

$\sin^4 2x + \cos^4 2x = \sin 2x\cdot\cos 2x$ changes to

$2(y + 1)\left(y - \frac{1}{2}\right) = 0$ by substituting,

$\sin 2x\cdot\cos 2x = y$.

Train Your Brain

Example 18: Solve $3\cos x + 4\sin x = 5$

Sol. $\because 3\cos x + 4\sin x = 5$ ...(*I*)

$\because \cos x = \dfrac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$ and $\sin x = \dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$\therefore$ equation (*I*) becomes

$$\Rightarrow 3\left(\frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + 4\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) = 5 \quad ...(II)$$

Let $\tan\frac{x}{2} = t$ $\therefore$ equation (*II*) becomes

$3\left(\frac{1 - t^2}{1 + t^2}\right) + 4\left(\frac{2t}{1 + t^2}\right) = 5 \Rightarrow 4t^2 - 4t + 1 = 0$

$\Rightarrow (2t - 1)^2 = 0 \Rightarrow t = \frac{1}{2}\left(\because t = \tan\frac{x}{2}\right)$

$\Rightarrow \tan\frac{x}{2} = \frac{1}{2} \Rightarrow \tan\frac{x}{2} = \tan\alpha$, where $\tan\alpha = \frac{1}{2}$

$\Rightarrow \frac{x}{2} = n\pi + \alpha \Rightarrow x = 2n\pi + 2\alpha$

Where, $\alpha = \tan^{-1}\left(\frac{1}{2}\right), n \in I$

Example 19: Solve the equation

$\sin 2x - 12(\sin x - \cos x) + 12 = 0$

Sol. Putting $\sin x - \cos x = t$ and using the identity $(\sin x - \cos x)^2 = 1 - 2\sin x\cos x$, we rewrite the original equation in the form $t^2 + 12t - 13 = 0$.

This equation has the roots $t_1 = -13$ and $t_2 = 1$. But $t = \sin x - \cos x = \sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$, and thus, $|t| \leq \sqrt{2}$. Consequently, the root $t_1 = -13$ must be discarded. Therefore, the original equation is reduced to the equation $\sin\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$.

$\Rightarrow x_1 = \pi + 2k\pi, x_2 = \frac{\pi}{2} + 2k\pi.$

Concept Application

16. Solve the equation

$\sin^{10} x + \cos^{10} x = \frac{29}{16}\cos^4 2x$

17. Solve the equation $1 + 2\operatorname{cosec} x = -\dfrac{\sec^2\frac{x}{2}}{2}$.

Type-VIII:

Solving equations with the use of the Boundness of the functions $\sin x$ & $\cos x$

Remember:

$-1 \le \sin x \le 1$, $-1 \le \cos x \le 1$, $\tan x \in R$, $\cot x \in R$.

$|\operatorname{cosec} x| \ge 1$, $|\sec x| \ge 1$.

Train Your Brain

Example 20: Solve for x: $\cos x + \cos 2x + \cos 3x = 3$.

Sol. $\cos x = 1$ and $\cos 2x = 1$ and $\cos 3x = 1$

when $\cos x = 1$

$\Rightarrow x = 2n\pi,\ n \in I$

when $\cos 2x = 1$

$\Rightarrow x = \dfrac{2n\pi}{2} = n\pi,\ n \in I$

when $\cos 3x = 1$

$\Rightarrow 3x = 2n\pi \Rightarrow x = \dfrac{2n\pi}{3},\ n \in I$

$2n\pi,\ n \in I$

Example 21: Solve for x:

$$\sin x\left(\cos\frac{x}{4} - 2\sin x\right) + \left(1 + \sin\frac{x}{4} - 2\cos x\right)\cos x = 0$$

Sol. $\sin\left(\dfrac{5x}{4}\right) + \cos x = 2$

$\Rightarrow \sin\left(\dfrac{5x}{4}\right) = 1 \Rightarrow \dfrac{5x}{4} = (4n+1)\dfrac{\pi}{2}$

$\Rightarrow x = (4n+1)\dfrac{2\pi}{5}$ and $\cos x = 1$

$x = 2m\pi$; $x = 0, \pm 2\pi, \pm 4\pi$

Period of the given equation is 8π.

$\therefore$ Consider $x \in [0, 8\pi)$

$\Rightarrow x = \dfrac{2\pi}{5},\ 2\pi,\ \dfrac{18\pi}{5},\ \dfrac{26\pi}{5},\ \dfrac{34\pi}{5}$

Common solution $= 2\pi$

$\therefore 8n\pi + 2\pi = 2\pi(4n+1)$

Example 22: If $x, y \in [0, 2\pi]$ then find the total number of order pairs (x, y) satisfying the equation $\sin x \cos y = 1$.

Sol. We have $\sin x \cos y = 1$

$\Rightarrow \sin x = 1, \cos y = 1$ or $\sin x = -1, \cos y = -1$

If $\sin x = 1$, $\cos y = 1 \Rightarrow x = \pi/2, y = 0, 2\pi$

If $\sin x = -1$, $\cos y = -1 \Rightarrow x = 3\pi/2, y = \pi$

Then the possible ordered pairs are

$\left(\dfrac{\pi}{2}, 0\right), \left(\dfrac{\pi}{2}, 2\pi\right), \left(\dfrac{3\pi}{2}, \pi\right)$.

Concept Application

18. If $x \in \left[-\dfrac{5\pi}{2}, \dfrac{5\pi}{2}\right]$, then the greatest positive solution of $1 + \sin^4 x = \cos^2 3x$ is
    (*a*) π (*b*) 2π
    (*c*) $\dfrac{5\pi}{2}$ (*d*) None of these
19. Solve $2\cos^2\dfrac{x}{2}\cdot\sin^2 x = x^2 + \dfrac{1}{x^2}$ $0 < x \le \dfrac{\pi}{2}$.
20. Solve the equation $\dfrac{1 - \tan x}{1 + \tan x} = 1 + \sin 2x$.

Type-IX:

Solution of trigonometric equation of the form $f(x) = \sqrt{\phi(x)}$.

(*i*) $f(x) \ge 0$, $\phi(x) \ge 0$ (*ii*) $f^2(x) = \phi(x)$

MISCELLANEOUS QUESTIONS

A trigonometric equation is of the form

$b_0\sin^n x + b_1\sin^{n-1} x.\cos x +$
$b_2\sin^{n-2} x.\cos^2 x + \ldots\ldots.. + b_n\cos^n x = 0$

where, $b_0, b_1, b_2, \ldots.. b_n \in R$ is a homogenous equation of $\sin x$ and $\cos x$, where $\cos x$ is non zero.

Rule: 1. Divide both the sides by highest power of $\cos x$.

2. The given equation can be reduced to
$b_0\tan^n x + b_1\tan^{n-1} x + \ldots.. + b_n = 0$

3. And then use the factorization method.

Train Your Brain

Example 23: Solve for x, $\sqrt{(1 - \cos x)} = \sin x$.

Sol. $\sin x \ge 0$ and $1 - \cos x \ge 0$ ...(*i*)

$\therefore$ $1 - \cos x = \sin^2 x = 1 - \cos^2 x$

$\Rightarrow$ $(1 - \cos x)[1 - (1 + \cos x)] = 0 \Rightarrow \cos x = 0, 1$

when $\cos x = 0$, $x = 2n\pi \pm \dfrac{\pi}{2}$ but $\sin x \ge 0$

$\Rightarrow$ $x = 2n\pi + \dfrac{\pi}{2}$

when $\cos x = 1$, $x = 2n\pi$

Both are satisfying in equality (*i*)

Example 24: Solve : $2\sin^2 x - 5\sin x\cos x - 8\cos^2 x = -2$

Sol. The given equation can be written as

$2\sin^2 x - 5\sin x\cos x - 8\cos^2 x = -2(\sin^2 x + \cos^2 x)$

$\Rightarrow 2\tan^2 x - 5\tan x - 8 = -2(\tan^2 x + 1)$

$\Rightarrow 4\tan^2 x - 5\tan x - 6 = 0 \Rightarrow (\tan x - 2)(4\tan x + 3) = 0$

$\Rightarrow \tan x = 2, \tan x = -\dfrac{3}{4} \Rightarrow x = n\pi + \alpha, x = m\pi + \beta$,

where $\alpha = \tan^{-1}(2)$, $\beta = \tan^{-1}\left(-\dfrac{3}{4}\right)$, $(n, m \in Z)$.

Concept Application

21. Values of x between 0 and 2π which satisfy the equation $\sin x\sqrt{8\cos^2 x} = 1$ are in A.P. whose common difference is:

(*a*) $\frac{\pi}{4}$ (*b*) $\frac{\pi}{3}$

(*c*) $\frac{\pi}{2}$ (*d*) $\frac{2\pi}{3}$

22. Solve $3\cos^2\theta - 2\sqrt{3}\,\sin\theta\cos\theta - 3\sin^2\theta = 0$

TRIGONOMETRIC INEQUALITIES AND SYSTEM OF INEQUALITY

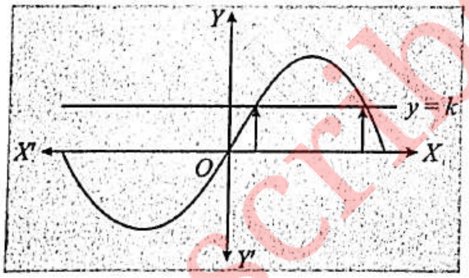
To solve the trigonometric inequalities of the type $f(x) \le a$ or $f(x) \ge a$, where $f(x)$ is some trigonometric ratio we take following steps

1. Draw the graph of $f(x)$ in an interval length equal to the fundamental period of $f(x)$.
2. Draw the line $y = a$.
3. Take the portion of the graph for which the inequality is satisfied.
4. To generalise, add pn, $n \in I$ and in the final solution where p is the fundamental period of $f(x)$.

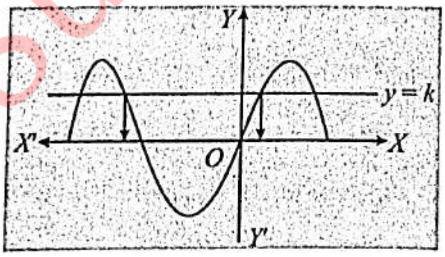
Suppose we have to solve $f(x) > k$ or $f(x) < k$.

When we solve the inequation, we often use the graphs of the functions $y = f(x)$ and $y = k$.

Then, the solution of the inequality $f(x) > k$ is the values of x for which the point $(x, f(x))$ of the graph of $y = f(x)$ lies above the straight line $y = k$.



Similarly, when we solve $f(x) < k$, then the solution of the inequalition $f(x) < k$ is the values of x for which the point $(x, f(x))$ of the graph of $y = f(x)$ lies below the straight line $y = k$.



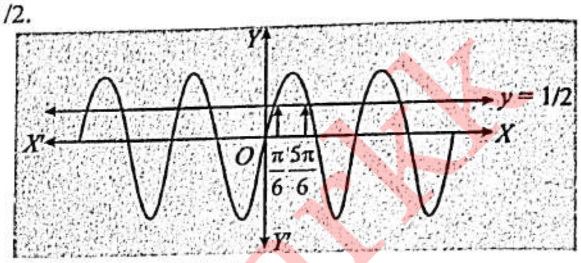
❖ An inequation is of the form $\sin x > k$, $\cos x < k$, $\cos x > k$, $\tan x > k$, $\tan x < k$.

Rule: Find the smallest values of x that satisfies the given inequation and then add $2n\pi$ with that values of x.

Train Your Brain

Example 25: Solve $\sin x > 1/2$.

Sol. Here, we should construct the graph of $y = \sin x$ and $y = 1/2$.

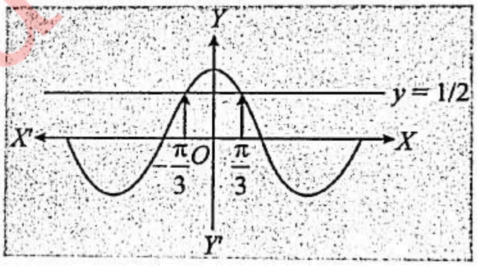


Hence, the solution set is

$$x = \underset{n\in I}{U}\left(2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}\right)$$

Example 26: Solve : $\cos x \ge \frac{1}{2}$.

Sol. Here, we should draw the graph of $y = \cos x$ and $y = 1/2$



Hence, the solution set is

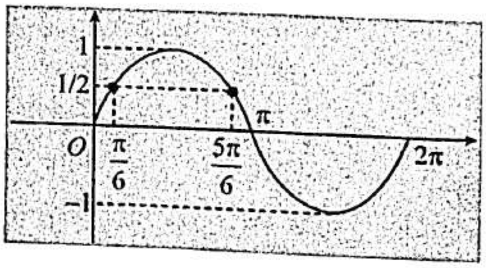
$$x = \underset{n\in I}{U}\left[2n\pi - \frac{\pi}{3}, 2n\pi + \frac{\pi}{3}\right]$$

Example 27: Solve : $2\sin^2\theta - \sin\theta \ge 0$, where $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$.

Sol. $\sin\theta\,(2\sin\theta - 1) \ge 0$ which is possible. only where

$$\sin\theta \ge \frac{1}{2} \quad \text{or} \quad \sin\theta \le 0$$

$$\sin\theta \ge \frac{1}{2} \Rightarrow \frac{\pi}{2} \le \theta \le \frac{5\pi}{6}$$



$$\sin\theta \le 0 \Rightarrow \pi \le \theta \le \frac{3\pi}{2}$$

$$\theta \in \left[\frac{\pi}{2}, \frac{5\pi}{6}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$$

Example 28: Solve: $\sin\theta + \sqrt{3}\cos\theta \ge 1,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

Sol. $\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta \ge \frac{1}{2}$

$$\sin\left(\theta + \frac{\pi}{3}\right) \ge \frac{1}{2}$$

$$\frac{\pi}{6} \le \theta + \frac{\pi}{3} \le \frac{5\pi}{6} \Rightarrow -\frac{\pi}{6} \le \theta \le \frac{\pi}{2}$$

Concept Application

23. Find the value of x in the interval $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ for which $\sqrt{2}\sin 2x + 1 \le 2\sin x + \sqrt{2}\cos x$

24. Solve $\sin 2x > \sqrt{2}\sin^2 x + \left(2-\sqrt{2}\right)\cos^2 x$

25. Solve : $\tan x \ge \frac{1}{\sqrt{3}}$

26. Solve for x:

(*i*) $\sin x > 0, x \in (0, 2\pi)$

(*ii*) $\sin x > 1/2; x \in (0, 2\pi)$

(*iii*) $\sin x > 1/2, x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

(*iv*) $\cos x < \frac{-1}{2}; x \in (0, 2\pi)$

IMPORTANT POINTS

1. Many trigonometrical equations can be solved by different methods. The form of solution obtained in different methods may be different. From these different forms of solutions, the students should not think that the answer obtained by one method are wrong and those obtained by another method are correct. The solutions obtained by different methods may be shown to be equivalent by some supplementary transformations.

   To test the equivalence of two solutions obtained from two methods, the simplest way is to put values of $n = ...-2, -1, 0, 1, 2, 3.......$ etc. and then to find the angles in $[0, 2\pi]$. If all the angles in both solutions are same, the solutions are equivalent.

2. While manipulating the trigonometrical equation, avoid the danger of losing roots. Generally, some roots are lost by cancelling a common factor from the two sides of an equation. For example, suppose we have the equation $\tan x = 2\sin x$. Here by dividing both sides by $\sin x$, we get $\cos x = \frac{1}{2}$. This is not equivalent to the original equation. Here the roots obtained by $\sin x = 0$, are lost. Thus in place of dividing an equation by a common factor, the students are advised to take this factor out as a common factor from all terms of the equation.

3. While equating one of the factors to zero, take care of the other factor that it should not become infinite. For example, if we have the equation $\sin x = 0$, which can be written as $\cos x \tan x = 0$. Here we cannot put $\cos x = 0$, since for $\cos x = 0$, $\tan x = \sin x/\cos x$ is infinite.

4. **Avoid squaring:** When we square both sides of an equation, some extraneous roots appear. Hence it is necessary to check all the solutions found by substituting them in the given equation and omit the solutions not satisfying the given equation.

   For example: Consider the equation,

   $\sin\theta + \cos\theta = 1$ ...(*i*)

   Squaring we get

   $1 + \sin 2\theta = 1$ or $\sin 2\theta = 0$ ...(*ii*)

   i.e. $2\theta = n\pi$ or $\theta = n\pi/2$,

   This gives $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, ...$

   Verification shows that π and $\frac{3\pi}{2}$ do not satisfy the equation

   as $\sin\pi + \cos\pi = -1, \ne 1$

   and $\sin\frac{3\pi}{2} + \cos\frac{3\pi}{2} = -1, \ne 1$.

   The reason for this is simple.

   The equation (*ii*) is not equivalent to (*i*) and (*ii*) contains two equations : $\sin\theta + \cos\theta = 1$

   and $\sin\theta + \cos\theta = -1$. Therefore we get extra solutions.

   Thus if squaring is must, verify each of the solution.

5. **Some necessary restrictions:** If the equation involves $\tan x$, $\sec x$, take $\cos x \ne 0$. If $\cot x$ or $\operatorname{cosec} x$ appear, take $\sin x \ne 0$. If log appear in the equation, i.e. $\log[f(\theta)]$ appear in the equation, use $f(\theta) > 0$ and base of $\log > 0, \ne 1$.

   Also note that $\sqrt{[f(\theta)]}$ is always positive, for example $\sqrt{\sin^2\theta} = |\sin\theta|$, not $\pm\sin\theta$.

6. **Verification :** Student are advice to check whether all the roots obtained by them satisfy the equation and lie in the domain of the variable of the given equation.

Aarambh (Solved Examples)

1. Find the value(s) of k for which the equation $\sin x + \cos(k+x) + \cos(k-x) = 2$ has real solutions.

(*a*) $n\pi - \frac{\pi}{7} \le k \le n\pi + \frac{\pi}{7}$ (*b*) $n\pi - \frac{\pi}{6} \le k \le n\pi + \frac{\pi}{6}$

(*c*) $n\pi - \frac{\pi}{4} \le k \le n\pi + \frac{\pi}{4}$ (*d*) $n\pi - \frac{\pi}{5} \le k \le n\pi + \frac{\pi}{5}$

Sol. $\sin x + \cos(k+x) + \cos(k-x) = 2$

$2\cos k.\cos x + \sin x = 2$

This equation is of the form $a\cos x + b\sin x = c$

Here $a = 2\cos k$, $b = 1$ and $c = 2$

Since for real solution, $|c| \le \sqrt{a^2+b^2}$

$$\therefore |2| \le \sqrt{1+4\cos^2 k} \Rightarrow 2 \le \sqrt{1+4\cos^2 k}$$

$$\Rightarrow \cos^2 k \ge \frac{3}{4} \Rightarrow \sin^2 k \le \frac{1}{4}$$

$$\Rightarrow \sin^2 k - \frac{1}{4} \le 0 \Rightarrow \left(\sin k + \frac{1}{2}\right)\left(\sin k - \frac{1}{2}\right) \le 0$$

$$\Rightarrow -\frac{1}{2} \le \sin k \le \frac{1}{2} \Rightarrow n\pi - \frac{\pi}{6} \le k \le n\pi + \frac{\pi}{6}$$

Therefore, option (*b*) is the correct answer.

2. Find most general value of θ for

$$\cos(2x+3y) = \frac{1}{2} \& \cos(3x+2y) = \frac{\sqrt{3}}{2}$$

Sol. $\cos(2x+3y) = \frac{1}{2} = \cos\frac{\pi}{3}$

$$\Rightarrow 2x+3y = 2n\pi \pm \frac{\pi}{3} \qquad ...(i)$$

$$\text{and } \cos(3x+2y) = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$$

$$\Rightarrow 3x+2y = 2m\pi \pm \frac{\pi}{6} \qquad ...(ii)$$

where $m, n \in I$

Solving (*i*) and (*ii*) we get

$$x = \frac{1}{5}\left[(6m-4n)\pi \pm \frac{\pi}{2} \mp \frac{2\pi}{3}\right]$$

$$\text{or } y = \frac{1}{5}\left[(6n-4m)\pi \pm \pi \mp \frac{\pi}{3}\right]$$

Therefore, option (*c*) is the correct answer.

3. For what a is the equation $\sin^2 x - \sin x \cos x - 2\cos^2 x = a$ solvable?

(*a*) $a \in \left[-\frac{\sqrt{10}+1}{2}, \frac{\sqrt{10}-1}{2}\right]$ (*b*) $a \in \left[\frac{-\sqrt{10}-1}{2}, \frac{\sqrt{10}+1}{2}\right]$

(*c*) $a \in [-2, 3]$ (*d*) $a \in [-1, 4]$

Sol. Multiplying the right member of the equation by $\sin^2 x + \cos^2 x = 1$ we reduce it to the form $(1-a)\sin^2 x - \sin x \cos x - (a+2)\cos^2 x = 0$. ...(*i*)

First let us assume that $a \ne 1$. Then from (*i*) it follows that $\cos x = 0$, since otherwise we have $\sin x = \cos x = 0$ which is impossible. Dividing both members of (*i*) by $\cos^2 x$ and putting $\tan x = t$ we get the equation $(1-a)t^2 - t - (a+2) = 0$. ...(*ii*)

Equation (*i*) is solvable if and only if the roots of equation (*ii*) are real, i.e. if its discriminant is non-negative $D = -4a^2 - 4a + 9 \ge 0$. ...(*iii*)

Solving inequality (3) we find $-\frac{\sqrt{10}+1}{2} \le a \le \frac{\sqrt{10}-1}{2}$ ..(*iv*)

Therefore, option (*a*) is the correct answer.

4. If $\sin(x-y) = \cos(x+y) = 1/2$ then the values of x & y lying between 0 and π are given by:

(*a*) $x = \pi/4, y = 3\pi/4$ (*b*) $x = \pi/4, y = \pi/12$

(*c*) $x = 5\pi/4, y = 5\pi/12$ (*d*) $x = 11\pi/12, y = 3\pi/4$

Sol. $\sin(x-y) = \frac{1}{2}$ and $\cos(x+y) = \frac{1}{2}$

$$\Rightarrow x - y = \frac{\pi}{6}, \frac{5\pi}{6} \text{ and } x + y = \frac{\pi}{3}, \frac{5\pi}{3}$$

Adding $2x = \frac{\pi}{2}$ or $\frac{7\pi}{6}$ or $\frac{11\pi}{6} \Rightarrow x = \frac{\pi}{4}$ or $\frac{7\pi}{12}$ or $\frac{11\pi}{12}$

when $x = \frac{\pi}{4}, y = \frac{\pi}{12}$

when $x = \frac{7\pi}{12}$ no value of y is possible.

when $x = \frac{11\pi}{12}, y = \frac{3\pi}{4}$.

Therefore, option (*b, d*) is the correct answers.

5. $\cos 15x = \sin 5x$ if

(*a*) $x = -\frac{\pi}{20} + \frac{n\pi}{5}, n \in I$ (*b*) $x = \frac{\pi}{40} + \frac{n\pi}{10}, n \in I$

(*c*) $x = \frac{3\pi}{20} + \frac{n\pi}{5}, n \in I$ (*d*) $x = -\frac{3\pi}{40} + \frac{n\pi}{10}, n \in I$

Sol. $\cos 15x = \sin 5x$

$$\cos 15x = \cos\left(\frac{\pi}{2} - 5x\right) \text{ or } \cos\left(\frac{3\pi}{2} + 5x\right)$$

$$15x = 2n\pi \pm \left(\frac{\pi}{2} - 5x\right) \text{ or } 15x = 2n\pi \pm \left(\frac{3\pi}{2} + 5x\right)$$

$$\Rightarrow x = \frac{n\pi}{10} + \frac{\pi}{40}, n \in I, x = \frac{n\pi}{5} + \frac{3\pi}{20}, n \in I$$

$$\text{and } x = \frac{n\pi}{5} - \frac{\pi}{20}, n \in I \text{ and } x = \frac{n\pi}{10} - \frac{3\pi}{40}, n \in \text{I}$$

Therefore, option (*a,b,c,d*) is the correct answer.

6. $\sin^2 x + 2\sin x \cos x - 3\cos^2 x = 0$ if

(*a*) $\tan x = 3$ (*b*) $\tan x = -1$

(*c*) $x = n\pi + \pi/4, n \in I$ (*d*) $x = n\pi + \tan^{-1}(-3), n \in I$

Sol. $\sin^2 x + 2\sin x\cos x - 3\cos^2 x = 0$

Case-I: $\cos x \neq 0 \quad \therefore \tan^2 x + 2\tan x - 3 = 0$

$\Rightarrow \tan x = -3, 1 \Rightarrow x = n\pi + \tan^{-1}(-3), n\pi + \frac{\pi}{4}$

Case-II: $\cos x = 0 \Rightarrow 1 + 0 - 0 = 0$ not true.

Therefore, option (c,d) is the correct answers.

7. $\sin x - \cos^2 x - 1$ assumes the least value for the set of values of x given by:

(*a*) $x = n\pi + (-1)^{n+1}\left(\frac{\pi}{6}\right), n \in I$

(*b*) $x = n\pi + (-1)^{n}\left(-\frac{\pi}{6}\right), n \in I$

(*c*) $x = n\pi + (-1)^{n}\left(\frac{\pi}{3}\right), n \in I$

(*d*) $x = n\pi - (-1)^{n}\left(\frac{\pi}{6}\right), n \in I$

Sol. Let $E = \sin x - \cos^2 x - 1$

$\Rightarrow E = \sin x - 1 + \sin^2 x - 1 = \sin^2 x + \sin x - 2$

$= \left(\sin x + \frac{1}{2}\right)^2 - \frac{9}{4}$ assumes least value

when $\sin x = -\frac{1}{2} \Rightarrow x = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$.

Therefore, option (a,b) is the correct answers.

8. $\sin x$, $\sin 2x$, $\sin 3x$ are in A.P if

(*a*) $x = n\pi/2, n \in I$ (*b*) $x = n\pi, n \in I$

(*c*) $x = 2n\pi, n \in I$ (*d*) $x = (2n+1)\pi, n \in I$

Sol. $2\sin 2x = \sin x + \sin 3x$

$\Rightarrow 2\sin 2x = 2\sin x 2x \cos x \Rightarrow \sin 2x = 0$ or $\cos x = 1$

$\Rightarrow 2x = n\pi$ or $x = 2m\pi \Rightarrow x = \frac{n\pi}{2}, 2m\pi$

options (*a*), (*b*), (*c*), (*d*) are all a part of $x = \frac{n\pi}{2}$.

Therefore, option (a,b,c,d) is the correct answers.

9. If $x \in \left[0, \frac{\pi}{2}\right]$, then find the number of solutions of the equation $\sin 7x + \sin 4x + \sin x = 0$.

Sol. $\sin 7x + \sin 4x + \sin x = 0$

$\Rightarrow 2\sin 4x\cos 3x + \sin 4x = 0$

$\Rightarrow \sin 4x = 0$ or $\cos 3x = -\frac{1}{2}$

$\Rightarrow 4x = n\pi$ or $3x = 2n\pi \pm \frac{2\pi}{3}$

$\Rightarrow x = \frac{n\pi}{4}, \frac{2n\pi}{3} \pm \frac{2\pi}{9} = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{9}, \frac{4\pi}{9}$.

Therefore, 5 is the correct answer.

10. The number of solution the following trigonometric equation: in $[0, \pi]$ $\sin^2 4x + \cos^2 x = 2\sin 4x\cos^4 x$

Sol. Given equation is: $\sin^2 4x + \cos^2 x = 2\sin 4x\cos^4 x$

$\Rightarrow \sin^2 4x - 2\sin 4x\cos^4 x + \cos^2 x = 0$

$\Rightarrow (\sin 4x - \cos^4 x)^2 + \cos^2 x - \cos^8 x = 0$

$\Rightarrow (\sin 4x - \cos^4 x)^2 + \cos^2 x(1 - \cos^6 x) = 0$

It is possible only when

$(\sin 4x - \cos^4 x) = 0, \cos^2 x(1 - \cos^6 x) = 0$

Now, $\cos x = 0, \cos^2 x = 1$

when $\cos x = 0$ then $x = (2n+1)\pi/2$

So, $\sin 4\left(n + \frac{1}{2}\right)\pi = 0$

which is true

when $\cos^2 x = 1$, then $x = n\pi$

which will not satisfy the equation

$\sin(4x) - \cos^4 x = 0$

Hence, the solution is $x = (2n+1)\frac{\pi}{2}$

Therefore, 1 is the correct answer.

11. The number solutions of given equation in $\left[0, \frac{3\pi}{2}\right]$

$$2\sin\left(3x + \frac{\pi}{4}\right) = \sqrt{1 + 8\sin 2x \cdot \cos^2 2x}$$

Sol. $2\sin\left(3x + \frac{\pi}{4}\right) = \sqrt{1 + 8\sin 2x\cos^2 2x}$

$\Rightarrow 2\sin\left(3x + \frac{\pi}{4}\right) = \sqrt{1 + 4\sin 4x\cos 2x}$

$\Rightarrow 4\sin^2\left(3x + \frac{\pi}{4}\right) = 1 + 4\sin 4x\cos 2x$

$\Rightarrow 2 - 2\cos\left(6x + \frac{\pi}{2}\right) = 1 + 2\sin 6x + 2\sin 2x$

$\Rightarrow \sin 2x = \frac{1}{2} \Rightarrow x = \frac{n\pi}{2} + (-1)^n\frac{\pi}{12}, n \in I$

$\Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

$\because \sin\left(3x + \frac{\pi}{4}\right) \geq 0 \text{ so } n = \frac{\pi}{12}, \frac{17\pi}{12}$ satisfies

Therefore, 2 is the correct answer.

12. If the number of solutions of the equations:

$2^{\cos x} = |\sin x|$, when $x \in [-2\pi, 2\pi]$ is α

and $x + 2\tan x = \frac{\pi}{2}$, when $x \in [0, 2\pi]$ is β then find $(\alpha + \beta)$.

Sol. We have $2^{\cos x}$

It is true only for $\cos x = 0$ and $|\sin x| = 1$

$\Rightarrow \cos x = \cos\frac{\pi}{2}$ and $\sin x = \pm 1 = \sin\left(\pm\frac{\pi}{2}\right)$

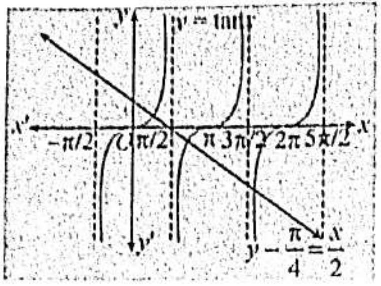
$\Rightarrow x = 2n\pi \pm \frac{\pi}{2}$

But, $x \in [-2\pi, 2\pi]$

$x = -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, -\frac{3\pi}{2}$

Hence, number of solution = 4.

We have, $x + 2\tan x = \frac{\pi}{2}$ or $\tan x = \frac{\pi}{4} - \frac{x}{2}$



Now the graph of the curve $y = \tan x$ and $y = \frac{\pi}{4} - \frac{x}{2}$, in the interval $[0, 2\pi]$ intersect at three points.

Hence, number of solution is three.

Therefore, 7 is the correct answer.

Comprehension (No. 13 to 15)

Let $a, b, c, d \in R$. Then the cubic equation of the type $ax^3 + bx^2 + cx + d = 0$ has either one root real or all three roots are real. But in case of trigonometric equations of the type a $\sin^3 x + b\sin^2 x + c\sin x + d = 0$ can possess several solutions depending upon the domain of x.

To solve an equation of the type $a\cos\theta + b\sin\theta = c$. The equation can be written as $\cos(\theta - \alpha) = c/\sqrt{(a^2 + b^2)}$.

The solution is $\theta = 2n\pi + (\alpha \pm \beta)$,

where $\tan\alpha = b/a$, $\cos\beta = c/\sqrt{(a^2 + b^2)}$.

13. On the domain $[-\pi, \pi]$ the equation $4\sin^3 x + 2\sin^2 x - 2\sin x - 1 = 0$ possess

(*a*) only one real root (*b*) three real roots

(*c*) four real roots (*d*) six real roots

Sol. $4\sin^3 x + 2\sin^2 x - 2\sin x - 1$

$= (2\sin x + 1)(2\sin^2 x - 1) = 0$

$\sin x = -\frac{1}{2}, \pm\frac{1}{\sqrt{2}}$

$\therefore$ there are 6 solutions.

Therefore, option (*d*) is the correct answer.

14. In the interval $[-\pi/4, \pi/2]$, the equation,

$$\cos 4x + \frac{10\tan x}{1 + \tan^2 x} = 3 \text{ has}$$

(*a*) no solution (*b*) one solution

(*c*) two solutions (*d*) three solutions

Sol. $3 = \cos 4x + \frac{10\tan x}{1 + \tan^2 x} = \cos 4x + 5\sin 2x$

i.e $3 = 1 - 2\sin^2 2x + 5\sin 2x$

i.e $\sin 2x = \frac{1}{2}$ $\therefore$ $2x = \frac{\pi}{6}, \frac{5\pi}{6}$

Thus there are two solutions.

Therefore, option (*c*) is the correct answer.

15. $|\tan x| = \tan x + \frac{1}{\cos x}$ $(0 \le x \le 2\pi)$ has

(*a*) no solution (*b*) one solution

(*c*) two solutions (*d*) three solutions

Sol. (*i*) when $\tan x \ge 0$, then the equation becomes

$\tan x = \tan x + \frac{1}{\cos x}$ i.e $\frac{1}{\cos x} = 0$ (not possible)

(*ii*) when $\tan x < 0$, then the equation becomes

$-\tan x = \tan x + \frac{1}{\cos x}$ i.e $\sin x = -\frac{1}{2}$

$\therefore x = \frac{11\pi}{6}$ is the only solution.

Therefore, option (*b*) is the correct answer.

16. Match the following for number of solutions in $[0, 2\pi]$

	Column-I		Column-II
(*a*)	$\sin^2\theta - \tan^2\theta = 1$	(*p*)	2
(*b*)	$\sin\theta + \cos\theta = 1$	(*q*)	0
(*c*)	$\tan\theta + \sec\theta = 2\cos\theta$	(*r*)	3
(*d*)	$3\sin^2\theta - 4\sin\theta + 1 = 0$	(*s*)	1

Sol. (*a*) $\sin^2\theta = \sec^2\theta$

$\Rightarrow \sin^2\theta\cos^2\theta = 1$

$\Rightarrow \sin^2\theta(1 - \sin^2\theta) = 1$

$\Rightarrow \sin^4\theta - \sin^2\theta + 1 = 0$

Let $\sin\theta = t$

$\therefore$ $t^2 - t + 1 = 0$; $D = 1 - 4 = -3$

Since $D < 0$, there is no real solution

(*b*) $\sin\theta + \cos\theta = 1$

If $\sin\theta = 0$, then $\cos\theta = 1$

$\therefore$ In $[0, 2\pi]$, Number of solution = 2

and if $\sin\theta = 1$ then $\cos\theta = 0$

$\therefore$ In $[0, 2\pi]$, Number of solution = 1

Total number of solution = 3

(*c*) $\tan\theta + \sec\theta = 2\cos\theta$

$\Rightarrow \frac{1}{\cos\theta}[\sin\theta + 1] = 2\cos\theta$

$\Rightarrow 2\sin^2\theta + \sin\theta - 1 = 0$

$\Rightarrow \sin\theta = \frac{-1 \pm 3}{4}$

$\therefore$ 2 real solutions

(*d*) $3\sin^2\theta - 4\sin\theta + 1 = 0$

If $\sin\theta = 1$, In $[0, 2\pi]$ one solution

& $\sin\theta = \frac{1}{3}$, in $[0, 2\pi]$, there are 2 solutions

Total number of solution = 3

Therefore, option (*a*) is the correct answer.

School Level Problems

SINGLE CORRECT TYPE QUESTIONS

1. If $\sin\theta$ and $\cos\theta$ are the roots of $ax^2 - bx + c = 0$, then the relation between a, b and c will be

(a) $a^2 + b^2 + 2ac = 0$ (b) $a^2 - b^2 + 2ac = 0$

(c) $a^2 + c^2 + 2ab = 0$ (d) $a^2 - b^2 - 3ac = 0$

2. If $a\cos x + b\sin x = c$, then the value of $(a\sin x - b\cos x)^2$ is

(a) $a^2 + b^2 + c^2$ (b) $a^2 - b^2 - c^2$

(c) $a^2 - b^2 + c^2$ (d) $a^2 + b^2 - c^2$

3. The value of $\cos^2 x + \cos^2 y - 2\cos x \cos y \cos(x + y)$ is

(a) $\sin(x + y)$ (b) $\sin^2(x + y)$

(c) $\sin^3(x + y)$ (d) $\sin^4(x + y)$

4. $\dfrac{\cos 7x + \cos 5x}{\sin 7x - \sin 5x} =$

(a) $\cot x$ (b) $\sin x$

(c) $\tan x$ (d) $\sec x$

5. In a triangle ABC, $\operatorname{cosec} A(\sin B\cos C + \cos B\sin C)$ equals

(a) $\dfrac{a}{c}$ (b) $\dfrac{c}{a}$

(c) 1 (d) None of these

6. $\tan^2\theta = 1 - a^2$ then the value of $\sec\theta + \tan^3\theta\operatorname{cosec}\theta$ is

(a) $(2 - a^2)$ (b) $(2 - a^2)^{1/2}$

(c) $(2 - a^2)^{3/2}$ (d) None of these

7. The equation $(\cos p - 1)x^2 + x\cos p + \sin p = 0$, where x is a variable, has real roots. Then the interval of p is given by

(a) $(0, \pi)$ (b) $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

(c) $(0, 2\pi)$ (d) $(-\pi, 0)$

8. If $\tan\theta = -\dfrac{5}{12}$, that what can be the value of $\sin\theta$?

(a) $\dfrac{5}{13}$ but cannot be $-\dfrac{5}{13}$

(b) $-\dfrac{5}{13}$ but cannot be $\dfrac{5}{13}$

(c) $\dfrac{5}{13}$ or $-\dfrac{5}{13}$

(d) None of these

9. Find general solution of equation $\cot x = \sqrt{3}$

(a) $x = n\pi + \dfrac{\pi}{3}$ (b) $x = n\pi + \dfrac{\pi}{6}$

(c) $x = n\pi + \dfrac{2\pi}{3}$ (d) $x = n\pi + \dfrac{5\pi}{6}$

VERY SHORT ANSWER TYPE QUESTIONS

10. If $\cos x = -\dfrac{1}{2}$ and x lies in quadrant III, find the value of $\sin\dfrac{x}{2}$

11. Find the principal solution of equation $\sin x = \dfrac{\sqrt{3}}{2}$.

12. Prove that $\sin(n + 1)x\sin(n + 2)x + \cos(n + 1)x\cos(n + 2)x = \cos x$

13. Prove that $\cos 70^\circ.\cos 10^\circ + \sin 70^\circ.\sin 10^\circ = \dfrac{1}{2}$

14. Prove that $\dfrac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ.\tan 66^\circ} = -1$

15. Show that $\tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$

16. Express $2\cos 4x\sin 2x$ as an algebraic sum of sines or cosine.

LONG ANSWER TYPE QUESTIONS

17. Prove $\dfrac{\cos(\pi + x).\cos(-x)}{\sin(\pi - x).\cos\left(\dfrac{\pi}{2} + x\right)} = \cot^2 x$

18. If in two circles, arcs of the same length subtend angles 60° and 75° at the center, find the ratio of their radii.

19. In any triangle ABC, prove that $a\sin(B - C) + b\sin(C - A) + c\sin(A - B) = 0$.

20. If $\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$, then prove that $\cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta) = 1 - 2a^2 - 2b^2$

21. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$

Show that $\cos(\alpha + \beta) = \dfrac{b^2 - a^2}{b^2 + a^2}$.

CASE STUDY BASED QUESTIONS

CASE STUDY-I

In a class test of class XI, a teacher asked students to consider $A + B = \dfrac{\pi}{4}$, where A and B are acute angles.

Based on the above information, answer the following questions.

22. Find the value of $(1 + \tan A)(1 + \tan B)$?

(a) −1 (b) 2 (c) −3 (d) 4

23. Find the value of $\sin(A + B) - \cos(A + B) + \tan(A + B)$.

(a) −3 (b) 2 (c) −4 (d) 1

CASE STUDY-II

If $\sin A = \dfrac{3}{5}$ and $\cos B = -\dfrac{5}{13}; 0 < A < \dfrac{\pi}{2}$ and $\pi < B < \dfrac{3\pi}{2}$.

Credit - Akshat

24. Find the value of $\cos A + \sin B$

(*a*) $\frac{-16}{65}$ (*b*) $\frac{-8}{65}$

(*c*) $\frac{8}{65}$ (*d*) $\frac{12}{65}$

25. Find the value of $\sin 2A$

(*a*) $\frac{14}{25}$ (*b*) $\frac{24}{25}$

(*c*) $\frac{-14}{25}$ (*d*) $\frac{-24}{25}$

Prarambh (Topicwise)

SOLUTION BY FACTORIZATION

1. The equation $2\sin\frac{x}{2}\cos^2 x - 2\sin\frac{x}{2}\sin^2 x = \cos^2 x - \sin^2 x$ has a root for which the true statement is

(*a*) $\sin 2x = 1$ (*b*) $\cos x = \frac{\pi}{6}$

(*c*) $\cos 2x = -\frac{\pi}{6}$ (*d*) None of these

2. If $2\tan^2\theta = \sec^2\theta$, then the general value of θ is [where $n \in I$]

(*a*) $n\pi + \frac{\pi}{4}$ (*b*) $n\pi - \frac{\pi}{4}$

(*c*) $n\pi \pm \frac{\pi}{4}$ (*d*) $2n\pi \pm \frac{\pi}{4}$

3. If $\sqrt{3}\tan 2\theta + \sqrt{3}\tan 3\theta + \tan 2\theta \tan 3\theta = 1$, then the general value of θ is [where $n \in$ I]

(*a*) $n\pi + \frac{\pi}{5}$ (*b*) $\left(n + \frac{1}{6}\right)\frac{\pi}{5}$

(*c*) $\left(2n \pm \frac{1}{6}\right)\frac{\pi}{5}$ (*d*) $\left(n + \frac{1}{3}\right)\frac{\pi}{5}$

4. If $\tan 2\theta \tan\theta = 1$, then the general value of θ is [where $n \in I$]

(*a*) $\left(n + \frac{1}{2}\right)\frac{\pi}{3}$ (*b*) $\left(n + \frac{1}{2}\right)\pi$

(*c*) $\left(2n \pm \frac{1}{2}\right)\frac{\pi}{3}$ (*d*) None of these

5. The solution of the equation $4\cos^2 x + 6\sin^2 x = 5$ [where $n \in I$]

(*a*) $x = n\pi \pm \frac{\pi}{2}$ (*b*) $x = n\pi \pm \frac{\pi}{4}$

(*c*) $x = n\pi \pm \frac{3\pi}{2}$ (*d*) Both (*a*) and (*c*)

6. If $\sin\left(\frac{\pi}{4}\cot\theta\right) = \cos\left(\frac{\pi}{4}\tan\theta\right)$, [where $n \in I$] then θ may be equal to

(*a*) $n\pi + \frac{\pi}{4}$ (*b*) $2n\pi \pm \frac{\pi}{4}$

(*c*) $n\pi - \frac{\pi}{4}$ (*d*) $2n\pi \pm \frac{\pi}{6}$

7. The general value of θ in the equation $2\sqrt{3}\cos\theta = \tan\theta$, is [where $n \in I$]

(*a*) $2n\pi \pm \frac{\pi}{6}$ (*b*) $2n\pi \pm \frac{\pi}{4}$

(*c*) $n\pi + (-1)^n\frac{\pi}{3}$ (*d*) $n\pi + (-1)^n\frac{\pi}{4}$

8. If $(2\cos x - 1)(3 + 2\cos x) = 0, 0 \le x \le 2\pi$, [where $n \in I$] then $x =$

(*a*) $\frac{\pi}{3}$ (*b*) $\frac{\pi}{3}, \frac{5\pi}{3}$

(*c*) $\frac{\pi}{2}, \frac{5\pi}{3}, \cos^{-1}\left(-\frac{3}{2}\right)$ (*d*) $\frac{5\pi}{3}$

9. If $\tan(\cot x) = \cot(\tan x)$, [where $n \in I$] then $\sin 2x =$

(*a*) $(2n+1)\frac{\pi}{4}$ (*b*) $\frac{4}{(2n+1)\pi}$

(*c*) $4\pi(2n + 1)$ (*d*) $\frac{\pi}{2}(2n-1)$

10. The solution set of the equation $4\sin\theta.\cos\theta - 2\cos\theta - 2\sqrt{3}\sin\theta + \sqrt{3} = 0$ in the interval $(0, 2\pi)$ is

(*a*) $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ (*b*) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$

(*c*) $\left\{\frac{3\pi}{4}, \pi, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$ (*d*) $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}\right\}$

11. All solutions of the equation $2\sin\theta + \tan\theta = 0$ are obtained by taking all integral values of m and n in:

(*a*) $2n\pi + \frac{2\pi}{3}$, $n \in$ I

(*b*) $n\pi$ or $2m\pi \pm \frac{2\pi}{3}$ where $n, m \in I$

(*c*) $n\pi$ or $m\pi \pm \frac{\pi}{3}$ where $n, m \in I$

(*d*) $n\pi$ or $2m\pi \pm \frac{\pi}{3}$ where $n, m \in I$

12. If $2\cos^2(\pi + x) + 3\sin(\pi + x)$ vanishes then the values of x lying in the interval from 0 to 2π are

(*a*) $x = \pi/6$ or $5\pi/6$ (*b*) $x = \pi/3$ or $5\pi/3$
(*c*) $x = \pi/4$ or $5\pi/4$ (*d*) $x = \pi/2$ or $5\pi/2$

13. If $\cos 2\theta + 3\cos\theta = 0$, then

(*a*) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{\sqrt{17}-3}{4}\right)$

(*b*) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{-\sqrt{17}-3}{4}\right)$

(*c*) $\theta = 2n\pi \pm \alpha$ where $\alpha = \cos^{-1}\left(\frac{\pm\sqrt{17}-3}{4}\right)$

(*d*) None of these

14. The general solution of the equation, $2\cos 2x = 6\cos^2 x - 4$ is

(*a*) $x = 2n\pi, n \in I$ (*b*) $x = n\pi, n \in I$
(*c*) $x = n\pi/4, n \in I$ (*d*) $x = n\pi/2, n \in I$

15. If $x \in (0, 1)$, then greatest root of the equation $\sin 2\pi x = \sqrt{2}\cos \pi x$ is

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{3}{4}$ (*d*) $\frac{1}{3}$

16. The general solution of $\sec\theta + \tan\theta = \sqrt{3}$ is

(*a*) $\theta = 2n\pi + \frac{\pi}{6}, 2n\pi - \frac{\pi}{2}, n \in I$

(*b*) $\theta = 2n\pi \pm \frac{\pi}{6}, n \in I$

(*c*) $2n\pi - \frac{\pi}{6}, n \in I$

(*d*) $2n\pi - \frac{\pi}{2}, n \neq 2k+1, n, k \in I$

17. The general solution of $\sin^2\theta\sec\theta + \sqrt{3}\tan\theta = 0$ is

(*a*) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3}, \theta = n\pi, n \in I$

(*b*) $\theta = n\pi, n \in I$

(*c*) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3}, n \in I$

(*d*) $\theta = \frac{n\pi}{2}, n \in I$

SOLVING SIMULTANEOUS EQUATION

18. If $\cos\theta = -\frac{1}{\sqrt{2}}$ and $\tan\theta = 1$, [where $n \in I$] then the general value of θ is

(*a*) $2n\pi + \frac{\pi}{4}$ (*b*) $(2n+1)\pi + \frac{\pi}{4}$

(*c*) $n\pi + \frac{\pi}{4}$ (*d*) $n\pi \pm \frac{\pi}{4}$

19. If $\tan\theta = -\frac{1}{\sqrt{3}}$ and $\sin\theta = \frac{1}{2}$, $\cos\theta = -\frac{\sqrt{3}}{2}$, [where $n \in I$] then the principal value of θ will be

(*a*) $\frac{\pi}{6}$ (*b*) $\frac{5\pi}{6}$

(*c*) $\frac{7\pi}{6}$ (*d*) $-\frac{\pi}{6}$

20. The most general solution of $\tan\theta = -1$ and $\cos\theta = \frac{1}{\sqrt{2}}$ is

(*a*) $n\pi + \frac{7\pi}{4}, n \in I$ (*b*) $n\pi + (-1)^n\frac{7\pi}{4}, n \in I$

(*c*) $2n\pi + \frac{7\pi}{4}, n \in I$ (*d*) None of these

21. The most general value of θ which will satisfy both the equations $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$ is [where $n \in I$]

(*a*) $n\pi + (-1)^n\frac{\pi}{6}$ (*b*) $n\pi + \frac{\pi}{6}$

(*c*) $2n\pi \pm \frac{\pi}{6}$ (*d*) $2n\pi + \frac{7\pi}{6}$

22. If $\sin A = \sin B$ and $\cos A = \cos B$, then

(*a*) $A = B + n\pi, n \in I$ (*b*) $A = B - n\pi, n \in I$
(*c*) $A = 2n\pi + B, n \in I$ (*d*) $A = n\pi - B, n \in I$

SOLUTION OF EQUATION REDUCIBLE TO QUADRATIC EQUATION

23. If $3(\sec^2\theta + \tan^2\theta) = 5$, [where $n \in I$] then the general value of θ is

(*a*) $2n\pi + \frac{\pi}{6}$ (*b*) $2n\pi \pm \frac{\pi}{6}$

(*c*) $n\pi \pm \frac{\pi}{6}$ (*d*) $n\pi \pm \frac{\pi}{3}$

24. If $\cot\theta + \tan\theta = 2\operatorname{cosec}\theta$, [where $n \in I$] the general value of θ is

(*a*) $n\pi \pm \frac{\pi}{3}$ (*b*) $n\pi \pm \frac{\pi}{6}$

(*c*) $2n\pi \pm \frac{\pi}{3}$ (*d*) $2n\pi \pm \frac{\pi}{6}$

25. The number of values of x for which $\sin 2x + \cos 4x = 2$ is

(*a*) Zero (*b*) 1 (*c*) 2 (*d*) Infinite

26. If $\tan^2\theta - (1+\sqrt{3})\tan\theta + \sqrt{3} = 0$, [where $n \in I$] then the general value of θ is

(*a*) $n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{3}$ (*b*) $n\pi - \frac{\pi}{4}, n\pi + \frac{\pi}{3}$

(*c*) $n\pi + \frac{\pi}{4}, n\pi - \frac{\pi}{3}$ (*d*) $n\pi - \frac{\pi}{4}, n\pi - \frac{\pi}{3}$

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27. If $4\sin^2\theta + 2(\sqrt{3}+1)\cos\theta = 4+\sqrt{3}$, [where $n \in I$] then the general value of θ is

(*a*) $2n\pi \pm \frac{\pi}{3}$ (*b*) $2n\pi + \frac{\pi}{4}$

(*c*) $n\pi \pm \frac{\pi}{3}$ (*d*) $n\pi - \frac{\pi}{3}$

28. The smallest positive angle which satisfies the equation $2\sin^2\theta + \sqrt{3}\cos\theta + 1 = 0$, is

(*a*) $\frac{5\pi}{6}$ (*b*) $\frac{2\pi}{3}$ (*c*) $\frac{\pi}{3}$ (*d*) $\frac{\pi}{6}$

29. If $12\cot^2\theta - 31\ \text{cosec}\ \theta + 32 = 0$, then the value of $\sin\theta$ is

(*a*) $\frac{3}{5}$ or 1 (*b*) $\frac{2}{3}$ or $\frac{-2}{3}$

(*c*) $\frac{4}{5}$ or $\frac{3}{4}$ (*d*) $\pm\frac{1}{2}$

SUM TO PRODUCT AND PRODUCT TO SUM

30. If $\cos 7\theta = \cos\theta - \sin 4\theta$, then the general value of θ is [where $n \in I$]

(*a*) $\frac{n\pi}{4}, \frac{n\pi}{3} + \frac{\pi}{18}$ (*b*) $\frac{n\pi}{3}, \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}$

(*c*) $\frac{n\pi}{4}, \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}$ (*d*) $\frac{n\pi}{6}, \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}$

31. If $\cos\theta + \cos 2\theta + \cos 3\theta = 0$, then the general value of θ is [where $m \in I$]

(*a*) $\theta = 2m\pi \pm \frac{2\pi}{3}$ (*b*) $\theta = m\pi \pm \frac{\pi}{4}$

(*c*) $\theta = m\pi \pm (-1)^m\frac{2\pi}{3}$ (*d*) Both (*a*) and (*b*)

Equation of the type a sinx + b cos x = C

32. The general solution of equation $\sin x + \sin 5x = \sin 2x + \sin 4x$ is :

(*a*) $\frac{n\pi}{2}$; $n \in I$ (*b*) $\frac{n\pi}{5}$; $n \in I$

(*c*) $\frac{n\pi}{3}$; $n \in I$ (*d*) $\frac{2n\pi}{3}$; $n \in I$

33. If $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$, then the most general value of θ is [where $n \in I$]

(*a*) $n\pi + (-1)^n\frac{\pi}{4}$ (*b*) $(-1)^n\frac{\pi}{4} - \frac{\pi}{3}$

(*c*) $n\pi + \frac{\pi}{4} - \frac{\pi}{3}$ (*d*) $n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{3}$

34. The equation $\sqrt{3}\sin x + \cos x = 4$ has

(*a*) Only one solution

(*b*) Two solutions

(*c*) Infinitely many solutions

(*d*) No solution

35. The equation $3\cos x + 4\sin x = 6$ has

(*a*) Finite solution (*b*) Infinite solution

(*c*) One solution (*d*) No solution

SOLUTION USING THE BOUNDNESS

36. The number of solutions of $\sin\theta + 2\sin 2\theta + 3\sin 3\theta + 4\sin 4\theta = 10$ in $(0, \pi)$ is

(*a*) 1 (*b*) 2 (*c*) 4 (*d*) 0

37. If $\sin x + \cos x = \sqrt{y + \frac{1}{y}}$, $x \in (0, \pi)$, then

(*a*) $x = \frac{\pi}{4}, y = 1$ (*b*) $y = 0$

(*c*) $y = 2$ (*d*) $x = \frac{3\pi}{4}$

38. One root of the equation $\cos x - x + \frac{1}{2} = 0$ lies in the inte

(*a*) $\left[0, \frac{\pi}{2}\right]$ (*b*) $\left[-\frac{\pi}{2}, 0\right]$

(*c*) $\left[\frac{\pi}{2}, \pi\right]$ (*d*) $\left[\pi, \frac{3\pi}{2}\right]$

39. If $\sec x \cos 5x + 1 = 0$, where $0 < x < 2\pi$, then $x =$

(*a*) $\frac{\pi}{6}$ (*b*) $\frac{\pi}{5}$ (*c*) $\frac{\pi}{4}$ (*d*) $\frac{\pi}{3}$

40. If the equation $1 + \sin^2 x\theta = \cos\theta$ has a non-zero solutio θ, then x must be

(*a*) An integer

(*b*) A rational number

(*c*) An irrational number

(*d*) None of these

41. The number of the solution of the equation $3\sin x + 4\cos x - x^2 - 16 = 0$ is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

42. The solution set of $\sin^4 x - \tan^8 x = 1$ is given by

(*a*) $x = 2n\pi + \frac{\pi}{8}, n \in I$ (*b*) $x = n\pi + \frac{\pi}{12}, n \in I$

(*c*) $x = 2n\pi + \frac{\pi}{24}, n \in I$ (*d*) None of these

TRIGONOMETRIC IN EQUALITIES

43. The set of all x in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ satisfying $|4\sin x - 1| < \sqrt{5}$ given by

(*a*) $\left(-\frac{\pi}{10}, \frac{3\pi}{10}\right)$ (*b*) $\left(\frac{\pi}{10}, \frac{3\pi}{10}\right)$

(*c*) $\left(\frac{\pi}{10}, \frac{3\pi}{10}\right)$ (*d*) None of these

44. If $4\sin^2 x - 12\sin x + 5 \le 0, 0 \le x \le 2\pi$, which of the following is the solution set for x?

(a) $\left[0, \frac{\pi}{6}\right]$ (b) $\left[0, \frac{5\pi}{6}\right]$

(c) $\left[\frac{5\pi}{6}, 2\pi\right]$ (d) $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

45. Which of the following is a set of values x for which $\sin x.\cos^3 x > \cos x.\sin^3 x$ in the interval $0 \le x \le 2\pi$?

(a) $(0, \pi)$ (b) $\left(0, \frac{\pi}{4}\right)$

(c) $\left(\frac{\pi}{4}, \pi\right)$ (d) $\left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

46. If $[\sin x] + [\sqrt{2}\cos x] = -3$, where $x \in [0, 2\pi]$ then find x (where $[\cdot]$ shows G.I.F.)

(a) $\left(\pi, \frac{5\pi}{4}\right)$

(b) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(c) $\left(\frac{5\pi}{4}, \frac{3\pi}{2}\right)$

(d) None of these

Prabal (JEE Main Level)

1. The number of solution of the equation $|\sin x| = |\cos 3x|$ in $[-2\pi, 2\pi]$ is equal to

(a) 32 (b) 28 (c) 24 (d) 30

2. If $\tan(\cot x) = \cot(\tan x)$, then $\operatorname{cosec} 2x =$

(a) $(2n+1)\frac{\pi}{4}$ (b) $\frac{4}{(2n+1)\pi}$

(c) $4\pi(2n+1)$ (d) None of these

3. The most general solution of $4^{\sin x} + 4^{\cos x} = 2^{1-\sqrt{2}}$ is

(a) $(2n+1)\pi + \frac{\pi}{4}$ (b) $n\pi - \frac{\pi}{4}$

(c) $n\pi + (-1)^n \frac{\pi}{4}$ (d) $2n\pi + \frac{\pi}{4}$

4. General solution of equation $\sec^2 x + \operatorname{cosec}^2 x + \cot^2 x - 4\cos^2 x = 3$ is

(a) $n\pi \pm \frac{\pi}{4}; n \in Z$ (b) $2n\pi \pm \frac{\pi}{4}; n \in Z$

(c) $n\pi + \frac{\pi}{3}; n \in Z$ (d) $n\pi - \frac{\pi}{6}; n \in Z$

5. Number of solutions of the equation

$$\cot(\theta) + \cot\left(\theta + \frac{\pi}{3}\right) + \cot\left(\theta - \frac{\pi}{3}\right) + \cot(3\theta) = 0$$

where $\theta \in (0, \pi)$ is

(a) Infinite (b) 0 (c) 1 (d) 2

6. The total number of solution of the equation $\max(\sin x, \cos x) = 1/2$ for $x \in (-2\pi, 6\pi)$ is equal to:

(a) 3 (b) 6 (c) 7 (d) 8

7. The number of solution of $\sin^4 x - \cos^2 x \sin x + 2\sin^2 x + \sin x = 0$ in $0 \le x \le 3\pi$ is

(a) 3 (b) 4

(c) 6 (d) 5

8. The number of solutions of $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x + \cos 3x, 0 \le x \le 2\pi$, is

(a) 7 (b) 5 (c) 4 (d) 6

9. The total number of solutions of $\cos x = \sqrt{1 - \sin 2x}$ in $[0, 2\pi]$ is equal to

(a) 2 (b) 3

(c) 5 (d) None of these

10. The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ satisfying the equation $(\sqrt{3})^{\sec^2\theta} = \tan^4\theta + 2\tan^2\theta$ is

(a) 2 (b) 4 (c) 0 (d) 1

11. The general solution of $8\tan^2 \frac{x}{2} = 1 + \sec x$ is

(a) $x = 2n\pi \pm \cos^{-1}\left(\frac{-1}{3}\right)$ (b) $x = 2n\pi \pm \frac{\pi}{6}$

(c) $x = 2n\pi \pm \cos^{-1}\left(\frac{1}{3}\right)$ (d) None of these

12. The general solution of $\tan\theta + \tan 4\theta + \tan 7\theta = \tan\theta.\tan 4\theta.\tan 7\theta$

(a) $\theta = \frac{n\pi}{4}, n \in Z$ (b) $\theta = 6\pi$

(c) $\theta = \frac{n\pi}{12}, n \in Z$ (d) None of these

13. Let $2\sin^2 x + 3\sin x - 2 > 0$ and $x^2 - x - 2 < 0$ (x is measured in radian). Then 'x' lies in the interval.

(*a*) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (*b*) $\left(-1, \frac{5\pi}{6}\right)$

(*c*) $(-1, 2)$ (*d*) $\left(\frac{\pi}{6}, 2\right)$

14. The general solution of the equation $\sin^{100} x - \cos^{100} x = 1$ is (where $n \in I$):

(*a*) $2n\pi + \frac{\pi}{2}$ (*b*) $n\pi + \frac{\pi}{2}$

(*c*) $2n\pi - \frac{\pi}{2}$ (*d*) $n\pi$

15. Number of solutions of $\sum_{r=1}^{5} \cos rx = 5$ in the interval $[0, 4\pi]$ is:

(*a*) 0 (*b*) 2

(*c*) 3 (*d*) 7

16. The general solution of the equation $\tan^2(x + y) + \cot^2(x + y) = 1 - 2x - x^2$ lie on the line is:

(*a*) $x = -1$ (*b*) $x = -2$

(*c*) $y = -1$ (*d*) $y = -2$

17. If $|k| = 5$ and $0° \le \theta \le 360°$, then the number of different solutions of $3\cos\theta + 4\sin\theta = k$ is

(*a*) Zero (*b*) Two

(*c*) One (*d*) Infinite

18. Total number of solutions of $\sin x = \frac{|x|}{10}$ is equal to

(*a*) 4 (*b*) 6

(*c*) 7 (*d*) None of these

19. $\cos 2x - 3\cos x + 1 = \frac{1}{(\cot 2x - \cot x)\sin(x - \pi)}$ holds, if

(*a*) $\cos x = 0$ (*b*) $\cos x = 1$

(*c*) $\cos x = 5/2$ (*d*) for no value of x

20. The number of solutions of $[\sin x + \cos x] = 3 + [-\sin x] + [-\cos x]$ (where [.] denotes the greatest integer function), $x \in [0, 2\pi]$, is

(*a*) 0 (*b*) 4

(*c*) Infinite (*d*) 1

21. The equation $\sin 3\theta = 4\sin\theta \cdot \sin 2\theta \cdot \sin 4\theta$ in $0 \le \theta \le \pi$ has

(*a*) 2 real solutions (*b*) 4 real solutions

(*c*) 6 real solutions (*d*) 8 real solutions

22. The arithmetic mean of the roots of the equation $4\cos^3 x - 4\cos^2 x - \cos(\pi + x) - 1 = 0$ in the interval $[0, 315]$ is equal to

(*a*) 49π (*b*) 50π

(*c*) 51π (*d*) 100π

23. If $\sin\theta + 7\cos\theta = 5$, then $\tan(\theta/2)$ is a root of the equation

(*a*) $x^2 - 6x + 1 = 0$ (*b*) $6x^2 - x - 1 = 0$

(*c*) $6x^2 + x + 1 = 0$ (*d*) $x^2 - x + 6 = 0$

24. $\frac{\sin 3\theta}{2\cos 2\theta + 1} = \frac{1}{2}$ if

(*a*) $\theta = n\pi + \frac{\pi}{6}, n \in I$

(*b*) $\theta = 2n\pi \pm \frac{\pi}{6}, n \in I$

(*c*) $\theta = n\pi + (-1)^n \frac{\pi}{6}, n \in I$

(*d*) $\theta = n\pi - \frac{\pi}{6}, n \in I$

25. The general solution of the equation $\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$ is

(*a*) $\frac{n\pi}{4} + \frac{\pi}{12}, n \in I$ (*b*) $\frac{n\pi}{3} + \frac{\pi}{6}, n \in I$

(*c*) $\frac{n\pi}{3} + \frac{\pi}{12}, n \in I$ (*d*) None of these

26. A triangle ABC is such that $\sin(2A + B) = \frac{1}{2}$. If A, B, C are in $A.P.$ then the angle A, B, C are respectively

(*a*) $\frac{5\pi}{12}, \frac{\pi}{4}, \frac{\pi}{3}$ (*b*) $\frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{12}$

(*c*) $\frac{\pi}{3}, \frac{\pi}{4}, \frac{5\pi}{12}$ (*d*) $\frac{\pi}{3}, \frac{5\pi}{12}, \frac{\pi}{4}$

27. The number of all possible triplets (a_1, a_2, a_3) such that : $a_1 + a_2\cos 2x + a_3\sin^2 x = 0$ for all x is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) Infinite

28. The value 'a' for which the equation $4\operatorname{cosec}^2[\pi(a + x)] + a^2 - 4a = 0$ has a real solution is :

(*a*) $a = 1$
(*b*) $a = 2$
(*c*) $a = 3$
(*d*) None of these

29. The solution of $|\cos x| = \cos x - 2\sin x$ is

(*a*) $x = n\pi, n \in I$

(*b*) $x = n\pi + \frac{\pi}{4}, n \in I$

(*c*) $x = n\pi + (-1)^n \frac{\pi}{4}, n \in I$

(*d*) $(2n+1)\pi + \frac{\pi}{4}, n \in I$

30. If $m, n \in N$ $(n > m)$, then number of solutions of the equation $n|\sin x| = m|\sin x|$ in $[0, 2\pi]$ is

(*a*) m (*b*) n

(*c*) mn (*d*) 3

31. The number of solutions of the equation $\cos(\pi\sqrt{x - 4})\cos(\pi\sqrt{x}) = 1$ is

(*a*) Zero (*b*) 1

(*c*) 2 (*d*) Infinite

32. The number of values of $\theta \in [0, 2\pi]$ satisfying $r \sin\theta = \sqrt{3}$ and $r + 4\sin\theta = 2(\sqrt{3}+1)$ is

(a) 4 (b) 5 (c) 6 (d) 7

33. The solution set of the equation $4\sin^4 x + \cos^4 x = 1$ is

(a) $x = n\pi + \frac{\pi}{6}$

(b) $x = n\pi \pm \theta$ where $\cos^2\theta = \frac{3}{5}$ or $x = n\pi, n \in I$

(c) $x = (2n+1)\frac{\pi}{2}$

(d) $x = 2n\pi \pm \frac{\pi}{3}$

34. The number of solutions of the equation $\sin x \,.\, \cos x\, (\cos x - \sin x)^2 \,.\, (\sin x + \cos x) = \lambda$, where $\lambda > \frac{1}{2\sqrt{2}}$ in the interval $[0, 4\pi]$, is

(a) 0 (b) 2 (c) 4 (d) 8

35. The complete set of values of x, $x \in \left(-\frac{\pi}{2}, \pi\right)$ satisfying the inequality $\cos 2x > |\sin x|$ is:

(a) $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{6}\right) \cup \left(-\frac{\pi}{6}, \frac{5\pi}{6}\right)$

(c) $\left(-\frac{\pi}{2}, -\frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \pi\right)$ (d) $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, \pi\right)$

36. Values of x between 0 and 2π which satisfy the equation $\sin x\sqrt{8\cos^2 x} = 1$ are in A.P. whose common difference is:

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

37. If α and β are distinct roots of the equation $a\cos\theta + b\sin\theta = c$, then $\cos(\alpha + \beta) =$

(a) $\frac{2ab}{a^2+b^2}$ (b) $\frac{2ab}{a^2-b^2}$

(c) $\frac{a^2-b^2}{a^2+b^2}$ (d) $\frac{a^2+b^2}{a^2-b^2}$

38. The solution of the equation $\log_{\cos x^2}(3-2x) < \log_{\cos x^2}(2x-1)$ is

(a) (1/2, 1) (b) $(-\infty, 1)$

(c) (1/2, 3) (d) $(1, \infty) - \sqrt{2n\pi}, n \in N$

39. If $\alpha, \beta, \gamma, \delta$ are the smallest positive angles in ascending order of magnitude which have their sines equal to the positive quantity k, then the value of $4\sin\left(\frac{\alpha}{2}\right) + 3\sin\left(\frac{\beta}{2}\right) + 2\sin\left(\frac{\gamma}{2}\right) + \sin\left(\frac{\delta}{2}\right)$ is equal to

(a) $2\sqrt{1-k}$ (b) $2\sqrt{1+k}$

(c) $\frac{\sqrt{1+k}}{2}$ (d) $\frac{\sqrt{1-k}}{2}$

40. The smallest positive value of x (in radians) satisfying the equation $\log_{\cos x}\left(\frac{\sqrt{3}}{2}\sin x\right) = 2 - \log_{\sec x}(\tan x)$ is

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{4}$

41. Suppose $a, b > 0$ and $\log_9(a) = \log_{12}(b) = \log_{16}(a+b)$ and $\frac{b}{a} = 2\sin\theta$, then θ lies is

(a) $\left(0, \frac{\pi}{4}\right)$ (b) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(c) $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ (d) $\left(\frac{3\pi}{4}, \pi\right)$

42. Smallest positive value of θ satisfying the equation $8\sin\theta\cos 2\theta\sin 3\theta\cos 4\theta = \cos 6\theta$ is

(a) $\frac{\pi}{18}$ (b) $\frac{\pi}{22}$

(c) $\frac{\pi}{24}$ (d) None of these

43. If $x \in (0, \pi)$ and $\sin x \cdot \cos^3 x > \cos x \cdot \sin^3 x$, then complete set of values of x is

(a) $x \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$ (b) $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{4}, \pi\right)$

(c) $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (d) None of these

44. Number of roots of $|\sin|x|| = x + |x|\ \forall\ x \subset [-2\pi, 2\pi]$ is

(a) 0 (b) 1 (c) 2 (d) 3

45. Number of degrees in the smallest positive angle x such that $8\sin x\cos^5 x - 8\sin^5 x\cos x = 1$, is

(a) 5° (b) 7.5° (c) 10° (d) 15°

46. Number of solution satisfying the equation, $\tan^2 2x = 2\tan 2x \cdot \tan 3x + 1$ in $[0, 2\pi]$ is

(a) 0 (b) 1 (c) 2 (d) 4

INTEGER TYPE QUESTIONS

47. If the equation $p\sin\theta + \cos 2\theta = 2p - 7$ possesses a solution, then find the number of integral value(s) of p.

48. If $4\sin^2 x - 8\sin x + 3 \le 0$, $0 \le x \le \pi$, then find the number of integral solution for x.

49. Solve for $x : 3^{\sin 2x + 2\cos^2 x} + 3^{1 - \sin 2x + 2\sin^2 x} = 28$. Then find $[x]$ where x is least positive solutions of given equation. {[ ] denotes the G.I.F}.

50. Given that $3\sin x + 4\cos x = 5$ where $x \in (0, \pi/2)$. Find the value of $2\sin x + \cos x + 4\tan x$.

Parikshit (JEE Advanced Level)

SINGLE CORRECT TYPE QUESTIONS

1. Find least value of $2^{\sin x}+2^{\cos x}\ \forall x\in R$

(a) $2^{1-\frac{1}{\sqrt{2}}}$ (b) $2^{1+\frac{1}{\sqrt{2}}}$

(c) $3^{1+\frac{1}{\sqrt{2}}}$ (d) $2^{1+\sqrt{2}}$

2. Let α, β be two distinct values of x lying in $[0,\pi]$ for which $\sqrt{5}\sin x, 10\sin x, 10(4\sin^2 x+1)$ are three consecutive terms of G.P. Then sum of minimum and maximum value of $|\alpha-\beta|$ is

(a) $\frac{\pi}{5}$ (b) $\frac{3\pi}{5}$ (c) $\frac{7\pi}{5}$ (d) π

3. Let $[x]$ be the G.I.F., then the equation

$\sin x=[1+\sin x]+[1-\cos x]$ has

(a) Exactly one solution $\forall x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

(b) Exactly one solution in $\left[\frac{\pi}{2},\pi\right]$

(c) Exactly one solution $\forall x\in R$

(d) No solution

4. Let $S=\left\{\theta\in[0,2\pi]:8^{2\sin^2\theta}+8^{2\cos^2\theta}=16\right\}$.

Then $n(S)+\sum\limits_{\theta\in S}\left(\sec\left(\frac{\pi}{4}+2\theta\right)\operatorname{cosec}\left(\frac{\pi}{4}+2\theta\right)\right)$ is equal to

(a) 0 (b) -2 (c) -4 (d) 12

5. If $4\sin\alpha\times\cos\beta+2\cos\beta-\sqrt{3}=2\sqrt{3}\sin\alpha$, where $\alpha,\beta\in[0,2\pi]$ and $L=\alpha-\beta$ then

(a) Maximum value of L is $\frac{11\pi}{6}$

(b) Minimum value of L is $-\frac{5\pi}{6}$

(c) Maximum value of L is $\frac{5\pi}{3}$

(d) Minimum value of L is $\frac{-11\pi}{6}$

6. Find the general solution of the equation

$$\frac{1+\sin x+\sin^2 x+\sin^3 x+\ldots\ldots+\sin^n x+\ldots\infty}{1-\sin x+\sin^2 x-\sin^3 x+\ldots(-1)^n x+\ldots\infty}=\frac{4}{1+\tan^2 x}$$

where $x\neq k\pi+\frac{\pi}{2}, k\in I$.

(a) $(-1)^n\left(\frac{\pi}{3}\right)+n\pi$ (b) $(-1)^n\left(\frac{\pi}{6}\right)+n\pi$

(c) $(-1)^{n+1}\left(\frac{\pi}{3}\right)+n\pi$ (d) $(-1)^{n-1}\left(\frac{\pi}{3}\right)+n\pi$

7. For $n\in Z$, the general solution of $(\sqrt{3}-1)\sin\theta\div(\sqrt{3}\div1)\cos\theta=2$ is $(n\in Z)$

(a) $\theta=2n\pi\pm\frac{\pi}{4}+\frac{\pi}{12}$ (b) $\theta=n\pi+(-1)^n\frac{\pi}{4}+\frac{\pi}{12}$

(c) $\theta=2n\pi\pm\frac{\pi}{4}$ (d) $\theta=n\pi+(-1)^n\frac{\pi}{4}-\frac{\pi}{12}$

8. The minimum value of the function

$$f(x)=\frac{\sin x}{\sqrt{1-\cos^2 x}}+\frac{\cos x}{\sqrt{1-\sin^2 x}}+\frac{\tan x}{\sqrt{\sec^2 x-1}}+\frac{\cot x}{\sqrt{\operatorname{cosec}^2 x-1}}$$

as x varies over all numbers in the largest possible domain of $f(x)$ is

(a) 4 (b) -2

(c) 0 (d) 2

MULTIPLE CORRECT TYPE QUESTIONS

9. Let α and β be non-zero real numbers such that $2(\cos\beta-\cos\alpha)+\cos\alpha\cos\beta=1$. Then which of the following is/are true?

(a) $\tan\left(\frac{\alpha}{2}\right)+\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

(b) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)+\tan\left(\frac{\beta}{2}\right)=0$

(c) $\tan\left(\frac{\alpha}{2}\right)-\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$

(d) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right)-\tan\left(\frac{\beta}{2}\right)=0$

10. Solve for x: $|\cos x|^{\sin^2 x-\frac{3}{2}\sin x+\frac{1}{2}}=1$

(a) $n\pi$ (b) $2n\pi$

(c) $n\pi+(-1)^n\frac{\pi}{6}$ (d) $2n\pi\pm\frac{\pi}{6}$

11. Solve the system of equations,

$$\sin x\sin y=\frac{\sqrt{3}}{4},\quad \cos x\cos y=\frac{\sqrt{3}}{4}$$

(a) $(2n+k)\frac{\pi}{2}+\frac{\pi}{3}\quad n,k\in I$

(b) $(2n+k)\frac{\pi}{2}+\frac{\pi}{6}\quad n,k\in I$

(c) $(k-2n)\frac{\pi}{2}+\frac{\pi}{3}\quad n,k\in I$

(d) $(k-2n)\frac{\pi}{2}+\frac{\pi}{6}\quad n,k\in I$

Credit - Akshat

12. For $0<\theta<\frac{\pi}{2}$, the solution(s) of $\sum_{m=1}^{6}\operatorname{cosec}\left(\theta+\frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta+\frac{m\pi}{4}\right)=4\sqrt{2}$ is (are)

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{12}$ (d) $\frac{5\pi}{12}$

13. Let $2\sin x+3\cos y=3$ and $3\sin y+2\cos x=4$ then

(a) $x+y=(4n+1)\pi/2,\ n\in I$

(b) $x+y=(2n+1)\pi/2,\ n\in I$

(c) x and y can be the two non-right angles of a 3-4-5 triangle with $x>y$

(d) x and y can be the two non-right angles of a 3-4-5 triangle with $y>x$

14. Which of the following set of value of 'x' satisfies the equation $2^{(3\sin^2 x-3\sin x+\cos^2 x)}+2^{(2-2\sin^2 x+3\sin x)}=9$.

(a) $x=n\pi\pm\frac{\pi}{6}, n\in I$ (b) $x=\frac{11\pi}{6}$

(c) $x=\frac{9\pi}{2}$ (d) $x=2n\pi+\frac{\pi}{2}, n\in I$

15. The value of θ in (0, 90°) satisfying equation

$$\frac{4\operatorname{cosec}\theta}{\sqrt{5}+1}+\frac{4\sqrt{5}\sec\theta}{\sqrt{10-2\sqrt{5}}}=8$$

(a) 18° (b) 54° (c) 36° (d) 72°

16. The value of 't' which satisfies $(t-[|\sin x|])!=3!\,5!\,7!$ is/are where $[\cdot]$ is GIF

(a) 9 (b) 10 (c) 11 (d) 12

17. Which of the following is/are correct.

(a) $(\tan x)^{\ln(\sin x)}>(\cot x)^{\ln(\sin x)}, \forall x\in(0,\pi/4)$

(b) $4^{\ln(\operatorname{cosec} x)}<5^{\ln(\operatorname{cosec} x)}, \forall x\in(0,\pi/2)$

(c) $(1/2)^{\ln(\cos x)}<(1/3)^{\ln(\cos x)}, \forall x\in(0,\pi/2)$

(d) $2^{\ln(\tan x)}>2^{\ln(\sin x)}, \forall x\in(0,\pi/2)$

18. The equation $2\sin\frac{x}{2}.\cos^2 x+\sin^2 x=2\sin\frac{x}{2}.\sin^2 x+\cos^2 x$ has a root for which

(a) $\sin 2x=1$

(b) $\sin 2x=-1$

(c) $\cos x=\frac{1}{2}$

(d) $\cos 2x=-\frac{1}{2}$

19. $5\sin^2 x+\sqrt{3}\sin x\cos x+6\cos^2 x=5$ if

(a) $\tan x=-1/\sqrt{3}$ (b) $\sin x=0$

(c) $x=n\pi+\pi/2,\ n\in I$ (d) $x=n\pi+\pi/6,\ n\in I$

20. The expression $\cos 3\theta+\sin 3\theta+(2\sin 2\theta-3)(\sin\theta-\cos\theta)$ is positive for all θ in

(a) $\left(2n\pi-\frac{3\pi}{4}, 2n\pi+\frac{\pi}{4}\right), n\in Z$

(b) $\left(2n\pi-\frac{\pi}{4}, 2n\pi+\frac{\pi}{6}\right), n\in Z$

(c) $\left(2n\pi-\frac{\pi}{3}, 2n\pi+\frac{\pi}{3}\right), n\in Z$

(d) $\left(2n\pi-\frac{\pi}{4}, 2n\pi+\frac{3\pi}{4}\right), n\in Z$

21. The value of x, between 0 and 2π, satisfying the equation $\cos 3x+\cos 2x=\sin\frac{3x}{2}+\sin\frac{x}{2}$

(a) $\frac{\pi}{7}$ (b) $\frac{5\pi}{7}$

(c) $\frac{9\pi}{7}$ (d) $\frac{13\pi}{7}$

22. Let $f(\theta)=\left(\cos\theta-\cos\frac{\pi}{8}\right)\left(\cos\theta-\cos\frac{3\pi}{8}\right)\left(\cos\theta-\cos\frac{5\pi}{8}\right)\left(\cos\theta-\cos\frac{7\pi}{8}\right)$ then

(a) Maximum value of $f(\theta)\ \forall\ \theta\in R$ is 1/4.

(b) Maximum value of $f(\theta)\ \forall\ \theta\in R$ is 1/8.

(c) $f(0)=\frac{1}{8}$.

(d) Number of principle solutions of $f(\theta)=0$ is 8.

23. The equation $2\sin^3\theta+(2\lambda-3)\sin^2\theta-(3\lambda+2)\sin\theta-2\lambda=0$ has exactly three roots in $(0, 2\pi)$, then λ can be equal to

(a) 0 (b) 1/2 (c) 1 (d) –1

24. The equation $2\sin^2\left(\frac{\pi}{2}\cos^2 x\right)=1-\cos(\pi\sin 2x)$ is satisfied by

(a) $x=(2n+1)\frac{\pi}{2}, n\in Z$ (b) $\tan x=\frac{1}{2}$

(c) $\tan x=-\frac{1}{2}$ (d) $x=\frac{n\pi}{2}, n\in Z$

25. If $(\sin\alpha)x^2-2x+b\geq 2$ for all the real values of $x\leq 1$ and $\alpha\in(0,\pi/2)\cup(\pi/2,\pi)$, then the possible real values of b is/are

(a) 2 (b) 3

(c) 4 (d) 5

26. The value of x in $(0, \pi/2)$ satisfying $\frac{\sqrt{3}-1}{\sin x}+\frac{\sqrt{3}+1}{\cos x}=4\sqrt{2}$ is/are

(a) $\frac{\pi}{12}$ (b) $\frac{5\pi}{12}$

(c) $\frac{7\pi}{24}$ (d) $\frac{11\pi}{36}$

27. Let the solution set of the equations

$\sec x + \tan x = \sqrt{3}$

$1 + \sin x = \sqrt{3}\cos x$

$\sec 4x + \sec 2x = 2$

$\cos 2x + \cos 4x = 2\cos 2x \cos 4x$

in $[0, 2\pi]$ be S_1, S_2, S_3, S_4 respectively. Then

(*a*) $S_1 = S_2$ (*b*) $S_1 \subset S_2$
(*c*) $S_3 = S_4$ (*d*) $S_3 \subset S_4$

COMPREHENSION BASED QUESTIONS

Comprehension – (No. 28 to 30): Equation of the form P (sin$x$ ± cos$x$, sin$x$ cos$x$)=0 where $P(y, z)$ is a polynomial, which can be solved by substituting : $\cos x \pm \sin x = t$; $1 \pm 2\sin x\cos x = t^2$ then reduce the given equation into $P\left(t, \frac{t^2-1}{2}\right) = 0$.

28. General solution of $\sin x + \cos x = 1 + \sin x \cos x$ is

(*a*) $\frac{\pi}{2} + 2\pi n$ & $2n\pi$ (*b*) $\frac{\pi}{4} + 2\pi n$ & $(2n + 1)\pi$
(*c*) $2n\pi$ (*d*) None of these

29. If $(\cos x - \sin x)\left(2\tan x + \frac{1}{\cos x}\right) + 2 = 0$, then x is

(*a*) $n\pi \pm \frac{\pi}{3}$ (*b*) $2n\pi \pm \frac{\pi}{3}$
(*c*) $n\pi \pm \frac{\pi}{6}$ (*d*) $2n\pi \pm \frac{\pi}{6}$

30. $\sin^4 x + \cos^4 x = \sin x \cos x$ then x is–

(*a*) $(6n + 1)\frac{\pi}{6}$ (*b*) $n\pi$
(*c*) $(4n + 1)\frac{\pi}{4}$ (*d*) None of these

Comprehension (Q. 31 to 33): If α is a root of the equation $2\sin x + \sin 2x = 1 + \cos x$, β is a root of the equation $3\cos^3 x - 10\cos x + 3 = 0$ and γ is a root of the equation $1 + \sin x = \cos x(1 + 2\sin x)$, $0 \le \alpha, \beta, \gamma \le \pi/2$, then on the basis of above information answer the following questions

31. $\cos\alpha \times \cos\beta \times \cos\gamma$ is equal to

(*a*) $\frac{1}{2\sqrt{3}}$ (*b*) $\frac{1}{\sqrt{3}}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{\sqrt{2}}$

32. $\sin\alpha + \sin\beta + \sin\gamma$ is equal to

(*a*) $\frac{14-3\sqrt{2}}{6\sqrt{2}}$ (*b*) $\frac{5}{6}$ (*c*) $\frac{3+4\sqrt{2}}{6}$ (*d*) $\frac{1+\sqrt{2}}{2}$

33. $\sin(\alpha - \beta)$ is equal to

(*a*) 1 (*b*) 0 (*c*) $\frac{1-2\sqrt{6}}{6}$ (*d*) $\frac{\sqrt{3}-2\sqrt{2}}{6}$

Comprehension (34 to 36): Consider the cubic equation $x^3 - (1 + \cos\theta + \sin\theta)x^2 + (\cos\theta\sin\theta + \cos\theta + \sin\theta)x - \sin\theta\cos\theta = 0$ where x_1, x_2, x_3 are roots.

34. The value of $x_1^2 + x_2^2 + x_3^2$ is

(*a*) 1 (*b*) $2\cos\theta$ (*c*) $2\sin\theta$ (*d*) 2

35. Number of values of θ in $[0, 2\pi]$ for which at least two roots are equal

(*a*) 3 (*b*) 4 (*c*) 5 (*d*) 6

36. Greatest possible difference between two of roots if $\theta \in [0, 2\pi]$ is

(*a*) $2\sqrt{2}$ (*b*) 1 (*c*) $\sqrt{2}$ (*d*) 2

Comprehension (37 to 38): Consider the equation $\sec\theta + \operatorname{cosec}\theta = a$, $\theta \in (0, 2\pi) - \{\pi/2, \pi, 3\pi/2\}$

37. If the equation has four distinct real roots, then

(*a*) $|a| > 2\sqrt{2}$ (*b*) $|a| < 2\sqrt{2}$
(*c*) $a \ge -2\sqrt{2}$ (*d*) $a < -2\sqrt{2}$

38. If the equation has two distinct real roots, then

(*a*) $|a| \ge 2\sqrt{2}$ (*b*) $a < 2\sqrt{2}$
(*c*) $|a| < 2\sqrt{2}$ (*d*) $|a| \le -2\sqrt{2}$

MATCH THE COLUMN TYPE QUESTIONS

39. Match the following columns

Column-I		Column-II	
A.	If $\cos 3x \cdot \cos^3 x = -\sin 3x . \sin^3 x$ then x is	p.	$\left(n\pi \pm \frac{\pi}{3}\right), n \in I$
B.	$\sin 3\alpha + 2\sin\alpha\ (\cos 2x - \cos 2\alpha) = 0$, then α is where $\alpha \ne n\pi$.	q.	$\left(n\pi + \frac{\pi}{4}\right), n \in I$
C.	If $\lvert 2\tan x - 1\rvert + \lvert 2\cot x - 1\rvert = 2$ then x is	r.	$\left(\frac{n\pi}{4} + \frac{\pi}{8}\right), n \in I$
D.	If $\sin^{10}x + \cos^{10}x = \frac{29}{16}\cos^4(2x)$, then x is	s.	$\left(\frac{n\pi}{2} \pm \frac{\pi}{4}\right), n \in I$

(*a*) A → (q,s); B → (p); C → (q); D → (r)
(*b*) A → (q); B → (r); C → (q,s); D → (p)
(*c*) A → (r); B → (p); C → (q); D → (q,s)
(*d*) A → (p); B → (r); C → (q,s); D → (q)

40. Match the following columns

Column-I		Column-II	
A.	$(2\sin x - \cos x)(1 + \cos x) = \sin^2 x$	p.	$\sin x = 1/2$
B.	$1 + \sin 2x = \cos x + \sin x$	q.	$\tan x = -1$
C.	$4x^4 + x^6 + \sin^2 5x = 0$	r.	$x = 0$
D.	$\tan x = 1/\sqrt{3}$	s.	$x = 19\pi/6$

(*a*) A → (p); B → (q); C → (r); D → (s)
(*b*) A → (s); B → (q); C → (r); D → (p)
(*c*) A → (r); B → (q); C → (p); D → (s)
(*d*) A → (p); B → (s); C → (r); D → (q)

NUMERICAL TYPE QUESTIONS

41. Find the number of solutions of the equation $(1 - \sin x)^3 - (\cos x - 1)^3 - (\sin x)^3 = (2 - \cos x - 2\sin x)^3$ in $[0, 2\pi]$.

42. Find the number of values of y in $[-2\pi, 2\pi]$ for which $|\sin(2x)| + |\cos(2x)| = |\sin(y)|$

43. Find the number of solutions of the equation $2\sin^2 x + \sin^2 2x = 2$; $\sin 2x + \cos 2x = \tan x$ in $[0, 4\pi]$ satisfying the condition $2\cos^2 x + \sin x \le 2$.

44. Number of roots of the equation $\cos^2 x + \frac{\sqrt{3}+1}{2}\sin x - \frac{\sqrt{3}}{4} - 1 = 0$ which lie in the interval $[-\pi, \pi]$ is

45. If $2\tan^2 x - 5\sec x - 1 = 0$ has 7 different roots in $\left[0, \frac{n\pi}{2}\right]$, $n \in N$. Find the greatest value of n.

46. The number of values of x satisfying the equation, $2^{\tan^2\left(x-\frac{\pi}{4}\right)} - 2(0.25)^{\frac{\sin^2\left(x-\frac{\pi}{4}\right)}{\cos 2x}} + 1 = 0, x \in [0, 2\pi]$ are

47. The number of ordered pair (x, y) satisfying the equation $\sin x + \sin y = \sin(x + y)$ and $|x| + |y| = 1$ are.

48. If the sum of all the roots of the equation, $\sin\sqrt{x} = -1$, which are less than $100\pi^2$ is $\frac{P\pi^2}{4}$ then value of P is

49. If the sum of all values of $\theta, 0 \le \theta \le 3\pi$ satisfying the equation $(8\cos 4\theta - 3)(\cot\theta + \tan\theta - 2)(\cot\theta + \tan\theta + 2) = 12$ is $3p\pi$, then p is equal to

50. The complete set of values of x satisfying $\frac{2\sin 6x}{\sin x - 1} < 0$ and $\sec^2 x - 2\sqrt{2}\tan x \le 0$ in $\left(0, \frac{\pi}{2}\right)$ is $[a, b) \cup (c, d]$, then find the value of $\left(\frac{cd}{ab}\right)$.

51. The number of solutions of $\sin^3 x\cos x + \sin^2 x\cos^2 x + \sin x\cos^3 x = 1$ in $[0, 2\pi]$ is

52. Find the number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$

53. If $\cos^2\theta - 2\cos\theta - 1 = 0$ is to be satisfied for exactly 4 distinct values of $\theta \in [0, n\pi]$, then find the least value of n.

54. If A and B are acute positive angles satisfying the equations $3\sin^2 A + 2\sin^2 B = 1$ and $3\sin 2A - 2\sin 2B = 0$, then find $\sin(A + 2B)$.

"POMODORO RELAXATION TECHNIQUE"

- After every 60-90 minutes of studying, take a 5-minute relaxation break using the Pomodoro Technique. During this break, practice deep breathing, do light stretches, or listen to calming music.
- This method balances intense focus with relaxation, preventing burnout and boosting productivity.
- Set a timer for 60 minutes of focused study. When the timer goes off, take 5 minutes to relax by doing a few yoga stretches or sipping tea while listening to soft music.

PYQ's (Past Year Questions)

INTRODUCTION, PRINCIPAL SOLUTION OF A TRIGONOMETRIC EQUATIONS, GENERAL SOLUTION OF EQUATIONS

1. Let $f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3\pi + \theta)\right) - 2(1 - \sin^2 2\theta)$ and $S = \left\{\theta \in [0, \pi] : f'(\theta) = -\frac{\sqrt{3}}{2}\right\}$. If $4\beta = \Sigma_{\theta \in S}\theta$, then $f(\beta)$ is equal to **[29 Jan, 2023 (Shift-I)]**

   (*a*) $\frac{11}{8}$ (*b*) $\frac{5}{4}$ (*c*) $\frac{9}{8}$ (*d*) $\frac{3}{2}$

2. If the solution of the equation $\log_{\cos x}\cot x + 4\log_{\sin x}\tan x = 1, x \in \left(0, \frac{\pi}{2}\right)$, is $\sin^{-1}\left(\frac{\alpha + \sqrt{\beta}}{2}\right)$, where α, β are integers, then $\alpha + \beta$ is equal to: **[30 Jan, 2023 (Shift-I)]**

   (*a*) 3 (*b*) 5 (*c*) 6 (*d*) 4

3. Let $S = \left\{\theta \in \left(0, \frac{\pi}{2}\right) : \sum_{m=1}^{9} \sec\left(\theta + (m-1)\frac{\pi}{6}\right) \sec\left(\theta + \frac{m\pi}{6}\right) = -\frac{8}{\sqrt{3}}\right\}$. Then **[27 July, 2022 (Shift-II)]**

   (*a*) $S = \left\{\frac{\pi}{12}\right\}$ (*b*) $S = \left\{\frac{2\pi}{3}\right\}$

   (*c*) $\sum_{\theta \in S}\theta = \left\{\frac{\pi}{12}\right\}$ (*d*) $\sum_{\theta \in S}\theta = \left\{\frac{3\pi}{4}\right\}$

4. If the sum of solutions of the system of equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is $k\pi$, then k is equal to _____ **[26 July, 2022 (Shift-II)]**

5. Match the following columns:

	Column-I		Column-II
A.	$\left\{x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right] : \cos x + \sin x = 1\right\}$	p.	Has two elements
B.	$\left\{x \in \left[-\frac{5\pi}{18}, \frac{5\pi}{18}\right] : \sqrt{3}\tan 3x = 1\right\}$	q.	Has three elements
C.	$\left\{x \in \left[-\frac{6\pi}{5}, \frac{6\pi}{5}\right] : 2\cos(2x) = \sqrt{3}\right\}$	r.	Has four elements
D.	$\left\{x \in \left[-\frac{7\pi}{4}, \frac{7\pi}{4}\right] : \sin x - \cos x = 1\right\}$	s.	Has five elements
		t.	Has six elements

The correct option is: **[JEE Adv, 2022]**

(*a*) A → p; B → s; C → p; D → s

(*b*) A → p; B → p; C → t; D → r

(*c*) A → q; B → p; C → t; D → s

(*d*) A → q; B → s; C → p; D → r

6. The sum of all values of x in $[0, 2\pi]$, for which $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$, is equal to: **[25 July, 2021 (Shift-I)]**

   (*a*) 11π (*b*) 9π (*c*) 8π (*d*) 12π

7. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval **[2 Sep, 2020 (Shift II)]**

   (*a*) $\left[-1, -\frac{1}{2}\right]$ (*b*) $\left[-\frac{3}{2}, -\frac{5}{4}\right]$

   (*c*) $\left(-\frac{1}{2}, -\frac{1}{4}\right]$ (*d*) $\left(-\frac{5}{4}, -1\right)$

8. Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha\sin x = 2\alpha - 7$ has a solution. Then S is equal to: **[12 April, 2019 (Shift-II)]**

   (*a*) [2, 6] (*b*) [3, 7] (*c*) R (*d*) [1, 4]

9. If $0 \le x < \frac{\pi}{2}$, then the number of values of x for which $\sin x - \sin 2x + \sin 3x = 0$, is **[9 Jan, 2019 (Shift-II)]**

   (*a*) 2 (*b*) 1 (*c*) 3 (*d*) 4

Comprehension for 10 - 11

Answer the following by appropriately matching the lists based on the information given in the paragraph

Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order: **[JEE Adv, 2019]**

$X = \{x : f(x) = 0\}$, $Y = \{x : f'(x) = 0\}$, $Z = \{x : g(x) = 0\}$, $W = \{x : g'(x) = 0\}$,

List-I contains the sets X, Y, Z and W. List-II contains some information regarding these sets.

List-I		List-II	
(I)	X	(P)	$\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II)	Y	(Q)	an arithmetic progression
(III)	Z	(R)	NOT an arithmetic progression
(IV)	W	(S)	$\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
		(T)	$\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
		(U)	$\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

10. Which of the following is the only CORRECT combination ?

(*a*) (II), (R), (S)   (*b*) (I), (P), (R)

(*c*) (II), (Q), (T)   (*d*) (I), (Q), (U)

11. Which of the following is the only CORRECT combination?

(*a*) (IV), (Q), (T)   (*b*) (IV), (P), (R), (S)

(*c*) (III), (R), (U)   (*d*) (III), (P), (Q), (U)

TRIGONOMETRIC INEQUALITIES AND SYSTEM OF INEQUALITY

12. Let the set of all $a \in R$ such that the equation $\cos 2x + a \sin x = 2a - 7$ has a solution be $[p, q]$ and $r = \tan 9° - \tan 27° - \frac{1}{\cot 63°} + \tan 81°$, then pqr is equal to ____.

[27 Jan, 2024 (Shift-I)]

13. Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is **[9 April, 2019 (Shift-I)]**

(*a*) $\frac{13\pi}{6}$ (*b*) π

(*c*) 2π (*d*) $\frac{5\pi}{3}$

14. All the pairs (x, y) that satisfy the inequality

$2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot \frac{1}{4^{\sin^2 y}} \le 1$ also satisfy the equation.

[10 April, 2019 (Shift-I)]

(*a*) $\sin x = |\sin y|$

(*b*) $\sin x = 2 \sin y$

(*c*) $2|\sin x| = 3\sin y$

(*d*) $2\sin x = \sin y$

SOLVING SYSTEM OF TRIGONOMETRIC EQUATIONS

15. If $2\tan^2\theta - 5\sec\theta = 1$ has exactly 7 solutions in the interval, $\left[0, \frac{n\pi}{2}\right]$ for the least value of $n \in N$ then $\sum_{k=1}^{n} \frac{k}{2^k}$ is equal to

[27 Jan, 2024 (Shift-II)]

(*a*) $\frac{1}{2^{15}}(2^{14} - 14)$ (*b*) $\frac{1}{2^{14}}(2^{15} - 15)$

(*c*) $1 - \frac{15}{2^{13}}$ (*d*) $\frac{1}{2^{13}}(2^{14} - 15)$

16. If $\alpha, -\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ is the solution of $4\cos\theta + 5\sin\theta = 1$, then the value of $\tan\alpha$ is **[29 Jan, 2024 (Shift-I)]**

(*a*) $\frac{10 - \sqrt{10}}{6}$ (*b*) $\frac{10 - \sqrt{10}}{12}$

(*c*) $\frac{\sqrt{10} - 10}{12}$ (*d*) $\frac{\sqrt{10} - 10}{6}$

17. The sum of the solutions $x \in R$ of the equation $\frac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$ is **[29 Jan, 2024 (Shift-II)]**

(*a*) 0   (*b*) 1   (*c*) –1   (*d*) 3

18. If $2\sin^3 x + \sin 2x \cos x + 4\sin x - 4 = 0$ has exactly 3 solutions in the interval $\left[0, \frac{n\pi}{2}\right], n \in N$, then the roots of the equation $x^2 + nx + (n - 3) = 0$ belong to: **[30 Jan, 2024 (Shift-I)]**

(*a*) $(0, \infty)$ (*b*) $(-\infty, 0)$

(*c*) $\left(-\frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}\right)$ (*d*) Z

19. For $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, let $3 \sin(\alpha + \beta) = 2 \sin(\alpha - \beta)$ and a real number k be such that $\tan \alpha = k \tan \beta$. Then the value of k is equal to:

[30 Jan, 2024 (Shift-II)]

(*a*) $-\frac{2}{3}$ (*b*) –5

(*c*) $\frac{2}{3}$ (*d*) 5

20. The number of solutions of the equation $4 \sin^2 x - 4 \cos^3 x + 9 - 4 \cos x = 0$; $x \in [-2\pi, 2\pi]$ is: **[1 Feb, 2024 (Shift-II)]**

(*a*) 1   (*b*) 3   (*c*) 2   (*d*) 0

21. Suppose $\theta \in \left[0, \frac{\pi}{4}\right]$ is a solution of $4\cos\theta - 3\sin\theta = 1$. Then $\cos\theta$ is equal to: **[05 April, 2024 (Shift-I)]**

(*a*) $\frac{4}{(3\sqrt{6} - 2)}$ (*b*) $\frac{6 - \sqrt{6}}{(3\sqrt{6} - 2)}$

(*c*) $\frac{6 + \sqrt{6}}{(3\sqrt{6} + 2)}$ (*d*) $\frac{4}{(3\sqrt{6} + 2)}$

22. The number of solutions of $\sin^2 x + (2 + 2x - x^2)\sin x - 3(x - 1)^2 = 0$, where, where $-\pi \le x \le \pi$, is

[05 April, 2024 (Shift-II)]

23. Let $S = \left\{x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) : 9^{\tan^2 x} + 9^{1 - \tan^2 x} = 10\right\}$ and $\beta = \sum_{x \in S} \tan^2\left(\frac{x}{3}\right)$, then $\frac{1}{6}(\beta - 14)^2$ is equal to

[10 April, 2023 (Shift-II)]

(*a*) 32   (*b*) 8

(*c*) 64   (*d*) 16

24. The number of solutions of the equation $\sin x = \cos^2 x$ in the interval (0, 10) is **[29 June, 2022 (Shift-II)]**

25. The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14 \operatorname{cosec}^2 x - 2\sin^2 x = 21 - 4 \cos^2 x$ holds, is______

[25 June, 2022 (Shift-I)]

26. Let $S = \left[-\pi, \frac{\pi}{2}\right) - \left\{-\frac{\pi}{2}, -\frac{\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}\right\}$. Then the number of elements in the set **[28 July, 2022 (Shift-II)]**

$A = \left\{\theta \in S : \tan\theta\left(1 + \sqrt{5}\tan(2\theta)\right) = \sqrt{5} - \tan(2\theta)\right\}$ is

27. The sum of solutions of the equation

$\frac{\cos x}{1+\sin x} = |\tan 2x|, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is:

[26 Aug, 2021 (Shift-I)]

(*a*) $-\frac{\pi}{15}$ (*b*) $\frac{\pi}{10}$

(*c*) $-\frac{7\pi}{30}$ (*d*) $-\frac{11\pi}{30}$

28. If $\sin\theta + \cos\theta = \frac{1}{2}$ then $16(\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$ is equal to **[27 July, 2021 (Shift-I)]**

(*a*) 27 (*b*) –23

(*c*) –27 (*d*) 23

29. The number of solutions of the equation

$32^{\tan^2 x} + 32^{\sec^2 x} = 81, 0 \le x \le \frac{\pi}{4}$ is:

[31 Aug, 2021 (Shift-II)

(*a*) 0 (*b*) 2 (*c*) 1 (*d*) 3

30. The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to:

[16 March, 2021 (Shift-I)]

(*a*) 3

(*b*) 4

(*c*) 8

(*d*) 2

31. The number of integral values of '*k*' for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, $k \in \mathbb{R}$ is

[26 Feb, 2021 (Shift-I)]

32. The number of solutions of the equation

$|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is_____

[18 March, 2021 (Shift-I)]

33. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta; \alpha, \beta \in [0, \pi]$, then $\cos(\alpha+\beta) - \cos(\alpha-\beta)$ is equal to:

[12 Jan, 2019 (Shift-II)]

(*a*) 0

(*b*) –1

(*c*) $\sqrt{2}$

(*d*) $-\sqrt{2}$

PW Challengers

1. Let *a*, *x* and *y* be real numbers with $x, y \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ such that $x^3 + \sin x = 2a$, $4y^3 + \sin y\cos y = -a$

Find the value of $\cos(x + 2y)$.

(*a*) 0 (*b*) 2 (*c*) 3 (*d*) 1

2. Which of the following options cannot be a solution for the equation

$$\cos(\pi\cdot 3^x) - 2\cos^2(\pi\cdot 3^x) + 2\cos(4\pi\cdot 3^x) - \cos(7\pi\cdot 3^x)$$
$$= \sin(\pi\cdot 3^x) + 2\sin^2(\pi\cdot 3^x) - 2\sin(4\pi\cdot 3^x)$$
$$+ 2\sin(\pi\cdot 3^{x+1}) - \sin(7\pi\cdot 3^x)?$$

(*a*) $x = \log_3\left(\frac{2k}{3} - \frac{1}{6}\right); k \in N$

(*b*) $x = \log_3\left(\frac{n}{2}\right); n \in N$

(*c*) $x = \log_3\left(\frac{1}{8} + \frac{m}{2}\right); m \in W$

(*d*) $x = \log_3\left(\frac{p}{4} + 1\right); p \in N$

3. Consider the equation $\int_0^x (t^2 - 8t + 13)\,dt = x\sin(a/x)$

The number of real values of *x* for which the equation has solution is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) Infinite

4. The side of triangle *ABC* are in A.P. (order being *a*, *b*, *c*) and satisfy $\frac{2!}{1!9!} + \frac{2!}{3!7!} + \frac{1}{5!5!} = \frac{8^a}{(2b)!}$, then the value of $\cos A + \cos B$ is

(*a*) $\frac{12}{7}$ (*b*) $\frac{13}{7}$ (*c*) $\frac{11}{7}$ (*d*) $\frac{10}{7}$

5. The solution of the equation $9\cos^{12} x + \cos^2 2x + 1 = 6\cos^6 x\cos 2x + 6\cos^6 x - 2\cos 2x$ is/are

(*a*) $x = n\pi + \frac{\pi}{2}, n \in I$

(*b*) $x = n\pi + \cos^{-1}\left(\sqrt[4]{\frac{2}{3}}\right), n \in I$

(*c*) $x = n\pi - \cos^{-1}\left(\sqrt[4]{\frac{2}{3}}\right), n \in I$

(*d*) None of the above

6. If in a ΔABC, $\sin^4 A + \sin^4 B + \sin^4 C = \sin^2 B \sin^2 C + 2\sin^2 C \sin^2 A + 2\sin^2 A \sin^2 B$, then $A =$

(*a*) $\frac{\pi}{6}, \frac{5\pi}{6}$ (*b*) $\frac{\pi}{3}, \frac{5\pi}{6}$

(*c*) $\frac{5\pi}{6}, \frac{2\pi}{3}$ (*d*) None of these

7. Consider the equation $(\cos\theta + 2)(\cos\theta + 3) \ldots (\cos\theta + (n+1)) = n!$, where $\theta \in [0, 10\pi]$. Then

(*a*) sum of the roots is 24π

(*b*) sum of the roots is 25π

(*c*) product of the roots is less than $2023\pi^5$

(*d*) product of the roots is greater than $2023\pi^5$

Paragraph for Question Number 8 to 10:

Let $a = \sin x + \sin 2x + \sin 3x$, $b = \cos x + \cos 2x + \cos 3x$ and $f(x) = (\cos^2 x + \cos^2 2x + \cos^2 3x) - (\sin^2 x + \sin^2 2x + \sin^2 3x)$.

On the basis of above information, answer the following questions:

8. The number of values of 'x' in $[0, 3\pi]$ for which $a = b$ is

(*a*) 3 (*b*) 5 (*c*) 9 (*d*) 10

9. If a, b are the roots of quadratic equation $y^2 - Ay + B = 0$, then general solution of x in the equation $A^2 - 2B = 0$ is

(*a*) $n\pi \pm \frac{\pi}{4}; n \in I$

(*b*) $2n\pi \pm \frac{2\pi}{3}; n \in I$

(*c*) $n\pi + (-1)^n \frac{\pi}{6}; n \in I$

(*d*) $2n\pi \pm \frac{\pi}{3}; n \in I$

10. The value of $(\sec^2 2x)\left(f\left(\frac{x}{2}\right)\right)^2$ is

(*a*) $\frac{a}{b}$ (*b*) $\frac{a^2}{b^2}$

(*c*) $a^2 + b^2$ (*d*) $b^2 - a^2$

ANSWER KEY

CONCEPT APPLICATION

1. $x = \phi$ 2. (*b*) 3. (*c*) 4. $(2n+1)\pi, n \in I$ 5. $2n\pi + \frac{\pi}{2}$ 6. $\pi + 2\pi n, \frac{\pi}{2} + 2\pi n$

7. (*d*) 8. $n\pi + \frac{\pi}{3}$ 9. (No solution) 10. (*a*)

11. $\frac{-n\pi}{2} + \frac{\pi}{6}, n \in I$ 12. (*c*) 13. $2m\pi, (2n+1)\frac{\pi}{4}, m, n \in I$ 14. [9] 15. $n\pi \pm \frac{\pi}{3}$

16. $\frac{n\pi}{4} \pm \frac{\pi}{8}, n \in I$ 17. $\frac{-\pi}{2} + 2n\pi, n \in I$ 18. (*b*) 19. (No solution)

20. $(n\pi, n \in I)$ 21. (*a*) 22. $n\pi + \frac{\pi}{3}$ 23. $x \in \left[\frac{-\pi}{4}, \frac{\pi}{6}\right] \cup \left[\frac{\pi}{4}, \frac{5\pi}{6}\right]$ 24. $\left(n\pi + \frac{\pi}{8}, n\pi + \frac{\pi}{4}\right), n \in I$

25. $\underset{n \in I}{U}\left[n\pi + \frac{\pi}{6}, n\pi + \frac{\pi}{2}\right)$ 26. (*i*) $(0, \pi)$ (*ii*) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ (*iii*) $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$ (*iv*) $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$

SCHOOL LEVEL PROBLEMS

1. (*b*) 2. (*d*) 3. (*b*) 4. (*a*) 5. (*c*) 6. (*c*) 7. (*a*) 8. (*c*) 9. (*b*) 22. (*b*)

23. (*d*) 24. (*b*) 25. (*b*)

PRARAMBH (TOPICWISE)

1. (*d*)	2. (*c*)	3. (*b*)	4. (*d*)	5. (*b*)	6. (*a*)	7. (*c*)	8. (*b*)	9. (*b*)	10. (*d*)
11. (*b*)	12. (*a*)	13. (*a*)	14. (*b*)	15. (*c*)	16. (*b*)	17. (*b*)	18. (*b*)	19. (*b*)	20. (*c*)
21. (*d*)	22. (*c*)	23. (*c*)	24. (*c*)	25. (*a*)	26. (*a*)	27. (*a*)	28. (*a*)	29. (*c*)	30. (*c*)
31. (*d*)	32. (*c*)	33. (*d*)	34. (*d*)	35. (*d*)	36. (*d*)	37. (*a*)	38. (*a*)	39. (*c*)	40. (*b*)
41. (*a*)	42. (*d*)	43. (*a*)	44. (*d*)	45. (*b*)	46. (*a*)				

PRABAL (JEE MAIN LEVEL)

1. (*c*)	2. (*a*)	3. (*a*)	4. (*a*)	5. (*d*)	6. (*d*)	7. (*b*)	8. (*d*)	9. (*a*)	10. (*a*)
11. (*c*)	12. (*c*)	13. (*d*)	14. (*b*)	15. (*c*)	16. (*a*)	17. (*b*)	18. (*b*)	19. (*a*)	20. (*c*)
21. (*d*)	22. (*b*)	23. (*b*)	24. (*c*)	25. (*c*)	26. (*b*)	27. (*d*)	28. (*b*)	29. (*d*)	30. (*d*)
31. (*b*)	32. (*a*)	33. (*b*)	34. (*a*)	35. (*d*)	36. (*c*)	37. (*c*)	38. (*a*)	39. (*b*)	40. (*b*)
41. (*b*)	42. (*a*)	43. (*a*)	44. (*d*)	45. (*b*)	46. (*a*)	47. [5]	48. [2]	49. [1]	50. [5]

PARIKSHIT (JEE ADVANCED LEVEL)

1. (*a*)	2. (*a*)	3. (*d*)	4. (*c*)	5. (*c*)	6. (*b*)	7. (*a*)	8. (*b*)	9. (*a,c*)	10. (*a,b,c*)
11. (*a,b,c,d*)	12. (*c,d*)	13. (*a,d*)	14. (*a,b,c,d*)	15. (*a,b*)	16. (*b,c*)	17. (*a,b,c,d*)	18. (*a,b,c,d*)	19. (*a,c*)	20. (*b,c*)
21. (*a,b,c,d*)	22. (*b,c,d*)	23. (*a,c,d*)	24. (*b,c*)	25. (*c,d*)	26. (*a,d*)	27. (*b,c*)	28. (*a*)	29. (*b*)	30. (*c*)
31. (*a*)	32. (*c*)	33. (*c*)	34. (*d*)	35. (*c*)	36. (*d*)	37. (*a*)	38. (*c*)	29. (*a*)	40. (*a*)
41. [5]	42. [4]	43. [8]	44. [4]	45. [15]	46. [0]	47. [6]	48. [765]	49. [6]	50. [6]]
51. [0]	52. [2]	53. [4]	54. $\frac{\pi}{2}$						

PYQ's (PAST YEAR QUESTIONS)

1. (*b*)	2. (*d*)	3. (*c*)	4. [3]	5. (*b*)	6. (*b*)	7. (*a*)	8. (*a*)	9. (*a*)	10. (*c*)
11. (*b*)	12. [48]	13. (*c*)	14. (*a*)	15. (*d*)	16. (*c*)	17. (*c*)	18. (*b*)	19. (*b*)	20. (*d*)
21. (*a*)	22. [2]	23. (*a*)	24. [4]	25. [4]	26. [5]	27. (*d*)	28. (*b*)	29. (*c*)	30. (*b*)
31. [11]	32. [1]	33. (*d*)							

PW CHALLENGERS

1. (*d*)	2. (*a,b,c*)	3. (*a*)	4. (*a*)	5. (*a,b,c*)	6. (*a*)	7. (*b,c*)	8. (*c*)	9. (*b*)	10. (*c*)

Notes

Credit - Akshat

Notes

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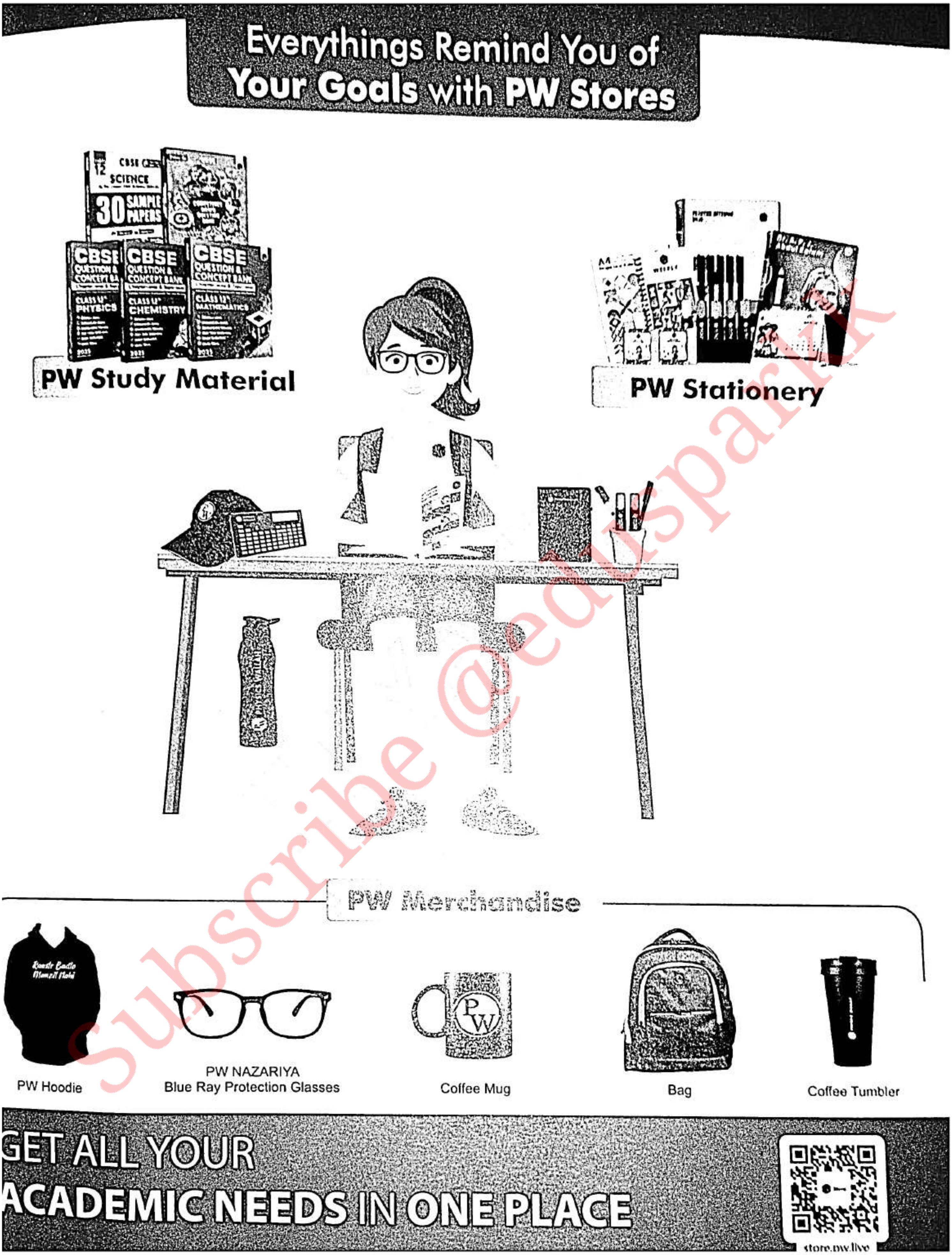

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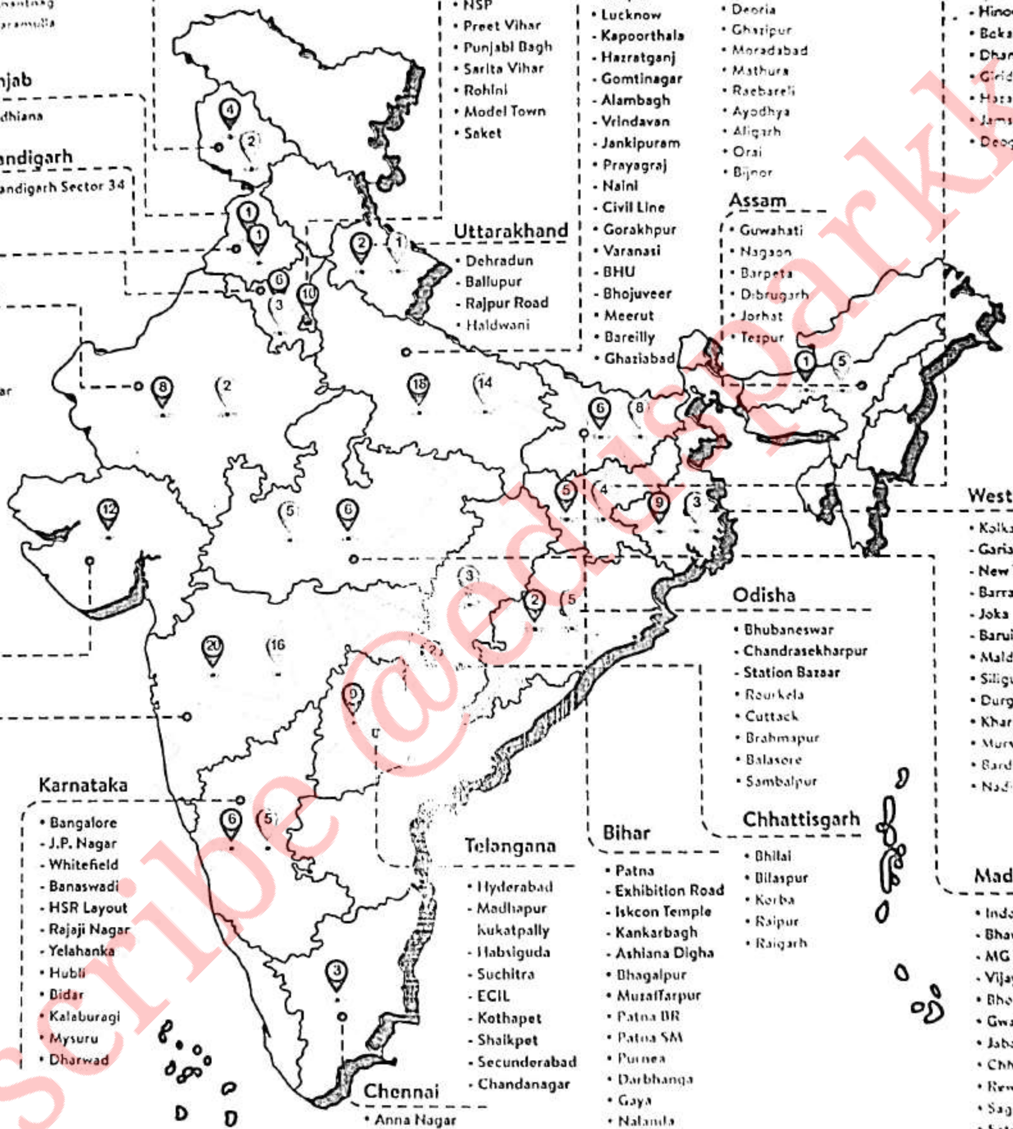

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  - Panvel
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